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UNIVERSITI SAINS MALAYSIA

Peperiksaan Semester Kedua  
Sidang Akademik 2005/2006

April/Mei 2006

**MSG 367 – Analisis Siri Masa**

Masa : 3 jam

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Sila pastikan bahawa kertas peperiksaan ini mengandungi LIMA BELAS muka surat yang bercetak sebelum anda memulakan peperiksaan ini.

**Arahan :** Jawab semua empat [4] soalan.

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1. (a) Walaupun ianya mudah, model Univariat Box dan Jenkins (UBJ) ARPB bagi suatu siri masa pegun adalah lebih berguna untuk ramalan jangka pendek berbanding ramalan jangka panjang. Beriuraian berkenaan kenyataan di atas menggunakan contoh seperti proses AR(1) atau PB(2).

[25 markah]

- (b) Langkah pertama dalam pemodelan UBJ ARPB melibatkan proses pengecaman berdasarkan sampel fungsi autokorelasi (fak) dan sampel fungsi autokorelasi separa (faks). Bincangkan perbezaan dalam proses pengecaman, berdasarkan fak dan faks, di antara siri masa tidak bermusim dan siri masa bermusim yang diketahui mempunyai tempoh 12.

[25 markah]

- (c) Penggunaan UBJ ARPB mempunyai kelemahan bagi kebanyakan siri masa kewangan kerana siri masa tersebut menunjukkan varians berubah mengikut masa yang mewakili risiko ataupun volatiliti sesuatu aset pengukur. Keadaan ini telah membawa kepada pembangunan model-model ARCH dan GARCH di pertengahan tahun 1980-an. Huraikan bagaimana kewujudan kesan ARCH boleh diuji.

[25 markah]

- (d) Tulis semula model-model berikut menggunakan pengoperasi anjak kebelakang  $B$  dan nyatakan bentuk ARKPB( $p, d, q$ ) atau bermusim ARKPB( $p, d, q$ )( $P, D, Q$ ). [ $p, d, q, P, D$ , dan  $Q$  adalah nombor-nombor positif terhingga]

$$\text{i) } Y_t = (1 + \phi_1)Y_{t-1} + (\phi_2 - \phi_1)Y_{t-2} + \varepsilon_t - \theta_1(\varepsilon_{t-1} - \varepsilon_{t-2}) - \theta_2\varepsilon_{t-2}$$

$$\text{ii) } Y_t = (2 + \phi_1)Y_{t-1} - (1 + 2\phi_1)Y_{t-2} + \phi_1Y_{t-3} + \varepsilon_t + \theta_1\varepsilon_{t-1}$$

$$\text{iii) } Y_t = Y_{t-12} + \varepsilon_t - \theta_1\varepsilon_{t-1} - \theta_2\varepsilon_{t-12} + \theta_1\theta_2\varepsilon_{t-13}$$

$$\text{iv) } Y_t = \varepsilon_t - 0.9\varepsilon_{t-1} + 0.81\varepsilon_{t-2} - \dots - 0.478\varepsilon_{t-7} + \dots + 0.185\varepsilon_{t-16} - \dots$$

[25 markah]

- 1.(a) *Despite its simplicity, the Univariate Box and Jenkins (UBJ) ARMA model for a stationary time series is more useful for short term than long term forecasts. Give explanation on the statement above using example such as an AR(1) or a MA(2) process.*

[25 marks]

- (b) *The first step in UBJ ARMA modeling involves identification process based on the sample autocorrelation function (acf) and sample partial autocorrelation function (pacf). Discuss the difference in the identification process, based on the acf and pacf, between a non-seasonal time series and a seasonal time series known to have a period of 12.*

[25 marks]

- (c) *The use of UBJ ARMA has its limitation for most of financial times series as they normally show time varying variance that represents risks or volatility of the underlying asset. This has led to the development of ARCH and GARCH models in the middle of 1980s. Explain how the presence of ARCH effect can be tested.*

[25 marks]

- (d) Rewrite each of the models below using the backward operator  $B$  and state the form of ARIMA( $p,d,q$ ) or SARIMA( $p,d,q$ )( $P,D,Q$ ). [ $p, d, q, P, D$ , and  $Q$  are positive finite numbers].

$$\begin{aligned}
 (i) \quad & Y_t = (1 + \phi_1)Y_{t-1} + (\phi_2 - \phi_1)Y_{t-2} + \varepsilon_t - \theta_1(\varepsilon_{t-1} - \varepsilon_{t-2}) - \theta_2\varepsilon_{t-2} \\
 (ii) \quad & Y_t = (2 + \phi_1)Y_{t-1} - (1 + 2\phi_1)Y_{t-2} + \phi_1Y_{t-3} + \varepsilon_t + \theta_1\varepsilon_{t-1} \\
 (iii) \quad & Y_t = Y_{t-12} + \varepsilon_t - \theta_1\varepsilon_{t-1} - \theta_2\varepsilon_{t-12} + \theta_1\theta_2\varepsilon_{t-13} \\
 (iv) \quad & Y_t = \varepsilon_t - 0.9\varepsilon_{t-1} + 0.81\varepsilon_{t-2} - \dots - 0.478\varepsilon_{t-7} + \dots + 0.185\varepsilon_{t-16} - \dots
 \end{aligned}$$

[25 marks]

2. (a) Tunjukkan bahawa fak bagi proses PB peringkat- $m$  yang diwakili oleh:

$$Y_t = \sum_{k=0}^m \left( \frac{\varepsilon_{t-k}}{m+1} \right)$$

boleh ditulis seperti:

$$\rho_k = \begin{cases} \frac{m+1-k}{m+1} & k = 0, 1, \dots, m \\ 0 & k > m \end{cases}$$

[20 markah]

- (b) Dapatkan suatu rumus am fungsi autokovarians, fungsi autokorelasi dan fungsi autokorelasi separa bagi proses ARPB(1,2) seperti diberi di bawah:

$$(1 - \phi_1 B)Y_t = (1 - \theta_1 B - \theta_2 B^2)\varepsilon_t$$

Seorang pelajar yang kurang berpengalaman telah mengumpul satu siri masa dengan 500 cerapan dan mempertimbangkan penyuaiian model ARPA(1,2) dengan koefisien-koefisien  $\phi_1 = 0.9$ ,  $\theta_1 = 0.10$ ,  $\theta_2 = 0.02$ . Hitung autokorelasi untuk  $k = 1, 2, 3, 4, 5$  dan autokorelasi separa untuk  $k = 1$  dan  $2$ . Komen terhadap corak yang diperoleh. Adakah fungsi autokorelasi dan fungsi autokorelasi separa mencadangkan satu model ARPB( $p,q$ )? [Diberi nilai fungsi autokorelasi susulan ke 6 hingga 8 masing-masing 0.38, 0.31 dan 0.24, dan nilai autokorelasi separa susulan 3 hingga 8 masing-masing -0.13, 0.04, 0.01, -0.02, -0.03 dan -0.05].

[30 markah]

- (c) Disebabkan kefahaman yang lebih mudah bagi proses autoregresif, pelajar tersebut sedang mempertimbangkan penyuaiian suatu model AR( $p$ ) terhadap data yang sama. Beri sebab yang mungkin telah membawa kepada pelajar tersebut untuk menyuaiikan suatu model AR(1).

[15 markah]

...4/-

- (d) Output bagi penyuaihan model AR(1) diberikan dalam Lampiran A. Lampiran A1 menunjukkan koefisien-koefisien teranggar bersama beberapa statistik berkaitan. Lampiran A2 menunjukkan fak dan faks bagi ralat dan ralat kuasa dua daripada model yang disuaikan.

Berdasarkan keputusan daripada analisis ralat pelajar tersebut sedang mempertimbangkan untuk menyuaikan suatu model yang lebih baik. Output bagi setiap langkah yang diambil dan keputusan-keputusan dari model terbaru ditunjukkan dalam Lampiran A3. Huraikan setiap output dalam Lampiran A3, berkemungkinan dengan alasan, dan tentukan sekiranya suatu model statistik yang lebih baik telah dihasilkan. Terutamanya, bincang dengan alasan bagi pengesahan dan kelebihan penyuaihan model GARCH.

[35 markah]

2. (a) Show that the acf of the  $m$ -th order MA process given by:

$$Y_t = \sum_{k=0}^m \left( \frac{\varepsilon_{t-k}}{m+1} \right)$$

can be written as:

$$\rho_k = \begin{cases} \frac{m+1-k}{m+1} & k = 0, 1, \dots, m \\ 0 & k > m \end{cases}$$

[20 marks]

- (b) Find a general formula for autocovariance, autocorrelation and partial autocorrelation functions for an ARMA(1,2) process as given below:

$$(1 - \phi_1 B) Y_t = (1 - \theta_1 B - \theta_2 B^2) \varepsilon_t$$

An inexperienced student has collected a time series of 500 observations and is considering fitting an ARMA(1,2) model with the following coefficients:  $\phi_1 = 0.9$ ,  $\theta_1 = 0.10$ ,  $\theta_2 = 0.02$ . Calculate the autocorrelation for  $k = 1, 2, 3, 4, 5$ , and partial autocorrelation for  $k = 1$  and  $2$ . Comment on the pattern observed. Does the acf and pacf suggest an ARMA( $p, q$ ) model? [Given the values of acf at lag 6 through to 8 are 0.38, 0.31 and 0.24 respectively, and pacf at lag 3 through to 8 are -0.13, 0.04, 0.01, -0.02, -0.03 and -0.05 respectively].

[30 marks]

- (c) Due to simpler understanding of an autoregressive process, the student is now considering fitting an AR( $p$ ) model to the same data set. Give reason which may have led the student to fit an AR(1) model.

[15 marks]

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- (d) The output of fitting the AR(1) model is given in Appendix A. Appendix A1 shows the estimated coefficients together with other related statistics. Appendix A2 shows the acf and pacf of the residuals and squared of the residuals from the fitted model.

Based on the results of residuals analysis the student is now attempt to fit a better model. The output of the steps taken and the results of the new model are presented in Appendix A3. Explain each of the output in Appendix A3, perhaps with reason, and determine whether a statistically better model has been obtained. In particular, discuss with reasons the validity and advantageous for fitting a GARCH model.

[35 marks]

3. (a) Pertimbangkan penyuaian suatu siri masa dengan model AR(1) yang diwakili oleh:

$$(1 - \omega_1 B)Z_t = \varepsilon_t.$$

Penyemakan diagnostik menunjukkan bahawa ralat  $\{\varepsilon_t\}$  mengikuti suatu proses AR(1):

$$(1 - \eta_1 B)\varepsilon_t = v_t, \quad v_t \sim N(0, \sigma_v^2)$$

Tunjukkan bahawa suatu model baru AR(2) berbentuk:

$$(1 - \phi_1 B - \phi_2 B^2)Z_t = v_t$$

berkemungkinan lebih baik disuaikan terhadap siri masa tersebut. Cari  $\phi_1$  dan  $\phi_2$  dalam sebutan  $\omega_1$  dan  $\eta_1$ .

[25 markah]

- (b) Suatu siri masa dengan 200 cerapan telah disuaikan dengan suatu model AR(1):

$$Z_t + 0.65Z_{t-1} = \varepsilon_t$$

Fak dan faks bagi sampel ralat ditunjukkan dalam jadual di bawah:

lag	1	2	3	4	5	6	7	8
acf	0.799	0.412	0.025	-0.228	-0.316	-0.287	-0.198	-0.111
pacf	0.799	-0.625	-0.044	0.038	-0.020	-0.077	-0.007	-0.061

Adakah nilai-nilai yang dipaparkan dalam jadual di atas setanding dengan andaian hingar putih bagi ralat? Sekiranya tidak, cadangkan suatu model yang lebih baik bagi  $\{Z_t\}$ , dan berikan anggaran bagi koefisien-koefisien.

[25 markah]

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- (c) Suatu set siri masa  $\{Z_t\}$  dipercayai dapat disuaikan yang terbaik dengan suatu model AR(1) dengan nilai lebih kurang  $\phi_1 = 0.65$ . Berapa panjangkah siri masa yang diperlukan untuk menganggar nilai sebenar  $\phi_1 = \rho_1$  dengan 95% keyakinan bahawa kemungkinan kesalahan terbesar yang dilakukan adalah 0.05?

[25 markah]

- (d) Suatu siri masa bermusim tidak pegun  $\{S_t\}$  mempunyai 500 cerapan dan mengikuti suatu model boleh songsang SARIMA $(0,0,1)(0,1,0)_{12}$  yang diberikan oleh:

$$S_t = S_{t-12} + \varepsilon_t + \lambda_1 \varepsilon_{t-1} , \quad \varepsilon_t \sim N(0, \sigma_\varepsilon^2)$$

Jadual 1 hingga Jadual 4 dalam Lampiran B menunjukkan fak dan faks bagi  $\{S_t\}$ ,  $\{\nabla S_t\}$ ,  $\{\nabla_{12} S_t\}$  dan  $\{\nabla \nabla_{12} S_t\}$ . Min dan varians bagi siri asal dan siri-siri yang telah dibezakan juga diberikan. Berdasarkan maklumat yang diberi, hitung nilai anggaran bagi  $\lambda_1$  dan varians ralat,  $\sigma_\varepsilon^2$ .

[25 markah]

3. (a) Consider fitting a time series with an AR(1) model as given by:

$$(1 - \omega_1 B) Z_t = \varepsilon_t$$

Diagnostic checking shows that the error  $\{\varepsilon_t\}$  follows an AR(1) process:

$$(1 - \eta_1 B) \varepsilon_t = v_t , \quad v_t \sim N(0, \sigma_v^2).$$

Show that a new model of AR(2) in the form:

$$(1 - \phi_1 B - \phi_2 B^2) Z_t = v_t$$

may be better fitted to the series. Find  $\phi_1$  and  $\phi_2$  in terms of  $\omega_1$  and  $\eta_1$ .

[25 marks]

- (b) A time series of 200 observations has been fitted to an AR(1) model:

$$Z_t + 0.65 Z_{t-1} = \varepsilon_t$$

The sample acf and pacf of the residuals are shown in the table below

lag	1	2	3	4	5	6	7	8
acf	0.799	0.412	0.025	-0.228	-0.316	-0.287	-0.198	-0.111
pacf	0.799	-0.625	-0.044	0.038	-0.020	-0.077	-0.007	-0.061

Are the values presented in the table above compatible with the assumption of white noise for the residuals? If not, suggest a better model for  $\{Z_t\}$ , giving estimates of the coefficients.

[25 marks]

...71-

- (c) A set of time series  $\{Z_t\}$  is believed to be best fitted with an AR(1) model with an approximately  $\phi_1 = 0.65$ . How long is the series that we need to estimate the true  $\phi_1 = \rho_1$  with 95% confidence that the possible error being made is at most 0.05.

[25 marks]

- (d) A non-stationary seasonal time series  $\{S_t\}$  has 500 observations and follows an invertible SARIMA  $(0,0,1)(0,1,0)_{12}$  model given by:

$$S_t = S_{t-12} + \varepsilon_t + \lambda_1 \varepsilon_{t-1} \quad , \quad \varepsilon_t \sim N(0, \sigma_\varepsilon^2)$$

Table 1 through to Table 4 in Appendix B show the acf and pacf of  $\{S_t\}$ ,  $\{\nabla S_t\}$ ,  $\{\nabla_{12} S_t\}$  and  $\{\nabla \nabla_{12} S_t\}$ . The mean and variance for the original and differenced series are also given. Based on the given information, calculate the estimate for  $\lambda_1$  and variance of the error,  $\sigma_\varepsilon^2$ .

[25 marks]

4. (a) (i) Suatu siri masa 300 cerapan dengan min bukan sifar telah disuaikan dengan model ARMA(1,2):

$$(1 - \phi_1 B)(Y_t - \mu) = (1 - \theta_1 B - \theta_2 B^2)\varepsilon_t$$

yang mana  $\{\varepsilon_t\}$  adalah suatu proses hingar putih dengan min 0 dan varians  $\sigma_\varepsilon^2$ .

Tunjukkan bahawa telahan 1-langkah dan 2-langkah kehadapan yang dilakukan at  $t = N$  masing-masing diberikan oleh:

$$\hat{Y}_N(1) = \mu(1 - \phi_1) + \phi_1 Y_N - \theta_1 \varepsilon_N - \theta_2 \varepsilon_{N-1}$$

$$\hat{Y}_N(2) = \mu(1 - \phi_1) + \phi_1 \hat{Y}_N(1) - \theta_2 \varepsilon_N$$

dan tunjukkan juga bahawa telahan  $m$ -langkah kehadapan diberikan oleh:

$$\hat{Y}_N(m) = \mu(1 - \phi_1) + \phi_1 \hat{Y}_N(m-1) \quad \text{untuk } m \geq 3$$

[10 markah]

- (ii) Sekiranya nilai-nilai teranggar bagi koefisien-koefisien adalah  $\hat{\phi}_1 = -0.9$ ,  $\hat{\theta}_1 = 0.5$ ,  $\hat{\theta}_2 = -0.06$ ,  $\hat{\mu} = 100$ ,  $s_\varepsilon^2 = 4$  dengan  $Y_{300} = 92$ ,  $\varepsilon_{300} = -4$ ,  $\varepsilon_{299} = 8$ , dapatkan nilai  $\hat{Y}_{300}(m)$  bagi  $m = 1, 2, \dots, 6$ . Bina selang telahan 95% bagi  $Y_{301}$ ,  $Y_{302}$  and  $Y_{303}$ . Komen terhadap 6 nilai telahan yang diperoleh. Apakah nilai berkemungkinan bagi nilai telahan pada  $t = 400$  dan nilai sepadan selang telahan 95%? Berikan penjelasan.

[30 markah]

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- (iii) Sekarang pada  $t = 301$  suatu cerapan baru dituliskan sebagai 96. Hitung nilai telahan kemaskini bagi  $Y_{302}, \dots, Y_{306}$ . Bandingkan nilai telahan terkini dengan nilai telahan yang dihitung dalam (ii) di atas dan bincangkan.

[10 markah]

- (iv) Pertimbangkan suatu kes khas apabila  $\hat{\phi}_1 = 0$ . Hitung enam nilai telahan sepadan dan tiga selang telahan sama seperti dalam (ii) di atas. Plot nilai-nilai telahan bagi kes khas ini bersama dengan nilai-nilai yang diperoleh dalam (ii) di atas. Bandingkan keputusan kamu dan bincang. Apakah yang boleh dikatakan mengenai telahan dan selang telahan bagi suatu proses purata bergerak?

[20 markah]

- (b) Pertimbangkan model bermusim SARMA(0,2)(1,0)<sub>4</sub> untuk suatu siri masa sukuan:

$$(1 - \phi_4 B^4)(Y_t - \mu) = (1 - \theta_1 B - \theta_2 B^2)\varepsilon_t.$$

Tunjukkan bahawa telahan 1-langkah dan 3-langkah kehadapan masing-masing diberikan oleh:

$$\hat{Y}_N(1) = \mu(1 - \phi_4) + \phi_4 Y_{N-3} - \theta_1 \varepsilon_{N-1} - \theta_2 \varepsilon_{N-2}$$

$$\hat{Y}_N(3) = \mu(1 - \phi_4) + \phi_4 Y_{N-1}$$

Tunjukkan juga bahawa telahan  $m$ -langkah kehadapan diberikan oleh rumus:

$$\hat{Y}_N(m) = \mu(1 - \phi_4) + \phi_4 \hat{Y}_N(m-4) \text{ untuk } m \geq 5$$

Suatu cerapan sukuan selama 25 tahun telah dikumpul dan telah disuaikan dengan model seperti di atas dan memberikan nilai-nilai koefisien teranggar:  $\hat{\phi}_4 = 0.9$ ,  $\hat{\theta}_1 = 0.5$ ,  $\hat{\theta}_2 = -0.06$ ,  $\hat{\mu} = 150$ ,  $s_\varepsilon^2 = 16$ ,  $Y_{100} = 142$ ,  $Y_{99} = 134$ ,  $Y_{98} = 154$ ,  $Y_{97} = 148$ ,  $\varepsilon_{100} = -4$ ,  $\varepsilon_{99} = 2$ . Hitung nilai-nilai telahan bagi  $m = 1, 2, \dots, 12$  dan selang telahan 95% bagi  $Y_{101}, \dots, Y_{108}$ . Apakah yang boleh diperkatakan mengenai telahan dan selang telahan bagi suatu proses bermusim?

[30 markah]

4. (a) (i) A time series of 300 observations with non-zero mean has been modeled with an ARMA(1,2) model:

$$(1 - \phi_1 B)(Y_t - \mu) = (1 - \theta_1 B - \theta_2 B^2)\varepsilon_t$$

where  $\{\varepsilon_t\}$  is a white noise process with mean 0 and variance  $\sigma_\varepsilon^2$ .

Show that the 1-step and 2-step ahead forecasts made at  $t = N$  are respectively given by:

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$$\hat{Y}_N(1) = \mu(1 - \phi_1) + \phi_1 Y_N - \theta_1 \varepsilon_N - \theta_2 \varepsilon_{N-1}$$

$$\hat{Y}_N(2) = \mu(1 - \phi_1) + \phi_1 \hat{Y}_N(1) - \theta_2 \varepsilon_N$$

and also show that the  $m$ -step-ahead forecast is given by:

$$\hat{Y}_N(m) = \mu(1 - \phi_1) + \phi_1 \hat{Y}_N(m-1) \quad \text{for } m \geq 3$$

[10 marks]

- (ii) If estimated values for the coefficients are  $\hat{\phi}_1 = -0.9$ ,  $\hat{\theta}_1 = 0.5$ ,  $\hat{\theta}_2 = -0.06$ ,  $\hat{\mu} = 100$ ,  $s_\varepsilon^2 = 4$  with  $Y_{300} = 92$ ,  $\varepsilon_{300} = -4$ ,  $\varepsilon_{299} = 8$ , obtain the value of  $\hat{Y}_{300}(m)$  for  $m = 1, 2, \dots, 6$ . Construct a 95% forecast interval for  $Y_{301}$ ,  $Y_{302}$  and  $Y_{303}$ . Comment on the 6 forecast values obtained above. What is the most likely forecast value at  $t = 400$  and its corresponding 95% forecast interval? Give explanation.

[30 marks]

- (iii) It is now observed at  $t = 301$  that a new observation is noted as 96. Calculate the updated values for  $Y_{302}, \dots, Y_{306}$ . Compare these new forecasts with those calculated in (ii) above and discuss.

[10 marks]

- (iv) Consider a special case when  $\hat{\phi}_1 = 0$ . Calculate the six corresponding forecasts and three forecast intervals similar to (ii) above. Plot the forecast values for this special case together with those obtained in (ii) above. Compare your results and discuss. What can you say about the forecasts and forecast intervals of a moving average process?

[20 marks]

- (b) Consider a seasonal model of SARMA(0,2)(1,0)<sub>4</sub> for a quarterly data set:

$$(1 - \phi_4 B^4)(Y_t - \mu) = (1 - \theta_1 B - \theta_2 B^2)\varepsilon_t$$

Show that the 1-step and 3-step ahead forecasts are respectively given by:

$$\hat{Y}_N(1) = \mu(1 - \phi_4) + \phi_4 Y_{N-3} - \theta_1 \varepsilon_{N-1} - \theta_2 \varepsilon_{N-2}$$

$$\hat{Y}_N(3) = \mu(1 - \phi_4) + \phi_4 Y_{N-1}$$

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Also show that the  $m$ -step-ahead forecast is given by:

$$\hat{Y}_N(m) = \mu(1 - \phi_4) + \phi_4 \hat{Y}_N(m-4) \quad \text{for } m \geq 5$$

A quarterly observations of 25 years have been collected and have been fitted to the model above and produces estimated coefficients:  $\hat{\phi}_4 = 0.9$ ,  $\hat{\theta}_1 = 0.5$ ,  $\hat{\theta}_2 = -0.06$ ,  $\hat{\mu} = 150$ ,  $s_\varepsilon^2 = 16$ ,  $Y_{100} = 142$ ,  $Y_{99} = 134$ ,  $Y_{98} = 154$ ,  $Y_{97} = 148$ ,  $\varepsilon_{100} = -4$ ,  $\varepsilon_{99} = 2$ . Calculate the forecast values for  $m = 1, 2, \dots, 12$  and 95% forecast interval for  $Y_{101}, \dots, Y_{108}$ . What can be said about the forecasts and forecast intervals of a seasonal process?

[30 marks]

**APPENDIX/LAMPIRAN A****A1**

Variable	Coefficient	Std. Error	t-Statistic	Prob.
AR(1)	0.869449	0.021848	39.79529	0.0000
R-squared	0.751556	Mean dependent var	2.129176	
Adjusted R-squared	0.751556	S.D. dependent var	10.86110	
S.E. of regression	5.413623	Akaike info criterion	6.217716	
Sum squared resid	14595.04	Schwarz criterion	6.226158	
Log likelihood	-1550.320	Durbin-Watson stat	2.066956	
Inverted AR Roots	.87			

**A2****A2(a): Residuals Analysis****A2(b): Squared-residuals analysis**

Lag	AC	PAC	Q-Stat	Prob	Lag	AC	PAC	Q-Stat	Prob
1	-0.037	-0.037	0.7004		1	0.167	0.167	13.965	
2	0.122	0.121	8.2143	0.004	2	0.326	0.307	67.510	0.000
3	-0.069	-0.062	10.649	0.005	3	0.253	0.186	99.789	0.000
4	-0.017	-0.037	10.800	0.013	4	0.268	0.152	136.01	0.000
5	-0.001	0.014	10.801	0.029	5	0.170	0.021	150.57	0.000
6	0.009	0.012	10.842	0.055	6	0.127	-0.045	158.82	0.000
7	0.029	0.025	11.276	0.080	7	0.253	0.143	191.30	0.000
8	-0.031	-0.032	11.753	0.109	8	0.106	-0.001	197.06	0.000
9	0.001	-0.006	11.753	0.163	9	0.227	0.108	223.25	0.000
10	-0.070	-0.059	14.231	0.114	10	0.094	-0.026	227.76	0.000
11	0.049	0.044	15.439	0.117	11	0.178	0.027	244.04	0.000
12	-0.025	-0.010	15.768	0.150	12	0.110	0.005	250.25	0.000
14	-0.024	-0.015	18.638	0.135	14	0.140	0.044	266.39	0.000
16	-0.127	-0.119	27.033	0.028	16	0.135	0.028	276.03	0.000
18	-0.097	-0.076	34.024	0.008	18	0.096	0.001	283.29	0.000
20	-0.027	-0.015	34.582	0.016	20	0.118	0.047	290.57	0.000
22	-0.048	-0.056	36.517	0.019	22	0.061	0.032	292.59	0.000
24	0.009	-0.004	39.072	0.019	24	0.013	-0.020	292.76	0.000
28	0.043	0.029	43.728	0.022	28	-0.068	-0.049	296.20	0.000
32	0.013	-0.005	44.760	0.052	32	-0.057	-0.002	299.19	0.000
36	0.025	0.017	49.466	0.053	36	-0.055	0.005	306.30	0.000
40	0.001	-0.003	51.067	0.093	40	-0.075	-0.041	313.28	0.000
44	-0.017	-0.001	51.464	0.176	44	-0.030	0.005	318.77	0.000
48	-0.010	-0.022	51.547	0.300	48	0.004	0.021	322.17	0.000

A3 (Step1)

## ARCH Test (LAG 1):

F-statistic	14.20277	Probability	0.000184
Obs*R-squared	13.86308	Probability	0.000197

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	24.40177	2.902013	8.408564	0.0000
RESID^2(-1)	0.166860	0.044276	3.768656	0.0002
R-squared	0.027838	Mean dependent var	29.29130	
Adjusted R-squared	0.025877	S.D. dependent var	58.69291	
S.E. of regression	57.92852	Akaike info criterion	10.96030	
Sum squared resid	1664434.	Schwarz criterion	10.97721	
Log likelihood	-2727.116	F-statistic	14.20277	
Durbin-Watson stat	2.102441	Prob(F-statistic)	0.000184	

## ARCH Test (LAG 12):

F-statistic	9.834046	Probability	0.000000
Obs*R-squared	97.07658	Probability	0.000000

## ARCH Test (LAG 24):

F-statistic	5.212767	Probability	0.000000
Obs*R-squared	103.3297	Probability	0.000000

A3 (Step2)

	Coefficient	Std. Error	z-Statistic	Prob.
AR(1)	0.853042	0.025887	32.95300	0.0000
Variance Equation				
C	0.845276	0.374792	2.255320	0.0241
ARCH(1)	0.217125	0.050494	4.299995	0.0000
GARCH(1)	0.767918	0.043400	17.69406	0.0000
R-squared	0.751275	Mean dependent var	2.129176	
Adjusted R-squared	0.749767	S.D. dependent var	10.86110	
S.E. of regression	5.433077	Akaike info criterion	5.974825	
Sum squared resid	14611.57	Schwarz criterion	6.008593	
Log likelihood	-1486.719	Durbin-Watson stat	2.030878	
Inverted AR Roots	.85			

Lag	AC	PAC	Q-Stat	Prob	Lag	AC	PAC	Q-Stat	Prob
12	-0.018	-0.013	7.4730	0.760	12	0.016	0.026	11.643	0.391
24	0.009	0.005	25.555	0.322	24	0.020	0.014	27.731	0.226
36	0.034	0.051	36.628	0.393	36	-0.014	-0.035	40.895	0.227
48	-0.032	-0.037	42.053	0.677	48	0.084	0.049	56.815	0.155

## ARCH Test (LAG1):

F-statistic	0.512236	Probability	0.474509
Obs*R-squared	0.513771	Probability	0.473511

## ARCH Test (LAG12):

F-statistic	1.034848	Probability	0.415120
Obs*R-squared	12.43304	Probability	0.411558

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**A3 (Step3): Alternative model**

Variable	Coefficient	Std. Error	t-Statistic	Prob.
AR(1)	0.833667	0.031589	26.39075	0.0000
MA(1)	0.017940	0.052286	0.343112	0.7317
MA(2)	0.159157	0.050069	3.178748	0.0016
R-squared	0.756365	Mean dependent var	2.129176	
Adjusted R-squared	0.755383	S.D. dependent var	10.86110	
S.E. of regression	5.371771	Akaike info criterion	6.206186	
Sum squared resid	14312.54	Schwarz criterion	6.231512	
Log likelihood	-1545.443	Durbin-Watson stat	2.010838	
Inverted AR Roots		.83		
Inverted MA Roots	-.01+.40i	-.01 -.40i		

## ARCH Test (LAG12):

F-statistic	8.461892	Probability	0.000000
Obs*R-squared	85.92116	Probability	0.000000

**A3 (Step4)**

	Coefficient	Std. Error	z-Statistic	Prob.
AR(1)	0.852525	0.032483	26.24511	0.0000
MA(1)	-0.017104	0.060461	-0.282895	0.7773
MA(2)	0.022631	0.054227	0.417345	0.6764
Variance Equation				
C	0.828468	0.377194	2.196398	0.0281
ARCH(1)	0.214460	0.050266	4.266487	0.0000
GARCH(1)	0.771015	0.043038	17.91494	0.0000
R-squared	0.752708	Mean dependent var	2.129176	
Adjusted R-squared	0.750200	S.D. dependent var	10.86110	
S.E. of regression	5.428375	Akaike info criterion	5.982185	
Sum squared resid	14527.36	Schwarz criterion	6.032838	
Log likelihood	-1486.555	Durbin-Watson stat	1.988781	
Inverted AR Roots	.85			
Inverted MA Roots	.01+.15i	.01 -.15i		

**A3 (Step5): Alternative Model**

	Coefficient	Std. Error	z-Statistic	Prob.
AR(1)	0.837952	0.051644	16.22566	0.0000
AR(2)	0.031538	0.060450	0.521725	0.6019
AR(3)	-0.017467	0.046907	-0.372378	0.7096
Variance Equation				
C	0.831508	0.377192	2.204467	0.0275
ARCH(1)	0.214943	0.050307	4.272646	0.0000
GARCH(1)	0.770274	0.043084	17.87824	0.0000
R-squared	0.749521	Mean dependent var	2.207496	
Adjusted R-squared	0.746971	S.D. dependent var	10.81097	
S.E. of regression	5.438137	Akaike info criterion	5.986315	
Sum squared resid	14520.51	Schwarz criterion	6.037123	
Log likelihood	-1481.599	Durbin-Watson stat	1.994327	
Inverted AR Roots	.85	.14	-.15	

**A3 (Step6): Alternative Model**

	Coefficient	Std. Error	z-Statistic	Prob.
AR(1)	-0.364539	0.205608	-1.772983	0.0762
AR(2)	0.694844	0.079119	8.782323	0.0000
AR(3)	0.276209	0.170973	1.615509	0.1062
MA(1)	1.219519	0.217874	5.597360	0.0000
MA(2)	0.349679	0.241993	1.444996	0.1485
MA(3)	0.042419	0.058054	0.730681	0.4650
Variance Equation				
C	0.903592	0.404345	2.234706	0.0254
ARCH(1)	0.214510	0.051307	4.180882	0.0000
GARCH(1)	0.767670	0.045874	16.73440	0.0000
R-squared	0.750577	Mean dependent var	2.207496	
Adjusted R-squared	0.746488	S.D. dependent var	10.81097	
S.E. of regression	5.443321	Akaike info criterion	5.977826	
Sum squared resid	14459.31	Schwarz criterion	6.054038	
Log likelihood	-1476.490	Durbin-Watson stat	2.023348	
Inverted AR Roots	.84	-.41	-.80	
Inverted MA Roots	-.17 -.14i	-.17+.14i	-.88	
	Coefficient	Std. Error	z-Statistic	Prob.
AR(1)	-0.434272	0.089748	-4.838778	0.0000
AR(2)	0.221274	0.084572	2.616396	0.0089
AR(3)	0.755364	0.063390	11.91606	0.0000
MA(1)	1.274567	0.098404	12.95241	0.0000
MA(2)	0.868327	0.067226	12.91660	0.0000
Variance Equation				
C	0.831455	0.373638	2.225296	0.0261
ARCH(1)	0.215859	0.050092	4.309223	0.0000
GARCH(1)	0.770144	0.043031	17.89752	0.0000
R-squared	0.748850	Mean dependent var	2.207496	
Adjusted R-squared	0.745255	S.D. dependent var	10.81097	
S.E. of regression	5.456545	Akaike info criterion	5.986268	
Sum squared resid	14559.43	Schwarz criterion	6.054012	
Log likelihood	-1479.588	Durbin-Watson stat	2.003247	
Inverted AR Roots	.86	-.65+.68i	-.65 -.68i	
Inverted MA Roots	-.64+.68i	-.64 -.68i		

Correlation matrix of the estimated parameters

	1	2	3	4
2	-0.980			
3	0.930	-0.984		
4	0.990	-0.971	0.923	
5	-0.921	0.973	-0.984	-0.925

**APPENDIX/LAMPIRAN B**Table 1: Series  $\{S_t\}$ , mean = -0.1502, variance = 7.614

Lag	1	2	3	4	5	6	7	8	9
acf	-0.468	0.203	-0.278	0.295	-0.240	0.129	-0.236	0.288	-0.281
pacf	-0.466	-0.012	-0.234	0.116	-0.059	-0.047	-0.177	0.096	-0.117
Lag	10	11	12	13	14	15	16	17	18
acf	0.207	-0.455	0.963	-0.455	0.195	-0.267	0.283	-0.231	0.124
pacf	0.005	-0.421	0.995	0.237	0.019	-0.026	-0.017	-0.049	-0.027
Lag	19	20	21	22	23	24	25	26	27
acf	-0.227	0.276	-0.272	0.201	-0.432	0.915	-0.431	0.187	-0.251
pacf	-0.187	0.060	-0.107	-0.001	-0.004	0.009	-0.024	0.019	0.024

Table 2: Series  $\{\nabla S_t\}$ , mean = -0.0054, variance = 22.344

Lag	1	2	3	4	5	6	7	8	9
acf	-0.727	0.395	-0.362	0.377	-0.306	0.250	-0.303	0.371	-0.357
pacf	-0.730	-0.299	-0.480	-0.201	-0.180	-0.049	-0.291	-0.053	-0.168
Lag	10	11	12	13	14	15	16	17	18
acf	0.388	-0.707	0.965	-0.703	0.381	-0.348	0.362	-0.290	0.240
pacf	0.223	-0.997	-0.238	-0.018	0.025	0.016	-0.015	0.018	0.049
Lag	19	20	21	22	23	24	25	26	27
acf	-0.284	0.352	-0.343	0.365	-0.657	0.916	-0.676	0.370	-0.327
pacf	-0.091	-0.003	0.017	-0.023	-0.079	-0.024	0.018	0.023	-0.014

Table 3: Series  $\{\nabla_{12} S_t\}$ , mean = 0.016, variance = 0.177

Lag	1	2	3	4	5	6	7	8	9
acf	-0.232	0.036	0.025	0.001	-0.010	-0.005	0.019	-0.063	0.005
pacf	-0.231	-0.016	0.033	0.016	-0.005	-0.009	0.018	-0.055	-0.019
Lag	10	11	12	13	14	15	16	17	18
acf	0.007	-0.031	0.050	-0.049	0.035	-0.033	-0.029	-0.015	-0.005
pacf	0.009	-0.023	-0.005	-0.051	0.018	-0.017	-0.044	-0.016	-0.005
Lag	19	20	21	22	23	24	25	26	27
acf	0.020	-0.001	0.050	-0.071	0.031	0.006	-0.005	-0.036	0.033
pacf	0.019	-0.006	0.038	-0.050	0.025	0.020	0.006	-0.043	0.055

Table 4: Series  $\{\nabla\nabla_{12} S_t\}$ , mean = -0.0011, variance = 0.435

Lag	1	2	3	4	5	6	7	8	9
acf	-0.607	0.112	0.005	-0.004	-0.007	-0.009	0.043	-0.058	0.025
pacf	-0.610	-0.409	-0.280	-0.202	-0.165	-0.165	-0.076	-0.103	-0.117
Lag	10	11	12	13	14	15	16	17	18
acf	0.016	-0.031	0.038	-0.058	0.063	-0.030	-0.004	0.058	0.052
pacf	-0.078	-0.089	-0.040	-0.103	-0.065	-0.036	-0.043	-0.050	-0.061
Lag	19	20	21	22	23	24	25	26	27
acf	-0.016	0.013	-0.038	-0.029	0.036	-0.030	0.008	-0.085	-0.067
pacf	-0.059	-0.124	-0.001	-0.087	-0.078	-0.056	-0.002	-0.092	-0.067

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