UNIVERSITI SAINS MALAYSIA

Second Semester Examination
Academic Session 2006/2007

April 2007

MSG 228 – Introduction to Modelling
[Pengenalan Pemodelan]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of NINE pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi SEMBILAN muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions: Answer **all five** [5] questions.

*[**Arahan:** Jawab **semua lima** [5] soalan.]*

...2/-

1. (a) Discuss whether each of the following set s of numbers is proportional or not.

(i) shoe size $\propto$ age

(ii) cost of cheese $\propto$ number of pounds (weight)

(iii) $y \propto x$, if $y = 3x, y = 3x+1, y = x^2, y = \frac{1}{x}$

(b) Consider the following difference equation

$$a_{n+1} = 0.5a_n + 1$$

together with the initial conditions

(i) $a_0 = 1.5$ (ii) $a_0 = 2.5$

What is the eventual behaviour of the equation?

(c) Take the case of two species, A and B, in an isolated environment with population a_n and b_n respectively. Assume that species B consumes the same food as A, but that species A does not consumes that same kind of food as species B does. From this information, build up a simple finite difference model for the population growth of species A and B

[100 marks]

2. (a) Explain under what situation do we need to use a simulation model instead of a mathematical model.

(b) Solve the following linear program using the simplex method.

Maximise: $z = 2x_1 + 3x_2$

Subject to:

$$x_1 + 2x_2 \leq 4$$
$$2x_1 - x_2 \leq 3$$
$$x_1, x_2 \geq 0$$

(c) A small solid metal ball was dropped from the top of a building of height 400 feet. It's flight to the ground was then photographed, its distance from the ground at certain time intervals was measured. The result is as follows

Time (sec)	0	1	2	3	4
Distance (feet)	400	384	336	256	144

From these data build an algebraic model for the velocity at any general time. From here find the velocity at times $t = 0, 1, 2, 3, 4$

[100 marks]

.../3-

1. (a) *Bincangkan jika setiap set nombor-nombor berikut berkadaran atau tidak.*
   (i) *saiz kasut* $\propto$ *umur*
   (ii) *harga keju* $\propto$ *bilangan paun (berat)*
   (iii) $y \propto x$, *jika* $y = 3x, y = 3x+1, y = x^2, y = \frac{1}{x}$

   (b) *Pertimbangkan persamaan beza berikut*

   $$a_{n+1} = 0.5\,a_n + 1$$

   bersamaan dengan syarat-syarat awal
   (i) $a_0 = 1.5$ (ii) $a_0 = 2.5$
   Apakah perlakuan persamaan seterusnya?

   (c) *Lihat kepada kes dua spesis, A dan B, yang berada pada satu persekitaran terpencil, dengan masing-masing populasi* a_n *dan* b_n. *Anggap spesis B memakan makanan yang sama seperti A, tetapi spesis A tidak makan makanan sama seperti B. Daripada maklumat ini, bangunkan satu model ringkas beza terhingga untuk perkembangan populasi A dan B.*

[100 markah]

2. (a) *Huraikan tentang situasi yang memerlukan kita mengguna model simulasi dan tidak model matematik.*

   (b) *Selesaikan aturcara linear berikut dengan menggunakan kaedah simpleks.*

   Maksimumkan: $z = 2x_1 + 3x_2$
   Tertakluk kepada:

   $$x_1 + 2x_2 \le 4$$
   $$2x_1 - x_2 \le 3$$
   $$x_1,\ x_2 \ge 0$$

   (c) *Sebiji bola kecil logam dilepaskan daripada atas sebuah bangunan dengan tinggi 400 kaki. Perjalanan bola tersebut ke bawah telah dirakamkan melalui kamera. Jarak bola daripada dataran kemudiannya diukur. Hasilnya adalah seperti berikut:*

Masa (saat)	*0*	*1*	*2*	*3*	*4*
Jarak (kaki)	*400*	*384*	*336*	*256*	*144*

   Daripada data ini, bangukan satu model aljabar untuk halaju pada sebarang masa. Dengan ini, dapatkan halaju pada masa-masa $t = 0,1,2,3,4$

[100 markah]

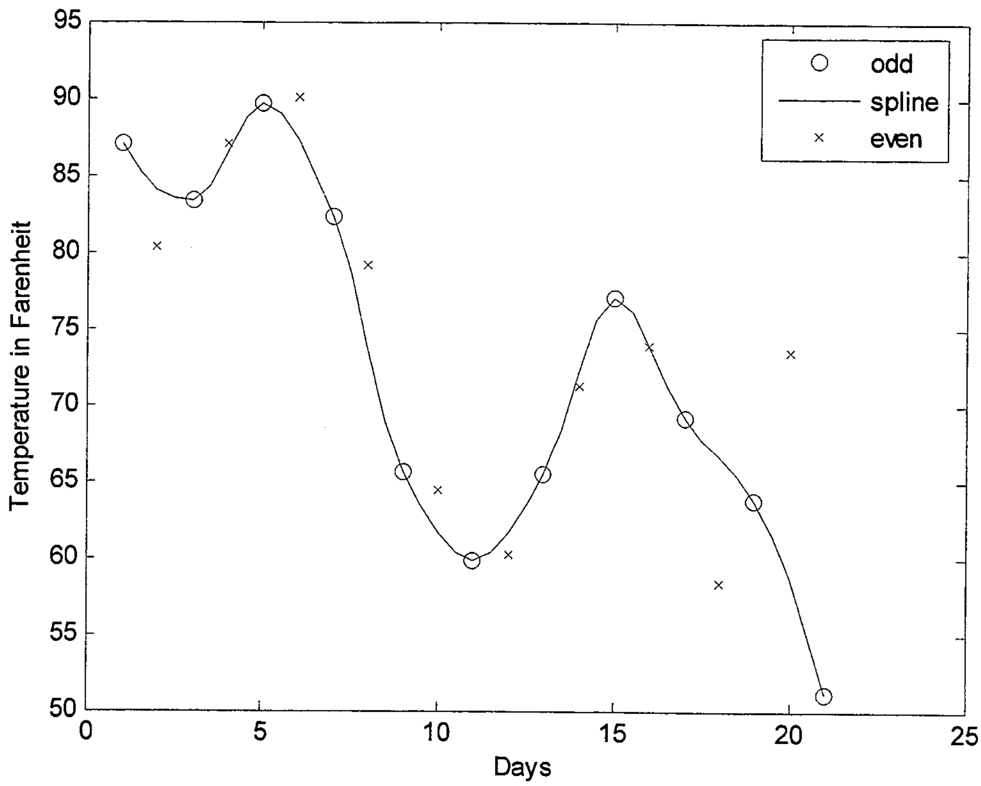
.../4-

3. (a) In an experiment, the following data points were at obtained.
(-0.50, 3.00), (2.00, 5.00), (3.00, 1.00), (3.5, 2.00), (4.00, -0.2)
Use a linear equation to model the above set of data

(b) Find a quadratic spline (degree 2) that interpolates the following data points.

x	0	1	2	3
y	-1	2	4	8

(c) In the graph below, the data points represent the temperature during the month of September of a town in the northern hemisphere. The 'o' indicates temperature on odd days and the 'x' that of even days. A cubic spline was drawn to interpolate the temperature on odd days. Give a plausible reason as to why the spline is not able to predict the last two even days temperature



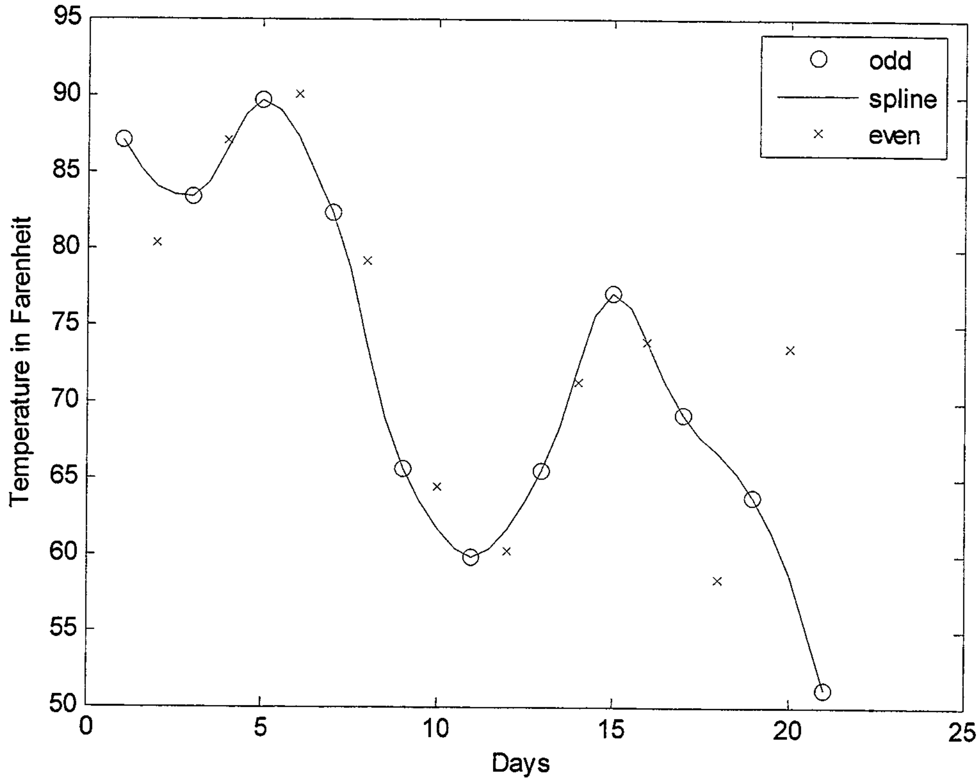
[100 marks]

3. (a) *Dalam satu ujikaji, data berikut telah diperolehi*
(-0.50,3.00), (2.00, 5.00), (3.00, 1.00), (3.5, 2.00), (4.00, -0.2)
Dapatkan satu persamaan linear sebagai modul untuk set data di atas.

(b) *Dapatkan satu splin kuadratik yang interpolasi titik-titik data berikut:*

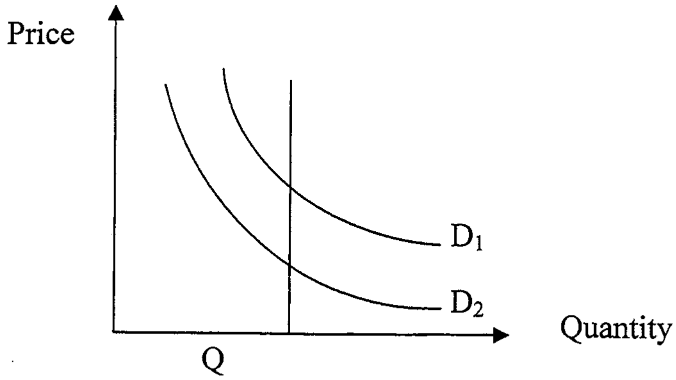
x	0	1	2	3
y	-1	2	4	8

(c) *Dalam graf berikut, titik-titik data mewakili suhu pada bulan September sebuah pekan di utara hemisfera. Titik 'o' mewakili suhu pada hari-hari ganjil dan 'x' mewakili suhu pada hari-hari genap. Satu splin kubik dilakarkan agar ianya interpolasi suhu pada hari-hari ganjil. Beri satu sebab mengapa splin tidak dapat meramalkan dua suhu terakhir hari genap.*



[100 markah]

4. (a) Consider the following supply demand curve



The supply curve is vertical. Give an example of the occurrence of such a curve and hence interpret the demand curves D_1, D_2.

(b) A simple SIR epidemics model can be written as

$$\frac{ds}{dt} = -\beta\, IS$$

$$\frac{dI}{dt} = \beta\, IS - vI$$

$$\frac{dR}{dt} = vI$$

$$N = S + I + R$$

where

S	is	number of susceptibles
I	is	number of invectives
R	is	number of recoveries
N	is	population number, a constant
β	and	v are constants

Assume the following initial conditions

$$S(0) = S_0, I(0) = I_0, R(0) = 0$$

(i) Solve for I in terms of S

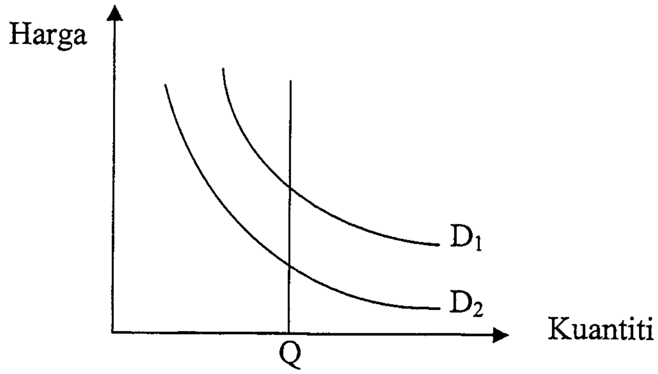
(ii) As $t \to \infty$, show that $I_\infty = 0$, i.e. the disease will eventually run its course.

(iii) Show that there is a portion of the population that will always not be infected.

(c) Consider the spread of a disease such as malaria. Assume that an infected person infect 10 mosquitoes and each mosquito infects 100 people. What is the average number of cases produced?

[100 marks]

4. (a) *Pertimbangkan lengkungan pembekalan permintaan berikut*



Lengkungan pembekalan adalah mencacang. Beri satu contoh di mana ini boleh berlaku dan dengan ini beri tafsiran kepada lengkungan-lengkungan permintaan D_1, D_2.

(b) *Satu model ringkas epidemik SIR boleh ditulis seperti*

$$\frac{ds}{dt} = -\beta IS$$

$$\frac{dI}{dt} = \beta IS - vI$$

$$\frac{dR}{dt} = vI$$

$$N = S + I + R$$

dengan

S	*bilangan yang boleh dijangkiti*
I	*bilangan yang dijangkiti*
R	*bilangan yang telah sembuh*
N	*populasi, satu pemalar*
β *dan* v	*pemalar-pemalar*

Anggap syarat-syarat awal berikut:

$$S(0) = S_0, I(0) = I_0, R(0) = 0$$

(i) *Selesaikan untuk I dalam sebutan-sebutan S*

(ii) *Apabila $t \to \infty$, tunjukkan bahawa $I_\infty = 0$, iaitu penyakit akan lambat-laun hapus.*

(iii) *Tunjukkan bahawa akan terdapat sebilangan daripada populasi yang tidak akan dijangkiti.*

(c) *Pertimbangkan merebaknya satu penyakit seperti malaria. Anggap bahawa seorang yang mendapat penyakit tersebut menjangkiti sepuluh ekor nyamuk dan setiap nyamuk ini akan menjangkiti 100 orang. Apakah bilangan purata kes yang terhasil?*

[100 markah]

5. (a) In the simple traffic light at an intersection model, drivers approaching the intersection are assumed to arrive at speed $\bar{v}$. A more realistic model will assume that the drivers approach at speeds between

$$\bar{v}-\left(\frac{\Delta v}{2}\right) \text{ and } \bar{v}+\left(\frac{\Delta v}{2}\right)$$

with equal probability. Calculate the average yellow cycle A in this case. You may consider

$$\bar{A}=\frac{\Delta v}{2}\int_{\bar{v}-\frac{\Delta v}{2}}^{\bar{v}+\frac{\Delta v}{2}} A\left(\bar{v}\right)^{2} d\bar{v}$$

(b) Suppose that we can regulate the concentration and flow upstream of a queue at a traffic light, subject to the Greenshields relation $q=u_f k\left(1-\frac{k}{k_j}\right)$, where u_f is the free flow speed, which we take to be 50 miles per hour. Find the value of k at which $u=u_s$ so that the stating wave moves at the same speed as the queue grows. Use $q_s=1500, k_s=150$.

[100 marks]

.../9-

5. (a) *Dalam satu model ringkas mengenai lampu trafik pada persimpangan jalan, pemandu yang menuju ke persimpangan akan sampai pada tahap kelajuan* $\bar{v}$. *Model yang lebih realistik akan menganggap pemandu akan sampai pada kelajua diantara.*

$$\bar{v}-\left(\frac{\Delta v}{2}\right) \; and \; \bar{v}+\left(\frac{\Delta v}{2}\right)$$

pada kebarangkalian sama. Hitungkan kitaran purata lampu kuning A. Anda boleh pertimbangkan

$$\bar{A}=\frac{\Delta v}{2}\int_{\bar{v}-\frac{\Delta v}{2}}^{\bar{v}+\frac{\Delta v}{2}} A\left(\bar{v}\right)^2 d\bar{v}$$

(b) *Andaikan kita boleh aturkan pemusatan dan hulu aliran satu giliran pada suatu lampu trafik, tertakluk kepada hubungan Greenshield* $q=u_f k\left(1-\frac{k}{k_j}\right)$, u_f *adalah kelajuan aliran bebas yang kita ambil sebagai 50 batu sejam. Dapatkan nilai k pada* $u=u_s$ *supaya gelombang permulaan bergerak pada kelajuan sama seperti giliran berkembang. Gunakan* $q_s=1500, k_s=150$.

[100 markah]

- ooo O ooo -