UNIVERSITI SAINS MALAYSIA

Second Semester Examination
2014/2015 Academic Session

June 2015

MSS 301 Complex Analysis
[Analisis Kompleks]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of FIVE pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi LIMA muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions: Answer **all seven** [7] questions.

*[Arahan: Jawab **semua tujuh** [7] soalan.]*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai.]

1. (a) Define a complex number as well as the operations addition and multiplication between two complex numbers.
   (b) Show that every complex number z can be expressed in the Cartesian form $z = x + iy$, where i is the unit imaginary number.
   (c) Show that no order exists on the field of complex numbers that will make it an ordered field.

[20 marks]

1. (a) Takrifkan nombor kompleks serta operasi hasil tambah dan hasil darab dua nombor kompleks.
   (b) Tunjukkan setiap nombor kompleks z dapat diungkapkan dalam bentuk Cartesan $z = x + iy$, dengan i sebagai nombor unit khayalan.
   (c) Tunjukkan bahawa tidak wujud tertib pada medan nombor kompleks yang dapat menjadikannya sebagai medan bertertib.

[20 markah]

2. (a) For real numbers x and y, show that $2|x||y| \leq |x|^2 + |y|^2$.
   (b) For a complex number z, show that

$$|z| \leq |\Re z| + |\Im z| \leq \sqrt{2}|z|.$$

   (c) Give an example of a nonzero z to illustrate that the equality $|\Re z| + |\Im z| = \sqrt{2}|z|$ can occur.

[15 marks]

2. *(a) Untuk nombor nyata x dan y, tunjukkan bahawa $2|x||y| \leq |x|^2 + |y|^2$.*
   (b) Untuk nombor kompleks z, tunjukkan bahawa

$$|z| \leq |\Re z| + |\Im z| \leq \sqrt{2}|z|.$$

   (c) Beri satu contoh z tak kosong yang mengilustrasikan persamaan $|\Re z| + |\Im z| = \sqrt{2}|z|$ boleh berlaku.

[15 markah]

3. Let $f(z) = u(x, y) + iv(x, y)$ be analytic in a domain D.

(a) What are the relationship among the first order partial derivatives of u and v?
(b) Show that $f''(z) = v_{xy} - iu_{xy} = -u_{yy} - iv_{yy}$.
(c) If $u(x, y)$ is constant, show that f is a constant in D.

[25 marks]

3. *Andaikan $f(z) = u(x, y) + iv(x, y)$ analisis pada suatu domain D.*

(a) Apakah hubungan di antara terbitan separa peringkat pertama u and v?
(b) Tunjukkan bahawa $f''(z) = v_{xy} - iu_{xy} = -u_{yy} - iv_{yy}$.
(c) Jika $u(x, y)$ malar, tunjukkan bahawa f ialah pemalar dalam D.

[25 markah]

4. (a) Suppose that $\lim_{z\to z_0} f(z) = 0$ and g is bounded at a neighborhood of z_0. Show that $\lim_{z\to z_0} f(z)g(z) = 0$.
(b) Show that $f(z)$ is continuous at $z = z_0$ if and only if $\overline{f(z)}$ is continuous at $z = z_0$.
(c) Consider $f : [0, 2\pi] \to \mathbb{C}$, $f(t) = e^{it}$. Prove that there exists no point c on the line segment $[0, 2\pi]$ such that

$$\frac{f(2\pi) - f(0)}{2\pi - 0} = f'(c).$$

[20 marks]

4. *(a) Andaikan $\lim_{z\to z_0} f(z) = 0$ dan g terbatas pada jiranan sekitar z_0. Tunjukkan bahawa $\lim_{z\to z_0} f(z)g(z) = 0$.*
(b) Tunjukkan bahawa $f(z)$ selanjar pada $z = z_0$ jika dan hanya jika $\overline{f(z)}$ selanjar pada $z = z_0$.
(c) Pertimbangkan $f : [0, 2\pi] \to \mathbb{C}$, $f(t) = e^{it}$. Buktikan bahawa tidak terdapat titik c pada garis tembereng $[0, 2\pi]$ sedemikian

$$\frac{f(2\pi) - f(0)}{2\pi - 0} = f'(c).$$

[20 markah]

5. (a) Evaluate the following integrals over the given positively oriented simple closed curves.

(i) $\displaystyle\oint_{|z|=2} \frac{z+5}{z^2-3z-4}\,dz$ (ii) $\displaystyle\oint_{|z|=1} \frac{\Im z}{z}\,dz$

(b) Evaluate the integral

$$\oint_{|z|=1} \frac{e^{az}}{z^{n+1}}\,dz,$$

where a is a real number and n is a natural number. Then show that

$$\int_0^{2\pi} e^{a\cos t}\cos(a\sin t - nt)\,dt = \frac{2\pi a^n}{n!}.$$

[20 marks]

5. (a) Nilaikan kamiran berikut pada lengkung tertutup ringkas berarah positif yang diberikan.

(i) $\displaystyle\oint_{|z|=2} \frac{z+5}{z^2-3z-4}\,dz$ *(ii)* $\displaystyle\oint_{|z|=1} \frac{\Im z}{z}\,dz$

(b) Nilaikan kamiran

$$\oint_{|z|=1} \frac{e^{az}}{z^{n+1}}\,dz,$$

dengan a ialah nombor nyata dan n ialah nombor tabii. Kemudian tunjukkan bahawa

$$\int_0^{2\pi} e^{a\cos t}\cos(a\sin t - nt)\,dt = \frac{2\pi a^n}{n!}.$$

[20 markah]

6. (a) Show that $w = \sin z$ is an unbounded entire function.
   (b) Evaluate the integral

$$\int_0^{\pi} \frac{d\theta}{5 + 4\cos\theta}.$$

[20 marks]

6. (a) Tunjukkan bahawa $w = \sin z$ merupakan fungsi seluruh yang tak terbatas.
   (b) Nilaikan kamiran

$$\int_0^{\pi} \frac{d\theta}{5 + 4\cos\theta}.$$

[20 markah]

7. Consider the function

$$f(z) = \frac{z^2 + z - 1}{(z-3)(z^2+2)}.$$

   (a) Find three Laurent series expansion for f in powers of z.
   (b) Classify all the singular points of f.
   (c) Find the residue of f for each of the singular points in part (b).

[30 marks]

7. Pertimbangkan fungsi

$$f(z) = \frac{z^2 + z - 1}{(z-3)(z^2+2)}.$$

   (a) Dapatkan tiga perkembangan siri Laurent bagi f dalam kuasa z.
   (b) Klasifikasikan semua titik singular untuk f.
   (c) Dapatkan reja untuk f pada setiap titik singular di bahagian (b).

[30 markah]

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