UNIVERSITI SAINS MALAYSIA

Second Semester Examination
Academic Session 2006/2007

April 2007

MSG 252 – Linear and Integer Programming
[Pengaturcaraan Linear dan Integer]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of NINE pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi SEMBILAN muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions: Answer **all nine** [9] questions.

*[**Arahan:** Jawab **semua sembilan** [9] soalan.]*

...2/-

1. Consider the following LP.

$$\begin{aligned} \text{Maximize} \quad & z = 2x_1 + 7x_2 + 4x_3 \\ \text{Subject to} \quad & x_1 + 2x_2 + x_3 \le 10 \\ & 3x_1 + 3x_2 + 2x_3 \le 1 \\ & x_1, x_2, x_3 \ge 0 \end{aligned}$$

Use duality theory to demonstrate that the optimal value of the primal problem cannot exceed 6.

[10 marks]

2. Consider the following LP.

$$\begin{aligned} \text{Maximize} \quad & z = 4x_1 + 3x_2 + x_3 + 2x_4 \\ \text{Subject to} \quad & 4x_1 + 2x_2 + x_3 + x_4 \le 5 \\ & 3x_1 + x_2 + 2x_3 + x_4 \le 4 \\ & x_1, x_2, x_3, x_4 \ge 0 \end{aligned}$$

You are given the facts that the basic variables in the optimal solution are x_2 and x_4. Use the information given to identify the optimal solution.

[10 marks]

3. Consider the following LP.

$$\begin{aligned} \text{Maximize} \quad & z = 2x_1 - 4x_2 \\ \text{Subject to} \quad & x_1 - x_2 \le 1 \\ & x_1, x_2 \ge 0 \end{aligned}$$

(a) Construct the dual and find its optimal solution by inspection.

(b) Use complementary slackness conditions to find the optimal solution for the primal problem.

(c) Suppose that c_1, the coefficient of x_1 in the primal objective function, can have any value in the model. For what values of c_1 does the dual problem have no feasible solution?

[10 marks]

4. Solve the following 0-1 problem.

$$\begin{aligned} \text{Maximize} \quad & z = 2x_1 - x_2 + 5x_3 - 3x_4 + 4x_5 \\ \text{Subject to} \quad & 3x_1 - 2x_2 + 7x_3 - 5x_4 + 4x_5 \le 6 \\ & x_1 - x_2 + 2x_3 - 4x_4 + 2x_5 \le 0 \\ & x_1, x_2, x_3, x_4, x_5 = 0, 1 \end{aligned}$$

[10 marks]

.../3-

1. *Pertimbangkan masalah PL berikut.*

Maksimumkan $z = 2x_1 + 7x_2 + 4x_3$

terhadap $x_1 + 2x_2 + x_3 \leq 10$

$3x_1 + 3x_2 + 2x_3 \leq 1$

$x_1, x_2, x_3 \geq 0$

Gunakan teori kedualan untuk menunjukkan bahawa nilai matlamat bagi masalah primal tidak melebihi 6.

[10 markah]

2. *Pertimbangkan masalah PL berikut.*

Maksimumkan $z = 4x_1 + 3x_2 + x_3 + 2x_4$

terhadap $4x_1 + 2x_2 + x_3 + x_4 \leq 5$

$3x_1 + x_2 + 2x_3 + x_4 \leq 4$

$x_1, x_2, x_3, x_4 \geq 0$

Anda diberitahu yang x_2 dan x_4 *adalah pembolehubah asas dalam penyelesaian optimum. Gunakan maklumat yang diberi untuk menentukan penyelesaian optimum.*

[10 markah]

3. *Pertimbangkan masalah PL berikut.*

Maksimumkan $z = 2x_1 - 4x_2$

terhadap $x_1 - x_2 \leq 1$

$x_1, x_{2,3} \geq 0$

(a) *Dapatkan masalah dual dan tentukan penyelesaian optimumnya dengan cara pemerhatian.*

(b) *Gunakan syarat-syarat kelalaian lengkap untuk mencari penyelesaian optimum bagi masalah primal.*

(c) *Katakan* c_1, *pekali* x_1 *di dalam fungsi matlamat, boleh mengambil sebarang nilai. Apakah nilai-nilai* c_1 *yang menyebabkan dual tak tersaur?*

[10 markah]

4. *Selesaikan masalah 0-1 berikut.*

Maksimumkan $z = 2x_1 - x_2 + 5x_3 - 3x_4 + 4x_5$

terhadap $3x_1 - 2x_2 + 7x_3 - 5x_4 + 4x_5 \leq 6$

$x_1 - x_2 + 2x_3 - 4x_4 + 2x_5 \leq 0$

$x_1, x_2, x_3, x_4, x_5 = 0, 1$

[10 markah]

5. Consider the following LP.

Maximize $z = 3x_1 + x_2 + 4x_3$

Subject to

$$6x_1 + 3x_2 + 5x_3 \leq 25$$
$$3x_1 + 4x_2 + 5x_3 \leq 20$$
$$x_1, x_2, x_3 \geq 0$$

The corresponding final set of equations yielding the optimal solution is

$$z + 0x_1 + 2x_2 + 0x_3 + \tfrac{1}{5}x_4 + \tfrac{3}{5}x_5 = 17$$
$$x_1 - \tfrac{1}{3}x_2 + 0x_3 + \tfrac{1}{3}x_4 - \tfrac{1}{3}x_5 = \tfrac{5}{3}$$
$$0x_1 + x_2 + x_3 - \tfrac{1}{5}x_4 + \tfrac{2}{5}x_5 = 3$$

where x_4 and x_5 are the slack variables corresponding to the 1st and 2nd constraints respectively.

(a) Identify the optimal solution to the dual problem from the final set of equations.

(b) Suppose that the original problem is changed to

Maximize $z = 3x_1 + 3x_2 + 4x_3$

Subject to

$$6x_1 + 2x_2 + 5x_3 \leq 25$$
$$3x_1 + 4x_2 + 5x_3 \leq 20$$
$$x_1, x_2, x_3 \geq 0$$

Use duality theory to determine whether the previous optimal solution is still optimal.

(c) Suppose that the only change in the original problem is that a new variable x_{new} has been introduced into the model as follows:

Maximize $z = 3x_1 + x_2 + 4x_3 + 2x_{new}$

Subject to

$$6x_1 + 3x_2 + 5x_3 + 3x_{new} \leq 25$$
$$3x_1 + 4x_2 + 5x_3 + 2x_{new} \leq 20$$
$$x_1, x_2, x_3, x_{new} \geq 0$$

Use duality theory to determine whether the optimal solution to the original problem is still optimal.

(d) Suppose the only change in the original problem is that a new constraint $x_1 + x_3 \geq 8$ has been introduced into the model. Find the new optimal solution if the original optimal solution changes.

[20 marks]

5. *Pertimbangkan masalah PL berikut.*

Maksimumkan $z = 3x_1 + x_2 + 4x_3$

terhadap

$$6x_1 + 3x_2 + 5x_3 \leq 25$$
$$3x_1 + 4x_2 + 5x_3 \leq 20$$
$$x_1, x_2, x_3 \geq 0$$

Persamaan-persamaan yang menghasilkan penyelesaian optimum ialah

$$z + 0x_1 + 2x_2 + 0x_3 + \tfrac{1}{5}x_4 + \tfrac{3}{5}x_5 = 17$$
$$x_1 - \tfrac{1}{3}x_2 + 0x_3 + \tfrac{1}{3}x_4 - \tfrac{1}{3}x_5 = \tfrac{5}{3}$$
$$0x_1 + x_2 + x_3 - \tfrac{1}{5}x_4 + \tfrac{2}{5}x_5 = 3$$

yang mana x_4 *dan* x_5 *masing-masing adalah pembolehubah lalai daripada kekangan 1 and 2.*

(a) *Camkan penyelesaian optimum bagi masalah dual berdasarkan persamaan-persamaan tersebut.*

(b) *Katakan masalah asal ditukar kepada*

Maksimumkan $z = 3x_1 + 3x_2 + 4x_3$

terhadap

$$6x_1 + 2x_2 + 5x_3 \leq 25$$
$$3x_1 + 4x_2 + 5x_3 \leq 20$$
$$x_1, x_2, x_3 \geq 0$$

Gunakan teori kedualan untuk menentukan penyelesaian sebelumnya masih optimum.

(c) *Katakan perubahan dalam masalah asal hanyalah penambahan pembolehubah baru* x_{new} *yang dimasukkan ke dalam model seperti berikut:*

Maksimumkan $z = 3x_1 + x_2 + 4x_3 + 2x_{new}$

terhadap

$$6x_1 + 3x_2 + 5x_3 + 3x_{new} \leq 25$$
$$3x_1 + 4x_2 + 5x_3 + 2x_{new} \leq 20$$
$$x_1, x_2, x_3, x_{new} \geq 0$$

Gunakan teori kedualan untuk menentukan sama ada penyelesaian bagi masalah asal masih optimum.

(d) *Katakan perubahan yang dibuat di dalam masalah asal ialah penambahan kekangan* $x_1 + x_3 \geq 8$ *ke dalam model. Dapatkan penylesaian optimum baru jika penylesaian optimum yang asal berubah.*

[20 markah]

6. Solve the following goal programming problem graphically.

Minimize $z = P_1u_1 + P_2u_2 + P_3v_3$

Subject to

$$x_1 + 2x_2 + u_1 - v_1 = 10$$

$$2x_1 + 3x_2 + u_2 - v_2 = 12$$

$$6x_1 + 18x_2 + u_3 - v_3 = 54$$

$$x_1, x_2, u_1, u_2, u_3, v_1, v_2, v_3 \geq 0$$

[10 marks]

7. Consider the following IP.

Maximize $z = 3x_1 + 2x_2$

Subject to

$$-x_1 + 2x_2 \leq 4$$

$$3x_1 + 2x_2 \leq 14$$

$$x_1 - x_2 \leq 3$$

$$x_1, x_2 \geq 0 \text{ and integer}$$

The optimal solution for this IP's linear programming relaxation is given in the following tableau:

Basic	x_1	x_2	x_3	x_4	x_5	**Solution**
z	0	0	0	1	0	14
x_5	0	0	$5/8$	$-1/8$	1	$15/4$
x_2	0	1	$3/8$	$1/8$	0	$13/4$
x_1	1	0	$-1/4$	$1/4$	0	$5/2$

where x_3, x_4 and x_5 are the slack variables corresponding to the 1st, 2nd and 3rd constraints respectively. Find the optimal integer solution by using the cutting plane method.

[10 marks]

6. *Selesaikan masalah pengaturcaraan gol berikut secara bergraf.*

Minimumkan $z = P_1u_1 + P_2u_2 + P_3v_3$

terhadap

$$x_1 + 2x_2 + u_1 - v_1 = 10$$

$$2x_1 + 3x_2 + u_2 - v_2 = 12$$

$$6x_1 + 18x_2 + u_3 - v_3 = 54$$

$$x_1, x_2, u_1, u_2, u_3, v_1, v_2, v_3 \geq 0$$

[10 markah]

7. *Pertimbangkan masalah PI berikut.*

Maksimumkan $z = 3x_1 + 2x_2$

terhadap

$$-x_1 + 2x_2 \leq 4$$

$$3x_1 + 2x_2 \leq 14$$

$$x_1 - x_2 \leq 3$$

$x_1, x_2, \geq 0$ *dan integer*

Penyelesaian optimum jika syarat integer diabaikan diberikan dalam tablo berikuti:

Asas	x_1	x_2	x_3	x_4	x_5	***Penyelesaian***
z	0	0	0	1	0	14
x_5	0	0	$5/8$	$-1/8$	1	$15/4$
x_2	0	1	$3/8$	$1/8$	0	$13/4$
x_1	1	0	$-1/4$	$1/4$	0	$5/2$

yang mana x_3, x_4 *dan* x_5 *masing-masing adalah pembolehubah lalai dari kekangan pertama, kedua dan ketiga. Dapatkan penyelesaian optimum yang integer dengan menggunakan kaedah satah potongan.*

[10 markah]

8. Consider the following integer programming.

Minimize $z = 6x_1 + 8x_2$

Subject to $3x_1 + x_2 \geq 4$

$x_1 + 2x_2 \geq 4$

$x_1, x_2 \geq 0,\ \ x_2$ integer

The optimal tableau for this IP's linear programming relaxation is as follows:

Basic	x_1	x_2	x_3	x_4	Solution
z	0	0	$-4/5$	$-18/5$	$88/5$
x_1	1	0	$-2/5$	$1/5$	$4/5$
x_2	0	1	$1/5$	$-3/5$	$8/5$

where x_3 and x_4 are the surplus variables corresponding to the 1st and 2nd constraints respectively. Use the branch and bound method to find the optimal solution.

[10 marks]

9. Write the dual of the following problem.

Minimize $z = \sum_i \sum_j c_{ij} x_{ij}$

Subject to $\sum_j x_{ij} \geq a_i \qquad i = 1,2,\ldots,m$

$\sum_i x_{ij} \leq b_j \qquad j = 1,2,\ldots,n$

$x_{ij} \geq 0$

[10 marks]

8. *Pertimbangkan masalah pengaturcaraan integer berikut.*

Minimumkan $z = 6x_1 + 8x_2$

terhadap $3x_1 + x_2 \geq 4$

$x_1 + 2x_2 \geq 4$

$x_1, x_2 \geq 0, \ x_2$ integer

Tablo optimum jika syarat integer diabaikan ialah:

Asas	x_1	x_2	x_3	x_4	***Penyelesaian***
z	0	0	$-4/5$	$-18/5$	$88/5$
x_1	1	0	$-2/5$	$1/5$	$4/5$
x_2	0	1	$1/5$	$-3/5$	$8/5$

yang mana x_3 *dan* x_4 *masing-masing adalah pembolehubah lebihan daripada kekangan pertama dan kedua. Gunakan kaedah cabang dan batas untuk mendapatkan penyelesaian optimum.*

[10 markah]

9. *Tuliskan dual bagi masalah berikut.*

Minimumkan $z = \sum_i \sum_j c_{ij} x_{ij}$

terhadap $\sum_j x_{ij} \geq a_i \qquad i = 1, 2, \ldots, m$

$\sum_i x_{ij} \leq b_j \qquad j = 1, 2, \ldots, n$

$x_{ij} \geq 0$

[10 markah]

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