UNIVERSITI SAINS MALAYSIA

Peperiksaan Semester Kedua
Sidang Akademik 2006/2007

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MSG 367 – ANALISIS SIRI MASA

Masa : 3 jam

Sila pastikan bahawa kertas peperiksaan ini mengandungi **DUA PULUH TIGA** muka surat yang bercetak sebelum anda memulakan peperiksaan ini.

Jawab **semua empat** soalan.

1. (a) *Dua langkah pertama dalam pembinaan model UBJ melibatkan pengecaman model dan penganggaran koefisien-koefisien bagi model. Selepas itu model tertakluk kepada ujian kecukupan melalui penyemakan diagnostik terhadap ralat. Terangkan dengan menggunakan contoh, bagaimana keputusan dan hasil daripada penyemakan diagnostik tersebut boleh digunakan untuk mendapatkan suatu model yang baru dan yang lebih baik.*

[20 markah]

(b) *Apabila model tentatif siri masa menjalani penyemakan diagnostik, dua daripada elemen yang penting adalah memeriksa kecukupan peringkat model ARMA dan juga kemungkinan wujud kesan ARCH. Terangkan ujian-ujian bagi dua elemen tersebut.*

[20 markah]

(c) *Adalah diketahui bahawa fungsi autokorelasi (fak) bagi suatu proses yang tak pegun menyusut dengan begitu perlahan. Pertimbangkan suatu proses tak pegun $IMA(1,1)$ yang boleh ditulis sebagai:*

$$Y_t = Y_{t-1} + \varepsilon_t - \theta_1 \varepsilon_{t-1}$$

Tunjukkan bahawa proses di atas boleh ditulis semula sebagai:

$$Y_t = \varepsilon_t + (1-\theta_1)\varepsilon_{t-1} + (1-\theta_1)\varepsilon_{t-2} + \ldots + (1-\theta_1)\varepsilon_0$$

dan fungsi autokovarians bersandar terhadap masa, t diberi oleh:

$$Cov(Y_t Y_{t-k}) = \left[1 + \theta_1^2 + (t-k)(1-\theta_1)^2\right]\sigma_\varepsilon^2$$

Akhir sekali, tunjukkan bahawa fak bagi proses $IMA(1,1)$ diberi oleh:

$$Corr(Y_t Y_{t-k}) \approx \sqrt{\frac{t-k}{t}}$$

Secara ringkas, terangkan, sekiranya fak yang ditunjukkan di atas menyokong fakta bahawa fak bagi suatu proses tak pegun menyusut dengan begitu perlahan.

[40 markah]

(d) *Tulis semula model-model berikut menggunakan pengoperasi anjak kebelakang B dan nyatakan bentuk ARKPB(p,d,q) atau bermusim ARKPB(p,d,q)(P,D,Q). [p, d, q, P, D, dan Q adalah nombor-nombor positif terhingga]*

(i) $Y_t = (1+\phi_1)Y_{t-1} + (\phi_2 - \phi_1)Y_{t-2} - \phi_2 Y_{t-3} + \varepsilon_t - \theta_1(\varepsilon_{t-1} - \varepsilon_{t-2})$

(ii) $Y_t = \varepsilon_t - (\theta_1 - \phi_1)\varepsilon_{t-1} - (\theta_1\phi_1 - \phi_1^2)\varepsilon_{t-2} - \ldots - (\theta_1\phi_1^{j-1} - \phi_1^j)\varepsilon_{t-j} - \ldots$

(iii) $Y_t = (1+\phi_1)Y_{t-1} - \phi_1 Y_{t-2} + Y_{t-12} - (1+\phi_1)Y_{t-13} + \phi_1 Y_{t-14} + \varepsilon_t + \theta_2\varepsilon_{t-12}$

(iv) $Y_t = \varepsilon_t - 0.8Y_{t-1} - 0.64Y_{t-2} - \ldots - 0.328Y_{t-5} - \ldots - 0.168Y_{t-8} - \ldots$

[20 markah]

1. (a) The first two steps of the UBJ modeling procedure involve identification of model and estimation of model coefficients. The model is then subjected to adequacy test through diagnostic checking of the residuals. Explain using example, how the results or the outcome of the diagnostic checking can be used to derive a new and better model.

[20 marks]

(b) When a tentative time series model is going through a diagnostic checking, two of the important elements are to check for adequacy of the order of the ARMA model and also the possible existence of ARCH effect. Explain the tests for the two elements.

[20 marks]

(c) It is known that the autocorrelation function (acf) of a non-stationary process tails off extremely slowly. Consider a non-stationary IMA(1,1) process that can be written as:

$$Y_t = Y_{t-1} + \varepsilon_t - \theta_1 \varepsilon_{t-1}$$

Show that the process can be rewritten as:

$$Y_t = \varepsilon_t + (1-\theta_1)\varepsilon_{t-1} + (1-\theta_1)\varepsilon_{t-2} + \ldots + (1-\theta_1)\varepsilon_0$$

and the autocovariance function depends on time, t is given by:

$$Cov(Y_t Y_{t-k}) = \left[1 + \theta_1^2 + (t-k)(1-\theta_1)^2\right]\sigma_\varepsilon^2$$

Finally, show that the acf of the IMA(1,1) process is given by:

$$Corr(Y_t Y_{t-k}) \approx \sqrt{\frac{t-k}{t}}$$

Very briefly, explain, if the acf as shown above supports the fact that acf of a non-stationary process tails off extremely slowly.

[40 marks]

(d) Rewrite each of the models below using the backward operator B and state the form of ARIMA(p,d,q) or SARIMA(p,d,q)(P,D,Q). [p, d, q, P, D, and Q are positive finite numbers].

(i) $Y_t = (1+\phi_1)Y_{t-1} + (\phi_2 - \phi_1)Y_{t-2} - \phi_2 Y_{t-3} + \varepsilon_t - \theta_1(\varepsilon_{t-1} - \varepsilon_{t-2})$

(ii) $Y_t = \varepsilon_t - (\theta_1 - \phi_1)\varepsilon_{t-1} - \left(\theta_1\phi_1 - \phi_1^2\right)\varepsilon_{t-2} - \ldots - \left(\theta_1\phi_1^{j-1} - \phi_1^j\right)\varepsilon_{t-j} - \ldots$

(iii) $Y_t = (1+\phi_1)Y_{t-1} - \phi_1 Y_{t-2} + Y_{t-12} - (1+\phi_1)Y_{t-13} + \phi_1 Y_{t-14} + \varepsilon_t + \theta_2\varepsilon_{t-12}$

(iv) $Y_t = \varepsilon_t - 0.8Y_{t-1} - 0.64Y_{t-2} - \ldots - 0.328Y_{t-5} - \ldots - 0.168Y_{t-8} - \ldots$

[20 marks]

2. (a) *Diberi suatu proses* $ARPB(1,1)$ *dan* $PB(\infty)$, *masing-masing diwakili oleh:*

$$Y_t = \phi_1 Y_{t-1} + \varepsilon_t - \theta_1 \varepsilon_{t-1} \qquad dan \qquad Y_t = \varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} + \ldots$$

Tulis syarat kepegunan yang perlu bagi dua proses di atas.

Tunjukkan bahawa proses $ARPB(1,1)$ *di atas boleh di tulis sebagai proses* $PB(\infty)$ *yang diwakili oleh:*

$$Y_t = \varepsilon_t + (\phi_1 - \theta_1) \sum_{j=1}^{\infty} \phi_1^{j-1} \varepsilon_{t-j}$$

Terangkan secara ringkas keperluan syarat kepegunan proses $ARPB(1,1)$ *bagi proses* $PB(\infty)$ *yang ditulis di atas.*

[25 markah]

(b) *Diberi suatu proses* $ARPB(1,2)$ *seperti di bawah:*

$$(1 - \phi_1 B)Y_t = (1 - \theta_1 B - \theta_2 B^2)\varepsilon_t$$

Dapatkan rumus umum bagi autokovarians, autokorelasi dan autokorelasi separa bagi proses tersebut.

Seorang graduan wanita daripada jurusan siri masa baru diambil bekerja sebagai pelatih dan sedang mempertimbangkan penyuaian model ARPB(p,q) bagi siri pulangan Indeks Komposite Kuala Lumpur dengan 1600 *cerapan. Beliau telah dimaklumkan bahawa siri pulangan yang terdahulu baik disuaikan dengan model* $ARPB(1,2)$ *dengan koefisien-koefisien* $\phi_1 = 0.25$, $\theta_1 = -0.60$ *dan* $\theta_2 = 0.27$.

Sebagai rakan kerja, beri bantuan kepada beliau untuk menhitung nilai-nilai autokorelasi (fak) bagi susulan $k = 1, 2, 3, 4$ *dan* 5, *dan autokorelasi separa (faks) bagi susulan* $k = 1$ *dan* 2.

Daripada nilai-nilai fak dan faks yang dihitung, beliau berpendapat bahawa model ini mempunyai lebihan parameter dan siri tersebut boleh boleh disuaikan dengan model $PB(1)$ *yang lebih parsimoni. Bincang secara ringkas pendapat beliau dan namakan model tersebut.*

[Diberi nilai fak bagi susulan 6 *hingga* 8 *masing-masing adalah* 0.009, -0.044 *dan* 0.039, *dan nilai faks bagi susulan* 3 *hingga* 8 *masing-masing adalah* -0.249, -0.163, -0.139, -0.105 *dan* -0.116*]*

[35 markah]

2. (a) Given an ARMA(1,1) and $MA(\infty)$ processes, respectively given by:

$$Y_t = \phi_1 Y_{t-1} + \varepsilon_t - \theta_1 \varepsilon_{t-1} \qquad \text{and} \qquad Y_t = \varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} + \ldots$$

Write down the necessary stationarity condition for the two processes.

Show that the ARMA(1,1) process above can be written as a $MA(\infty)$ process represented by:

$$Y_t = \varepsilon_t + (\phi_1 - \theta_1) \sum_{j=1}^{\infty} \phi_1^{j-1} \varepsilon_{t-j}$$

Briefly explain the necessity of the stationarity condition of the ARMA(1,1) process for the $MA(\infty)$ process written above.

[25 marks]

(b) Given an ARMA(1,2) process:

$$(1 - \phi_1 B) Y_t = \left(1 - \theta_1 B - \theta_2 B^2\right) \varepsilon_t$$

Find a general formula for autocovariance, autocorrelation and partial autocorrelation functions of the process.

A recently graduated female student of time series course has been employed as a trainee and is considering fitting an ARMA(p,q) model to a series of Kuala Lumpur Composite Index returns with 1600 observations. She was informed that historical series of the return is best fitted with an ARMA(1,2) model with the following coefficients: $\phi_1 = 0.25$, $\theta_1 = -0.60$ and $\theta_2 = 0.27$.

As a teammate, help her in calculating the values of autocorrelation for lag k = 1, 2, 3, 4, 5, and partial autocorrelation for lag k = 1 and 2.

From the calculated acf and pacf she believes that the model has been overparameterized and the series may be fitted by a more parsimonious MA(1) model. Briefly explain on her belief.

[Given the values of acf at lag 6 through to 8 are 0.009, -0.044 and 0.039 respectively, and pacf at lag 3 through to 8 are –0.249, –0.163, -0.139, - 0.105, -0.152 and -0.116 respectively].

[35 marks]

(c) *[Bersambung daripada scenario di bahagian (b) di atas]. Dengan adanya komputer dan pakej statistik, sekarang beliau mengkaji jika model* $PB(1)$ *dapat menyuai siri tersebut dengan cukup.*

Output daripada penyuaian model $PB(1)$ *diberikan dalam Lampiran A. Lampiran A1 menunjukkan nilai-nilai koefisien teranggar bersama statistik-statistik lain yang berkenaan.*

Berdasarkan daripada keputusan analisis ke atas ralat, pelatih tersebut sekarang mencuba untuk menyuaikan suatu model yang lebih baik. Output bagi setiap langkah dan keputusan-keputusan bagi model yang baru diberikan dalam Lampiran A2. Terangkan dengan alasan setiap output dalam Lampiran A, dan tentukan sekiranya suatu model statistik yang lebih baik telah dapat dihasilkan. Terutamanya, bincang dengan alasan bagi pengesahan dan kelebihan penyuaian model GARCH.

Perlu diketahui bahawa, sebaik sahaja Langkah 1 *di Lampiran A2 disiapkan, pelatih tersebut telah dimaklumkan bahawa model* $ARPB(2,2)$ *seperti di bawah juga mungkin boleh menyuaikan siri tersebut dengan baik:*

$$\left(1-0.3B-0.28B^2\right)Y_t=\left(1+0.7B-0.18B^2\right)\varepsilon_t$$

Terangkan dengan alasan mengapa pelatih tersebut tidak terperanjat apabila mendapati model yang diperoleh di Langkah 2 di Lampiran A2 *adalah sangat hampir dan mencukupi secara statistik dengan model yang diperoleh di Langkah 1.*

[40 markah]

(c) [Continuing the scenario from part (b) above]. With the computer and statistical package is now available, she is now investigating if a MA(1) model is adequately fit the series.

The output of fitting the MA(1) model is given in Appendix A. Appendix A1 shows the estimated coefficients together with other related statistics.

Based on the results of residuals analysis the trainee is now attempt to fit a better model. The output of the steps taken and the results of the new model are presented in Appendix A2. Explain with reason each of the output in Appendix A, and determine whether a statistically better model has been obtained. In particular, discuss with reasons the validity and advantage for fitting a GARCH model.

Note that, just after completing Step 1 in Appendix A2, the trainee has been informed that the following ARMA(2,2) model may also fit the series well:

$$\left(1-0.3B-0.28B^2\right)Y_t=\left(1+0.7B-0.18B^2\right)\varepsilon_t$$

Explain with reason why she is not surprised to find out that the model obtained in Step 2 in Appendix A2 is very closely and statistically adequate to the model obtained in Step 1.

[40 marks]

3. (a) *Pertimbangkan model* $ARPB(1,1)$ *yang diwakili oleh:*

$$(1-\phi_1 B)Y_t = (1-\theta_1 B)\varepsilon_t.$$

(i) *Dengan menggunakan kaedah momen, tunjukkan bahawa anggaran bagi* ϕ_1 *diberikan oleh* $\hat{\phi}_1 = \frac{\hat{\rho}_2}{\hat{\rho}_1}$ *dan anggaran bagi* θ_1 *boleh diperoleh dengan menyelesaikan persamaan kuadratik:*

$$\hat{\rho}_1 = \frac{(1-\theta_1\hat{\phi}_1)(\hat{\phi}_1-\theta_1)}{1-2\hat{\phi}_1\theta_1+\theta_1^2}$$

Terangkan secara ringkas mengapa seseorang perlu berhati-hati dengan penyelesaian daripada persamaan kuadratik tersebut.

[25 markah]

(ii) *Tunjukkan bahawa varians-varians dan korelasi bagi nilai-nilai anggaran diberikan oleh:*

$$Var(\hat{\phi}_1) = \frac{1-\phi_1^2}{N}\left(\frac{1-\phi_1\theta_1}{\phi_1-\theta_1}\right)^2 \qquad Var(\hat{\theta}_1) = \frac{1-\theta_1^2}{N}\left(\frac{1-\phi_1\theta_1}{\phi_1-\theta_1}\right)^2$$

$$Kor(\hat{\phi}_1,\hat{\theta}_1) = \frac{\sqrt{\left[(1-\phi_1^2)(1-\theta_1^2)\right]}}{1-\phi_1\theta_1}$$

[25 markah]

(iii) *Suatu siri masa dengan* 400 *cerapan telah disuaikan dengan suatu model* $AR(1)$*:*

$$Y_t + 0.65Y_{t-1} = \varepsilon_t$$

Sampel fak dan faks bagi ralat ditunjukkan dalam jadual di bawah:

Susulan	1	2	3	4	5	6	7	8
fak	0.800	-0.063	-0.044	0.038	-0.020	-0.077	-0.007	-0.061
faks	0.800	0.412	0.025	-0.228	-0.316	-0.287	-0.198	-0.111

Adakah nilai-nilai yang dipaparkan dalam jadual di atas setanding dengan andaian hingar putih bagi ralat?

Tunjukkan bahawa $ARPB(1,1)$ *adalah suatu model yang lebih baik bagi* $\{Y_t\}$ *dan cari anggaran bagi koefisien-koefisien serta nilai sisihan piawai yang sepadan.*

Adakah model $ARPB(1,1)$ *yang dicadangkan signifikan dan stabil? Beri komen.*

[25 markah]

3. (a) Consider an ARMA(1,1) process as given by:

$$(1-\phi_1 B)Y_t = (1-\theta_1 B)\varepsilon_t$$

(i) Using method of moments, show that the estimate of ϕ_1 is given by: $\hat{\phi}_1 = \frac{\hat{\rho}_2}{\hat{\rho}_1}$ and the estimate of θ_1 can be obtained by solving the quadratic equation:

$$\hat{\rho}_1 = \frac{(1-\theta_1\hat{\phi}_1)(\hat{\phi}_1-\theta_1)}{1-2\hat{\phi}_1\theta_1+\theta_1^2}$$

Briefly explain why one needs to be cautious with the solutions of the quadratic equation?

[25 marks]

(ii) Show that the variances and correlation of the estimates are given by:

$$Var(\hat{\phi}_1) = \frac{1-\phi_1^2}{N}\left(\frac{1-\phi_1\theta_1}{\phi_1-\theta_1}\right)^2 \qquad Var(\hat{\theta}_1) = \frac{1-\theta_1^2}{N}\left(\frac{1-\phi_1\theta_1}{\phi_1-\theta_1}\right)^2$$

$$Corr(\hat{\phi}_1, \hat{\theta}_1) = \frac{\sqrt{\left[(1-\phi_1^2)(1-\theta_1^2)\right]}}{1-\phi_1\theta_1}$$

[25 marks]

(iii) A time series of 400 observations has been fitted to an AR(1) model:

$$Y_t + 0.65Y_{t-1} = \varepsilon_t$$

The sample acf and pacf of the residuals are shown in the table below

lag	1	2	3	4	5	6	7	8
acf	0.800	-0.063	-0.044	0.038	-0.020	-0.077	-0.007	-0.061
pacf	0.800	0.412	0.025	-0.228	-0.316	-0.287	-0.198	-0.111

Are the values presented in the table above compatible with the assumption of white noise for the residuals?

Show that an ARMA(1,1) is a better model for $\{Y_t\}$ and find the estimates of the coefficients together with its corresponding standard errors.

Is the suggested ARMA(1,1) model significant and stable? Give comment.

[25 marks]

(b) *Suatu siri masa suhu bulanan, dalam sukatan Celcius di lapangan terbang Bayan Lepas $\{S_t\}$ dengan cerapan 696 telah dikumpul dan diberi kepada Mr. Y. Beliau telah disuruh untuk menyuaikan model siri masa yang sesuai ke atas siri tersebut dan mengkaji kehadiran pemanasan global.*

Berdasarkan daripada maklumat yang diberikan dalam Appendix B, bincang setiap langkah-langkah yang diambil oleh Mr. Y dalam analisis beliau. Adakah model terakhir menyuaikan dengan cukup siri tersebut? Adakah terdapat bukti pemanasan global? Sekiranya ada, apakah nilai purata kenaikan suhu jangka panjang setiap bulan?

[25 markah]

4. *Pertimbangkan model ARPB(2,1) bagi siri pegun dengan min bukan sifar:*

$$(1-\phi_1 B)(1-\phi_2 B)(Y_t-\mu)=(1-\theta_1 B)\varepsilon_t$$

Tulis semula model tersebut dalam perwakilan PB(∞): $(Y_t-\mu)=\sum_{j=0}^{\infty}\varphi_j\varepsilon_{t-j}$ *dan gunakannya untuk menjawab (a) dan (c).*

(a) *Pertimbangkan suatu kes khas dengan* $\phi_2=\theta_1=0$.

(i) *Tunjukkan bahawa koefisien PB diberikan oleh:* $\varphi_k=\phi_1^k$

Tunjukkan bahawa telahan m-langkah kehadapan yang dibuat pada waktu t = N diberikan oleh:

$$\hat{Y}_N(m)=\mu(1-\phi_1)+\phi_1\hat{Y}_N(m-1)$$

dan boleh ditulis semula sebagai:

$$\hat{Y}_N(m)=\mu\left(1-\phi_1^m\right)+\phi_1^m Y_N$$

(ii) *Tunjukkan bahawa varians bagi ralat telahan yang sepadan diberikan oleh:*

$$Var[\upsilon_N(m)]=\sigma_\varepsilon^2\left(\frac{1-\phi_1^{2m}}{1-\phi_1^2}\right)$$

(iii) *Akhir sekali, tunjukkan apabila* $m\to\infty$

$\hat{Y}_N(m)\to\mu$ *dan* $Var[\upsilon_N(m)]\to\gamma_0$ [i.e $Var(Y_t-\mu)$]

[30 markah]

(b) A time series of monthly temperature, in Celcius at Bayan Lepas airport $\{S_t\}$ of 696 observationse are collected and given to Mr. Y. He has been asked to fit a suitable time series model to the series and to investigate the presence of global warming.

Based on the information provided in Appendix B, discuss each of the steps taken by Mr. Y in his analysis. Is the final model adequately fit the series? Is there evidence of global warming? If so, what is the long term average increase in temperature per month?

[25 marks]

4. Consider an ARMA(2,1) model for a stationary series with non-zero mean:

$$(1-\phi_1 B)(1-\phi_2 B)(Y_t-\mu)=(1-\theta_1 B)\varepsilon_t$$

Rewrite the model in $MA(\infty)$ representation: $(Y_t-\mu)=\sum_{j=0}^{\infty}\varphi_j\varepsilon_{t-j}$ and use it to answer (a) and (c).

(a) Consider a special case with $\phi_2=\theta_1=0$.

(i) Show that the MA coefficient is given by: $\varphi_k=\phi_1^k$.

Show that the m-step ahead forecasts made at time $t=N$ is given by:

$$\hat{Y}_N(m)=\mu(1-\phi_1)+\phi_1\hat{Y}_N(m-1)$$

and that it can be rewritten as:

$$\hat{Y}_N(m)=\mu\left(1-\phi_1^m\right)+\phi_1^m Y_N$$

(ii) Show that the corresponding variance of forecast error is given by:

$$Var[\upsilon_N(m)]=\sigma_\varepsilon^2\left(\frac{1-\phi_1^{2m}}{1-\phi_1^2}\right)$$

(iii) Finally, show that as $m\to\infty$:

$\hat{Y}_N(m)\to\mu$ and $Var[\upsilon_N(m)]\to\gamma_0$ [i.e $Var(Y_t-\mu)$]

[30 marks]

(b) (i) *Tunjukkan bahawa telahan 1-langkah dan dua-langkah kehadapan yang dilakukan at $t = N$ masing-masing diberikan oleh:*

$$\hat{Y}_N(1) = \mu\left[1-(\phi_1+\phi_2)+\phi_1\phi_2\right]+(\phi_1+\phi_2)Y_N - \phi_1\phi_2 Y_{N-1} - \theta_1\varepsilon_N$$

$$\hat{Y}_N(2) = \mu\left[1-(\phi_1+\phi_2)+\phi_1\phi_2\right]+(\phi_1+\phi_2)\hat{Y}_N(1) - \phi_1\phi_2 Y_N$$

dan tunjukkan juga bahawa telahan m-langkah kehadapan diberikan oleh:

$$\hat{Y}_t(m) = \mu\left[1-(\phi_1+\phi_2)+\phi_1\phi_2\right]+(\phi_1+\phi_2)\hat{Y}_t(m-1)\text{-}\phi_1\phi_2\hat{Y}_N(m-2)$$

bagi $m \geq 3$

(ii) *Pertimbangkan N = 300. Sekiranya nilai-nilai teranggar bagi koefisien-koefisien adalah $\hat{\phi}_1 = -0.9$, $\hat{\phi}_2 = 0.6$, $\hat{\theta}_1 = 0.5$, $\hat{\mu} = 200$, $s_\varepsilon^2 = 8$ dengan $Y_{300} = 192$, $Y_{299} = 224$ dan $\varepsilon_{300} = -4$, dapatkan nilai $\hat{Y}_{300}(m)$ bagi m = 1, 2, ..., 6. Bina selang telahan 95% bagi Y_{301}, Y_{302}, Y_{303} dan Y_{304}. Beri komen bagi enam nilai telahan yang diperoleh.*

Apakah nilai berkemungkinan bagi nilai telahan pada waktu t = 400 dan nilai sepadan selang telahan 95%? Berikan penjelasan.

(iii) *Sekarang pada waktu t = 301 satu cerapan baru 196 diperoleh. Hitung nilai telahan kemaskini bagi $Y_{302},\ldots Y_{306}$. Bandingkan nilai telahan terkini dengan nilai telahan yang dihitung dalam (ii) di atas dan bincangkan.*

[45 markah]

(c) *Pertimbangkan suatu proses tak pegun dengan $\phi_2 = 1$ dan $\theta_1 = 0$.*

(i) *Tunjukkan bahawa ungkapan tak tersirat bagi koefisien PB diberikan oleh:*

$$\varphi_k = \frac{1-\phi_1^{k+1}}{1-\phi_1}$$

(ii) *Dengan menggunakan maklumat yang sama seperti di bahagian b(ii), hitung nilai $\hat{Y}_{300}(m)$ bagi m = 1, 2, ..., 6. Bina selang telahan 95% bagi Y_{301}, Y_{302}, Y_{303} dan Y_{304}. Beri komen bagi empat nilai telahan serta selang telahannya.*

[25 markah]

(b) (i) Show that the one-step and two-step ahead forecasts made at $t = N$ are respectively given by:

$$\hat{Y}_N(1) = \mu\left[1-(\phi_1+\phi_2)+\phi_1\phi_2\right]+(\phi_1+\phi_2)Y_N - \phi_1\phi_2 Y_{N-1} - \theta_1\varepsilon_N$$

$$\hat{Y}_N(2) = \mu\left[1-(\phi_1+\phi_2)+\phi_1\phi_2\right]+(\phi_1+\phi_2)\hat{Y}_N(1) - \phi_1\phi_2 Y_N$$

and also show that the m-step-ahead forecast is given by:

$$\hat{Y}_t(m) = \mu\left[1-(\phi_1+\phi_2)+\phi_1\phi_2\right]+(\phi_1+\phi_2)\hat{Y}_t(m-1)\text{-}\phi_1\phi_2\hat{Y}_N(m-2)$$

for $m \geq 3$

(ii) Consider $N = 300$. If estimated values for the coefficients are $\hat{\phi}_1 = -0.9$, $\hat{\phi}_2 = 0.6$, $\hat{\theta}_1 = 0.5$, $\hat{\mu} = 200$, $s_{\varepsilon}^2 = 8$ with $Y_{300} = 192, Y_{299} = 224$ $\varepsilon_{300} = -4$, obtain value of $\hat{Y}_{300}(m)$ for m = 1, 2, ..., 6. Construct a 95% forecast interval for Y_{301}, Y_{302}, Y_{303} and Y_{304}. Comment on the six forecast values obtained above.

What is the most likely forecast value at time t = 400 and its corresponding 95% forecast interval? Give explanation.

(iii) It is now observed at time $t = 301$ that a new observation is noted as 196. Calculate the updated values for $Y_{302}, \ldots Y_{306}$. Compare these new forecasts with those calculated in (ii) above and discuss.

[45 marks]

(c) Consider a non-stationary process such that $\phi_2 = 1$ and $\theta_1 = 0$.

(i) Show that the explicit expression of the MA coefficient is given by:

$$\varphi_k = \frac{1-\phi_1^{k+1}}{1-\phi_1}.$$

(ii) Using similar information as in part b(ii), calculate value of $\hat{Y}_{300}(m)$ for m = 1, 2, ..., 6. Construct a 95% forecast interval for Y_{301}, Y_{302}, Y_{303} and Y_{304}. Comment on the four forecast values and its corresponding forecast intervals.

[25 marks]

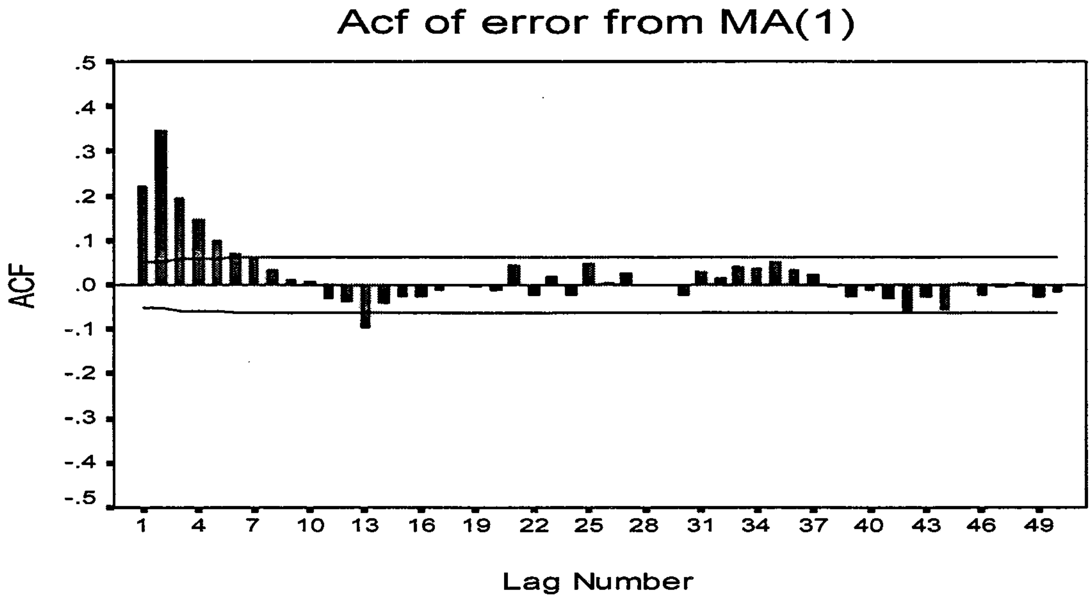
LAMPIRAN/APPENDIX A

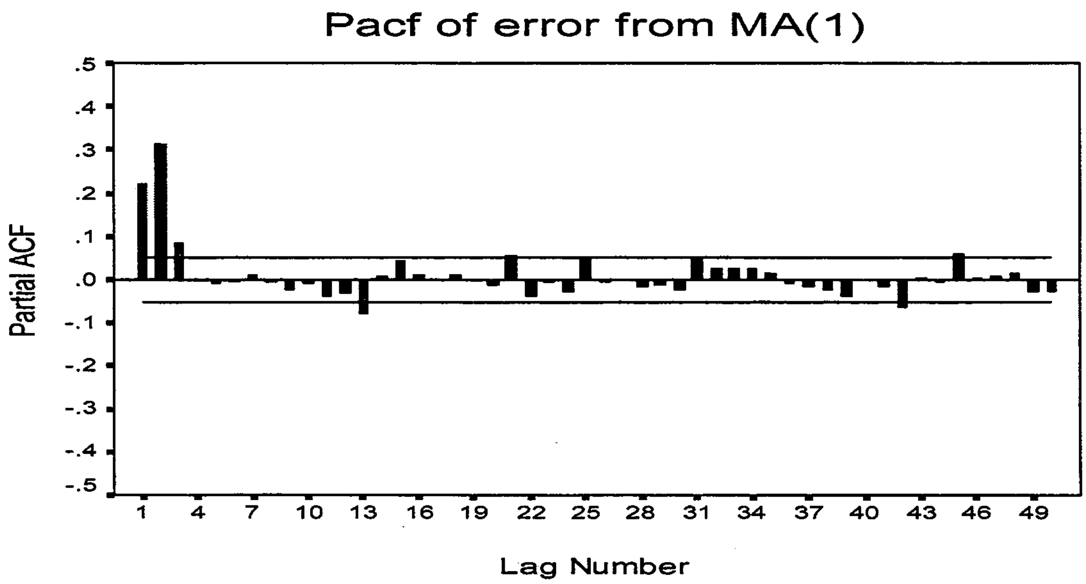
Lampiran/Appendix A1

Results of estimation: MA(1)

Dependent Variable: RT2
Method: Least Squares

Variable	Coefficient	Std. Error	t-Statistic	Prob.
MA(1)	0.884418	0.011640	75.98329	0.0000
Adjusted R-squared	0.539061	S.D. dependent var		1.744527
S.E. of regression	1.184403	Akaike info criterion		3.176982
Sum squared resid	2236.080	Schwarz criterion		3.180351
Log likelihood	-2532.643	Durbin-Watson stat		1.561755
Inverted MA Roots	-.88			





Q-statistics of error: MA(1)

Lag	AC	PAC	Q-Stat	Prob	Lag	AC	PAC	Q-Stat	Prob
1	0.219	0.219	76.674		20	-0.011	-0.013	417.71	0.000
2	0.346	0.313	268.25	0.000	22	-0.023	-0.036	421.67	0.000
3	0.193	0.084	327.88	0.000	24	-0.022	-0.027	422.95	0.000
4	0.148	0.001	362.79	0.000	27	0.026	0.001	427.97	0.000
5	0.100	-0.006	378.88	0.000	30	-0.021	-0.022	428.70	0.000
6	0.070	-0.003	386.81	0.000	33	0.041	0.024	433.35	0.000
7	0.060	0.013	392.64	0.000	36	0.031	-0.008	441.44	0.000
8	0.034	-0.003	394.52	0.000	39	-0.024	-0.039	443.25	0.000
9	0.011	-0.023	394.70	0.000	42	-0.062	-0.062	451.44	0.000
10	0.007	-0.009	394.77	0.000	45	0.005	0.057	457.50	0.000
11	-0.029	-0.035	396.16	0.000	48	0.005	0.015	458.37	0.000
12	-0.036	-0.030	398.22	0.000					
14	-0.041	0.008	415.12	0.000					
16	-0.025	0.012	417.26	0.000					
18	0.000	0.012	417.48	0.000					

Lampiran/Appendix A2

Langkah/Step 1:

Results of estimation: ARMA(3,1)

Dependent Variable: RT2
Method: Least Squares

Variable	Coefficient	Std. Error	t-Statistic	Prob.
AR(1)	0.124579	0.030057	4.144811	0.0000
AR(2)	0.299968	0.031544	9.509610	0.0000
AR(3)	0.084618	0.027451	3.082516	0.0021
MA(1)	0.883926	0.017688	49.97181	0.0000

Adjusted R-squared	0.606342	S.D. dependent var	1.746053
S.E. of regression	1.095511	Akaike info criterion	3.022829
Sum squared resid	1905.830	Schwarz criterion	3.036328
Log likelihood	-2402.172	Durbin-Watson stat	1.998744

Inverted AR Roots	.71	-.29 -.18i	-.29+.18i
Inverted MA Roots	-.88		

Q-statistics of error: ARMA(3,1)

Lag	AC	PAC	Q-Stat	Prob	Lag	AC	PAC	Q-Stat	Prob
1	0.000	0.000	3.E-05		20	-0.010	-0.006	18.026	0.322
2	0.000	0.000	0.0004		22	-0.029	-0.028	24.694	0.134
3	0.002	0.002	0.0088		24	-0.033	-0.035	26.493	0.150
4	0.002	0.002	0.0139		27	0.023	0.020	31.922	0.102
5	-0.008	-0.008	0.1229	0.726	30	-0.041	-0.040	35.334	0.105
6	-0.003	-0.003	0.1361	0.934	33	0.020	0.020	36.791	0.152
7	0.019	0.019	0.6914	0.875	36	0.021	0.019	40.678	0.140
8	0.007	0.007	0.7602	0.944	39	-0.025	-0.023	42.398	0.182
9	-0.002	-0.002	0.7672	0.979	42	-0.053	-0.065	47.529	0.138
10	0.013	0.013	1.0303	0.984	45	0.029	0.037	51.520	0.126
11	-0.007	-0.007	1.1086	0.993	48	0.024	0.030	52.644	0.174
12	-0.024	-0.024	2.0491	0.979					
14	-0.020	-0.021	17.305	0.068					
16	-0.006	-0.006	17.476	0.133					
18	0.012	0.010	17.703	0.221					

Langkah/Step 2:

Results of estimation: ARMA(2,2)

Dependent Variable: RT2
Method: Least Squares

Variable	Coefficient	Std. Error	t-Statistic	Prob.
AR(1)	0.342165	0.082668	4.139032	0.0000
AR(2)	0.280153	0.039568	7.080320	0.0000
MA(1)	0.667423	0.084438	7.904268	0.0000
MA(2)	-0.200818	0.067985	-2.953851	0.0032
R-squared	0.606939	Mean dependent var		0.021289
Adjusted R-squared	0.606197	S.D. dependent var		1.745577
S.E. of regression	1.095415	Akaike info criterion		3.022652
Sum squared resid	1906.696	Schwarz criterion		3.036144
Log likelihood	-2403.542	Durbin-Watson stat		2.002513
Inverted AR Roots	.73	-.39		
Inverted MA Roots	.23	-.89		

Q-statistics of error and error-squared: ARMA(2,2)

Lag	ACF (res)	PAC (res)	Q-Stat	Prob	Lag	ACF (res-sq)	PACF(res-sq)	Q-Stat	Prob
1	-0.001	-0.001	0.0027		1	0.062	0.062	6.1411	
2	0.002	0.002	0.0072		2	0.106	0.102	23.928	
3	0.012	0.012	0.2472		3	0.053	0.041	28.362	
4	-0.006	-0.006	0.3076		4	0.073	0.058	36.820	
5	-0.014	-0.014	0.6221	0.430	5	0.027	0.011	37.992	0.000
6	-0.001	-0.001	0.6249	0.732	6	0.029	0.013	39.352	0.000
7	0.013	0.013	0.8804	0.830	7	0.027	0.016	40.496	0.000
8	0.009	0.010	1.0180	0.907	8	0.046	0.035	43.872	0.000
9	-0.006	-0.006	1.0753	0.956	9	0.022	0.011	44.679	0.000
10	0.015	0.014	1.4215	0.965	10	0.021	0.008	45.389	0.000
11	-0.011	-0.011	1.6165	0.978	11	0.024	0.014	46.333	0.000
12	-0.022	-0.022	2.4018	0.966	12	0.029	0.017	47.646	0.000
14	-0.019	-0.020	18.354	0.049	14	0.024	0.013	49.471	0.000
16	-0.006	-0.004	18.483	0.102	16	0.026	0.015	50.785	0.000
18	0.014	0.011	18.819	0.172	18	0.018	0.009	51.551	0.000
20	-0.008	-0.006	19.150	0.261	20	0.015	0.006	52.255	0.000
22	-0.027	-0.027	25.337	0.116	22	0.017	0.009	53.109	0.000
24	-0.031	-0.035	26.989	0.136	24	0.008	0.000	53.202	0.000
27	0.021	0.018	32.100	0.098	27	0.020	0.018	53.824	0.000
30	-0.041	-0.040	35.608	0.099	30	0.032	0.025	56.227	0.001
33	0.021	0.021	37.167	0.142	33	0.007	0.003	56.551	0.002
36	0.021	0.019	41.148	0.129	36	0.003	-0.002	57.063	0.004
39	-0.025	-0.024	42.950	0.167	39	0.018	0.013	58.250	0.008
42	-0.053	-0.064	48.195	0.124	42	0.053	0.044	64.398	0.005
45	0.030	0.038	52.335	0.110	45	0.019	0.010	65.859	0.008
48	0.024	0.029	53.480	0.155	48	0.013	-0.003	70.539	0.007

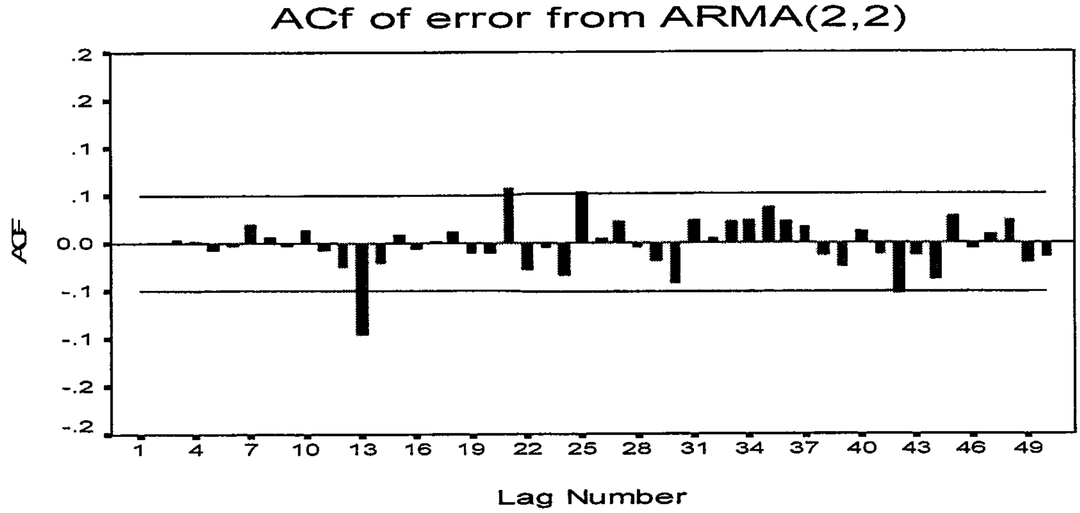
ARCH Lagrange-Multiplier test

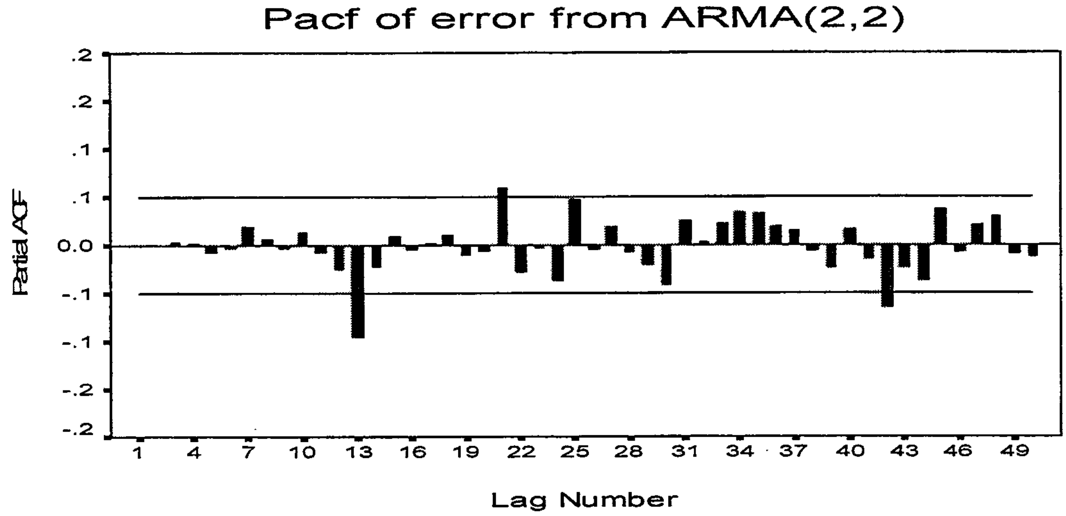
ARCH Test (Lag 1)

Obs*R-squared	6.126076	Probability	0.013320

ARCH Test (Lag 12)

Obs*R-squared	34.11039	Probability	0.000648





Langkah/Step 3:

Results of estimation: ARMA(2,2)-GARCH(1,1)

Dependent Variable: RT2
Method: ML - ARCH

	Coefficient	Std. Error	z-Statistic	Prob.
AR(1)	0.343317	0.073841	4.649441	0.0000
AR(2)	0.296306	0.035834	8.268821	0.0000
MA(1)	0.662747	0.077510	8.550466	0.0000
MA(2)	-0.197726	0.062207	-3.178509	0.0015
	Variance Equation			
C	-0.000325	0.000485	-0.669278	0.5033
ARCH(1)	0.017816	0.002056	8.663906	0.0000
GARCH(1)	0.982828	0.001538	638.8732	0.0000

Adjusted R-squared	0.605163	S.D. dependent var	1.745577
S.E. of regression	1.096852	Akaike info criterion	2.819016
Sum squared resid	1908.093	Schwarz criterion	2.842628
Log likelihood	-2238.346	Durbin-Watson stat	1.995757

Inverted AR Roots	.74	-.40
Inverted MA Roots	.22	-.89

Q-statistics of error and error-squared: ARMA(2,2)-GARCH(1,1)

Lag	ACF (res)	PAC (res)	Q-Stat	Prob	Lag	ACF (res-sq)	PACF (res-sq)	Q-Stat	Prob
1	0.008	0.008	0.0929		1	0.017	0.017	0.4668	
2	0.000	0.000	0.0931		2	0.034	0.033	2.2622	
3	0.012	0.012	0.3083		3	0.007	0.005	2.3299	
4	0.008	0.008	0.4146		4	0.014	0.012	2.6237	
5	-0.025	-0.025	1.4009	0.237	5	0.000	-0.001	2.6240	0.105
6	0.002	0.002	1.4074	0.495	6	0.002	0.001	2.6280	0.269
7	0.019	0.019	1.9752	0.578	7	-0.003	-0.003	2.6459	0.450
8	0.018	0.018	2.4798	0.648	8	0.003	0.003	2.6651	0.615
9	-0.007	-0.007	2.5605	0.767	9	0.006	0.006	2.7172	0.743
10	0.007	0.006	2.6318	0.853	10	-0.004	-0.004	2.7440	0.840
11	-0.016	-0.016	3.0185	0.883	11	0.001	0.001	2.7469	0.907
12	-0.016	-0.015	3.4139	0.906	12	0.005	0.005	2.7856	0.947
14	-0.006	-0.005	12.693	0.241	14	-0.007	-0.007	2.8949	0.984
16	0.004	0.005	13.090	0.363	16	0.004	0.005	3.0022	0.996
18	0.017	0.014	13.958	0.453	18	-0.004	-0.004	3.2409	0.999
20	-0.020	-0.017	14.679	0.548	20	-0.001	-0.001	3.2420	1.000
22	-0.026	-0.028	18.784	0.405	22	-0.001	-0.001	3.2662	1.000
24	-0.010	-0.013	18.967	0.524	24	-0.010	-0.009	3.6750	1.000
27	0.000	-0.001	22.622	0.483	27	-0.010	-0.008	4.6227	1.000
30	-0.025	-0.023	25.531	0.489	30	0.004	0.004	4.6469	1.000
33	0.026	0.023	26.688	0.589	33	-0.013	-0.012	5.8403	1.000
36	0.003	0.006	29.452	0.596	36	-0.016	-0.015	6.6385	1.000
39	-0.030	-0.033	31.254	0.650	39	-0.009	-0.008	7.1248	1.000
42	-0.040	-0.049	34.199	0.646	42	0.011	0.011	7.5066	1.000
45	0.033	0.040	37.836	0.612	45	-0.007	-0.006	8.0025	1.000
48	0.014	0.020	39.701	0.656	48	-0.009	-0.011	8.9728	1.000

ARCH Lagrange-Multiplier test-ARMA(2,2)-GARCH(1,1)

ARCH Test (Lag 1)

Obs*R-squared	0.465693	Probability	0.494975

ARCH Test (Lag 6)

Obs*R-squared	2.511471	Probability	0.867182

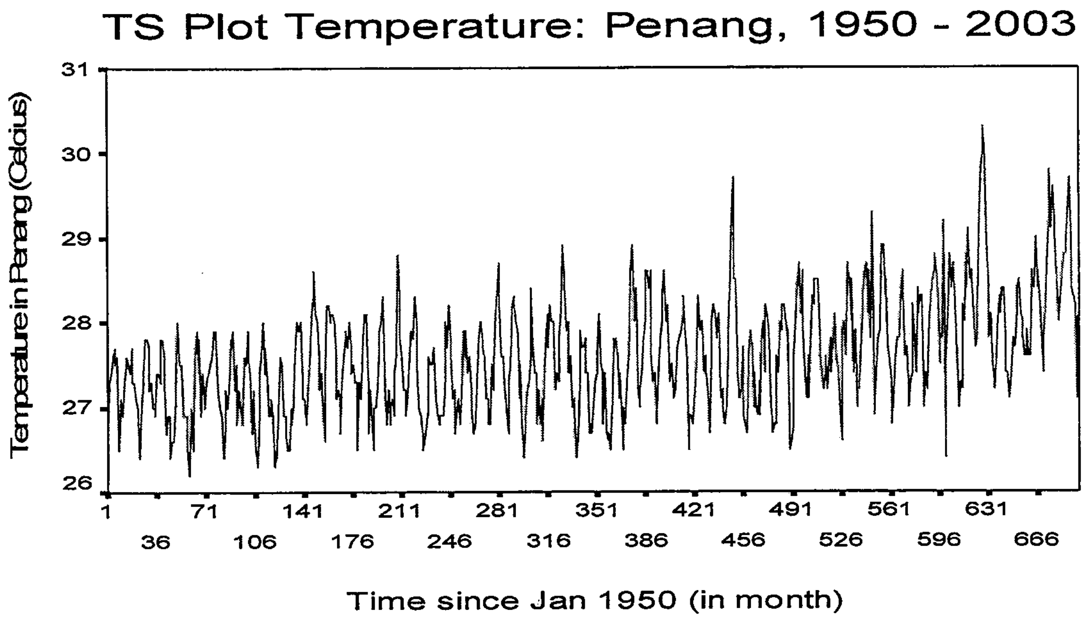
ARCH Test (Lag 12)

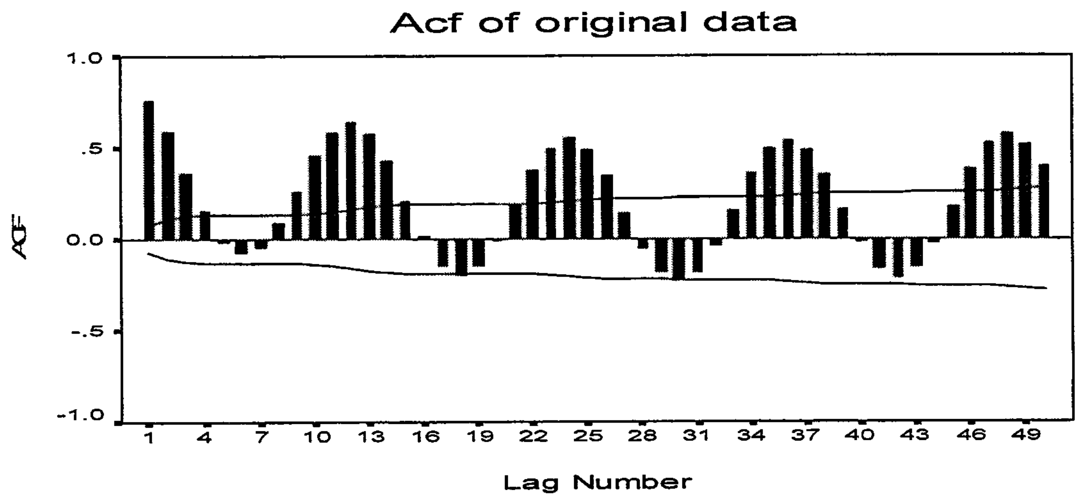
Obs*R-squared	2.630217	Probability	0.997639

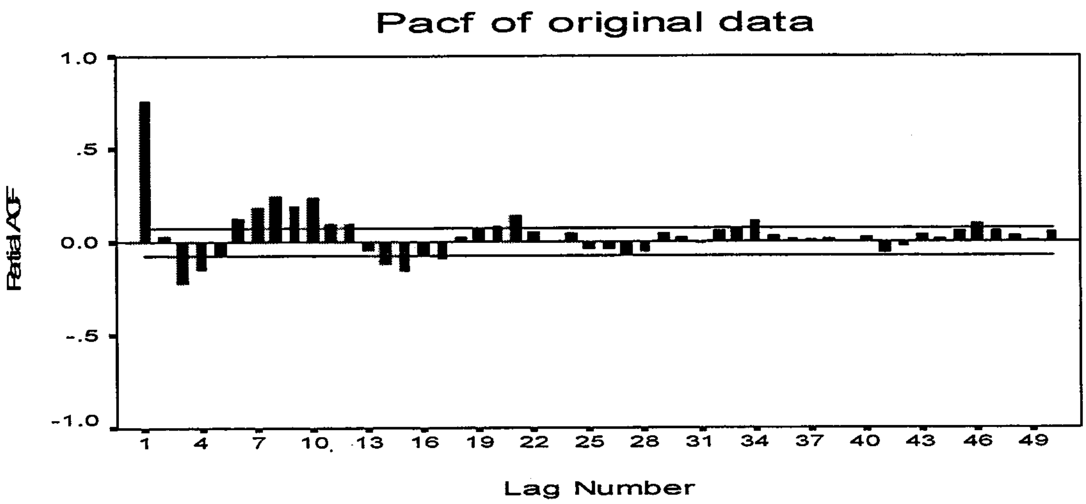
ARCH Test (lag 48)

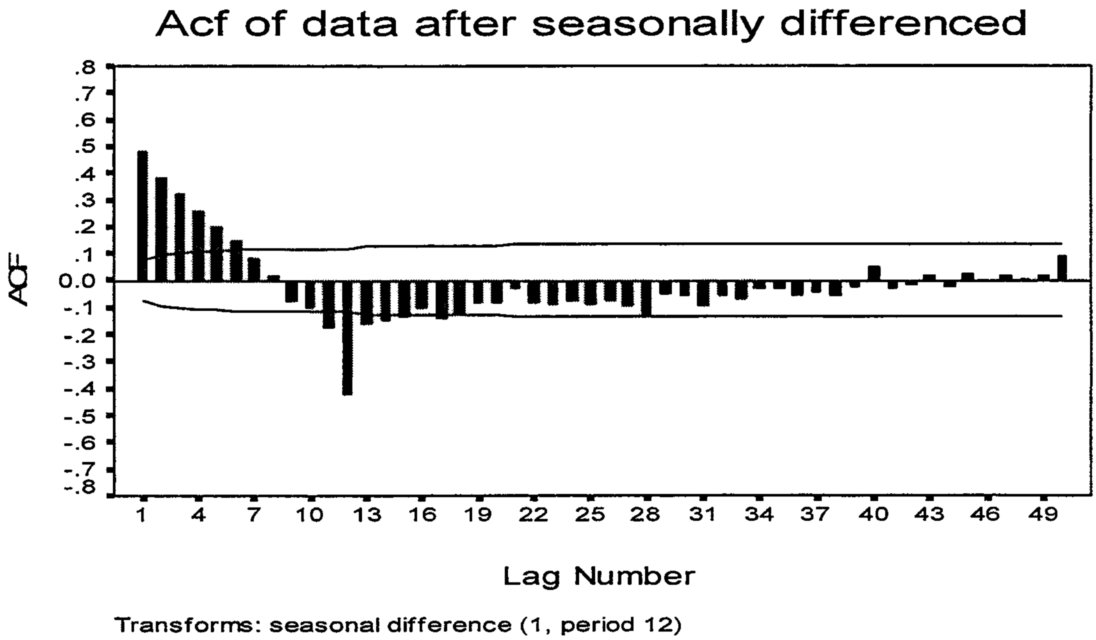
Obs*R-squared	8.587909	Probability	1.000000

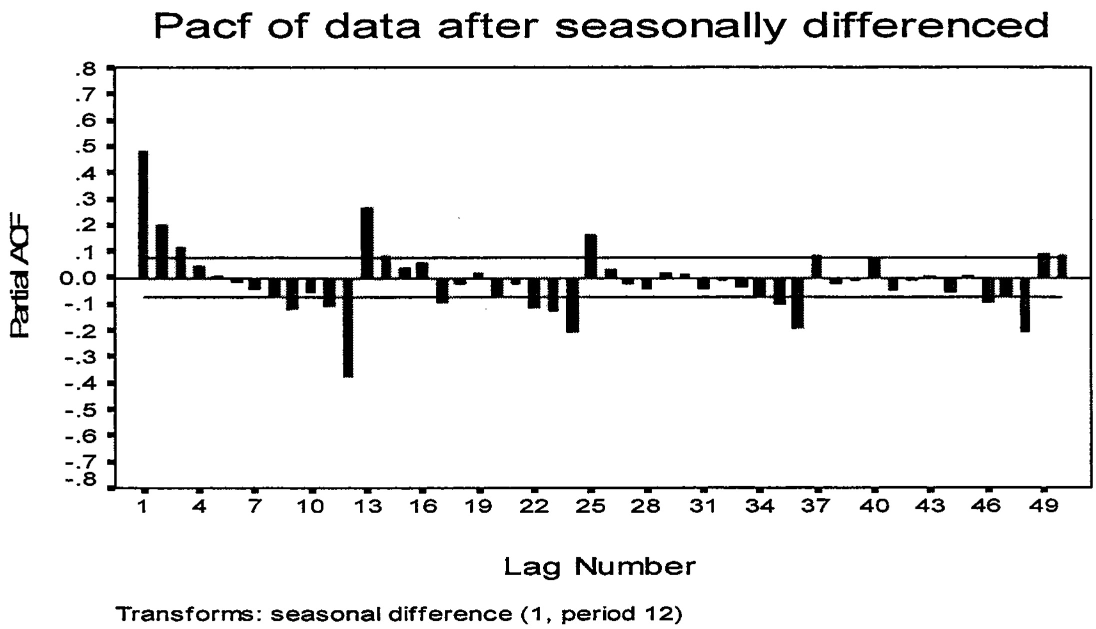
LAMPIRAN/APPENDIX B











ESTIMATED PARAMETERS: SARIMA(0,0,0)(0,1,1)

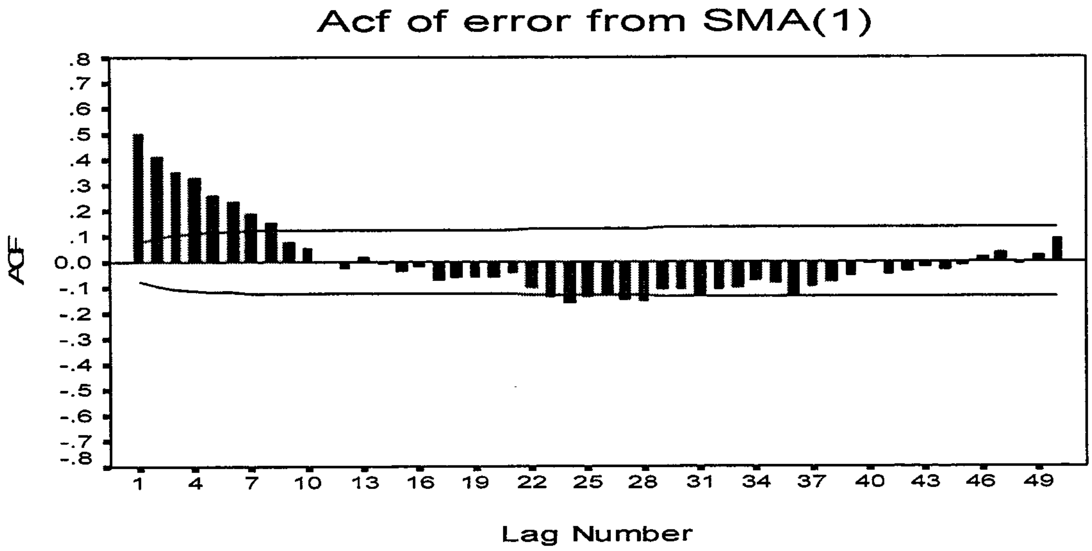
Number of residuals	684		
Standard error	.426047	Log likelihood	-392.324
AIC	786.649	SBC	791.177

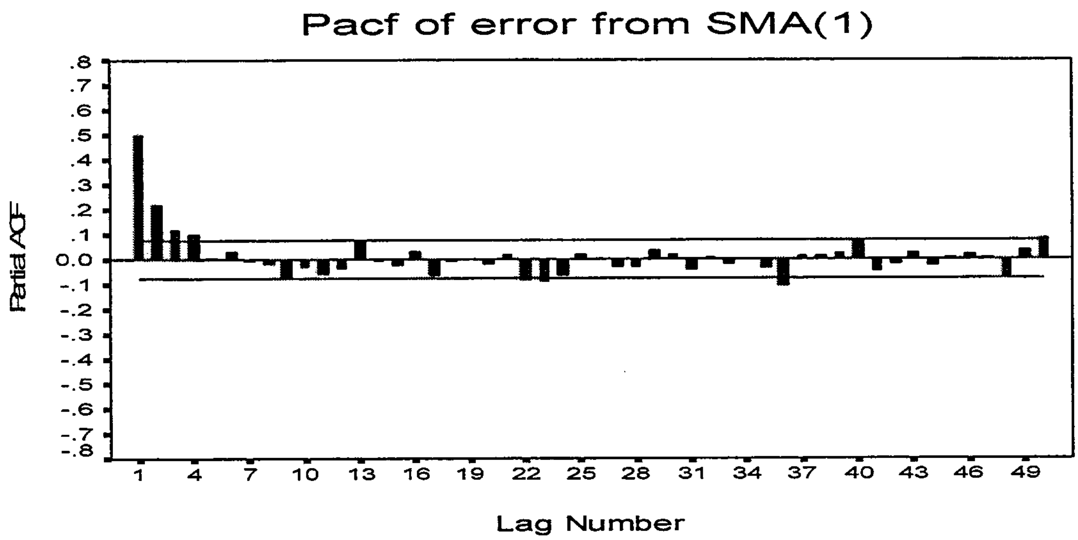
Analysis of Variance:

	DF Adj.	Sum of Squares	Residual Variance
Residuals	683	126.104	.181516

Variables in the Model:

	B	SEB	T-RATIO	APPROX. PROB.
SMA1	.788071	.026858	29.3424	.0000





ESTIMATED PARAMETERS: SARIMA(4,0,0)(0,1,1)

Number of residuals	684		
Standard error	.34175498	Log likelihood	-244.23282
AIC	498.46564	SBC	521.10543

Analysis of Variance:

	DF Adj.	Sum of Squares	Residual Variance
Residuals	679	81.772282	.11679647

Variables in the Model:

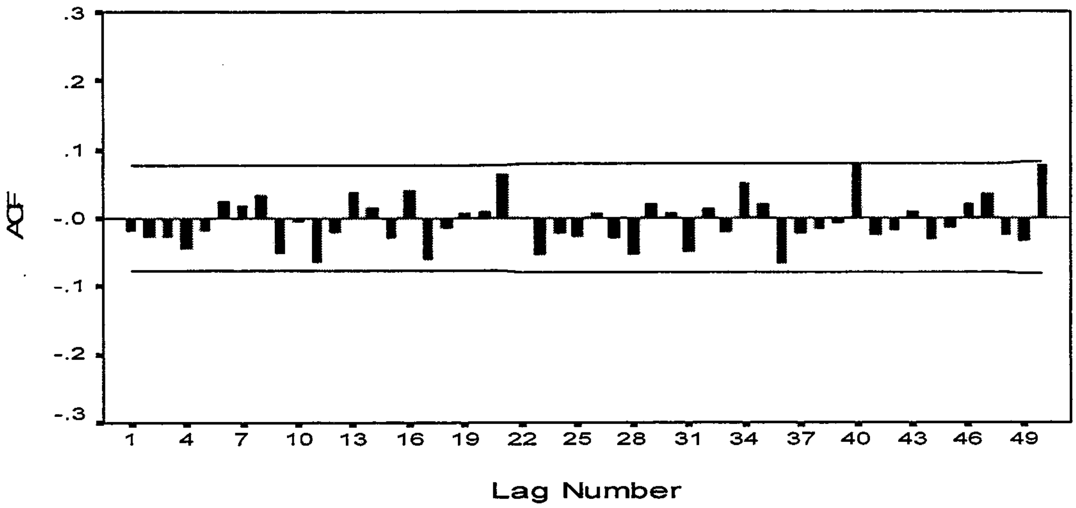
	B	SEB	T-RATIO	APPROX. PROB.
AR1	.36947093	.03745413	9.864624	.00000000
AR2	.18747937	.04020834	4.662698	.00000376
AR3	.08470844	.04008937	2.112990	.03496592
AR4	.13303313	.03793482	3.506887	.00048324
SMA1	.90666834	.02151901	42.133366	.00000000

```
Covariance Matrix:
              AR1         AR2         AR3         AR4          SMA1
AR1        .001403    -.000532    -.000295    -.000187      .000044
AR2       -.000531     .001617    -.000480    -.000279      .000093
AR3       -.000295    -.000478     .001607    -.000552      .000003
AR4       -.000187    -.000279    -.000552     .001439      .000085
SMA1       .000044     .000093     .000003     .000085      .000463
```

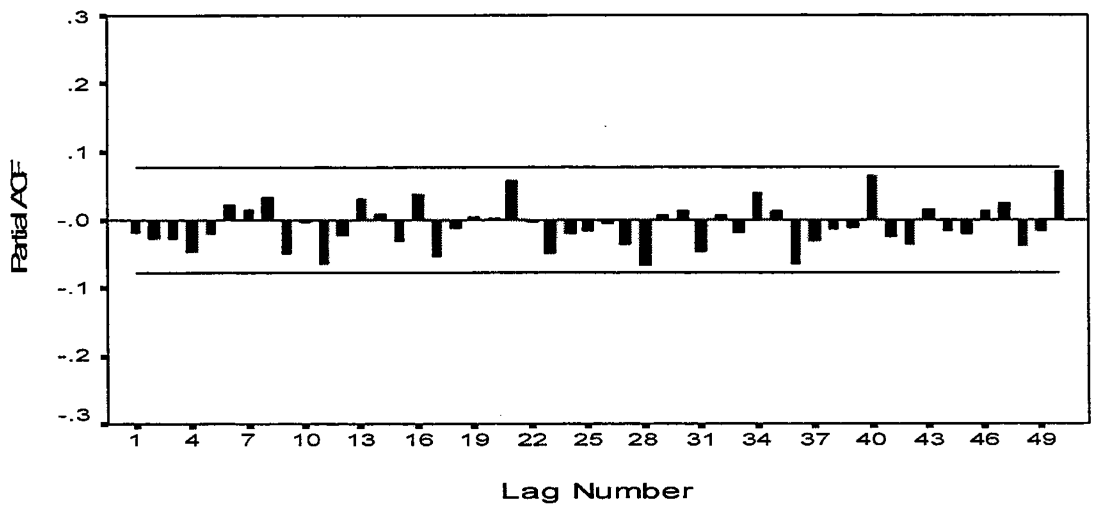
Descriptive Statistics of Errors from SARIMA(4,0,0)(0,1,1)

	N	Variance		
	Statistic	Statistic	Std. Error	Statistic
Error from SARIMA	684	.0372511	.0131056	.117

Acf from SARIMA(4,0,0)(0,1,1)



Pacf of error from SARIMA(4,0,0)(0,1,1)



FINAL PARAMETERS: SARIMA(4,0,0)(0,1,1) + Constant

Number of residuals	684		
Standard error	.336571	Log likelihood	-236.4159
AIC	484.8318	SBC	511.9995

Analysis of Variance:

	DF Adj.	Sum of Squares	Residual Variance
Residuals	678	79.908001	.11328028

Variables in the Model:

	B	SEB	T-RATIO	APPROX. PROB.
AR1	.350539	.037324	9.391829	.000000
AR2	.177781	.039778	4.469369	.000009
AR3	.063891	.039907	1.600983	.109846
AR4	.114842	.037672	3.048453	.002390
SMA1	.945213	.019583	48.265825	.000000
CONSTANT	.019731	.003580	5.511862	.000000

Covariance Matrix:

	AR1	AR2	AR3	AR4	SMA1
AR1	.001393	-.000505	-.000275	-.000153	.000006
AR2	-.000505	.001582	-.000454	-.000271	.000045
AR3	-.000275	-.000454	.001593	-.000521	-.000046
AR4	-.000153	-.000271	-.000521	.001419	.000023
SMA1	.000006	.000045	-.000046	.000023	.000383

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