UNIVERSITI SAINS MALAYSIA

Second Semester Examination
Academic Session 2005/2006

April/May 2006

MGM 511 – Linear Algebra
[Aljabar Linear]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of SIX pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi ENAM muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions : Answer **all eight** [8] questions.

*[**Arahan** : Jawab **semua lapan** [8] soalan].*

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1. Define dot product and cross product of vectors. Given the points $P(-2,0,1)$, $Q(3,1,2)$, $R(1,5,-7)$ and $S(1,-6,5)$ in $\mathbb{R}^3$, find the vectors $\overrightarrow{PQ}$ and $\overrightarrow{RS}$. Hence find the following:

(i) $\overrightarrow{PQ} \bullet \overrightarrow{RS}$,

(ii) $\overrightarrow{PQ} \times \overrightarrow{RS}$,

(iii) the projection vector of $\overrightarrow{PQ}$ on $\overrightarrow{RS}$.

[30 marks]

1. Takrifkan hasil darab noktah dan hasil darab langsung bagi vektor. Diberikan titik-titik $P(-2,0,1)$, $Q(3,1,2)$, $R(1,5,-7)$ *dan* $S(1,-6,5)$ *dalam* $\mathbb{R}^3$, *cari vektor-vektor* $\overrightarrow{PQ}$ *dan* $\overrightarrow{RS}$. *Dengan itu, cari yang berikut:*

(i) $\overrightarrow{PQ} \bullet \overrightarrow{RS}$,

(ii) $\overrightarrow{PQ} \times \overrightarrow{RS}$,

(iii) vektor unjuran $\overrightarrow{PQ}$ *atas* $\overrightarrow{RS}$.

[30 markah]

2. Define Hermition and skew-Hermition matrices and prove that any square matrix can be written as the sum of a Hermition and a skew-Hermition matrix. Hence, write the following two matrices as a sum of a Hermition and a skew-Hermition matrix:

$$A = \begin{bmatrix} 2+i & 3i \\ -4-4i & 4-2i \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}.$$

[40 marks]

2. Takrifkan matriks Hermition dan matriks Hermition pencong dan buktikan bahawa sebarang matriks segiempat sama dapat ditulis sebagai hasil tambah suatu matriks Hermition dengan suatu matriks Hermition pencong. Dengan itu, tuliskan matriks berikut sebagai hasil tambah suatu matriks Hermition dengan suatu matriks Hermition pencong:

$$A = \begin{bmatrix} 2+i & 3i \\ -4-4i & 4-2i \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}.$$

[40 markah]

3. Choose any non-zero distinct real values of your choice, a_i and b_i such that

$$L = \begin{bmatrix} 1 & 0 & 0 \\ a_1 & 1 & 0 \\ a_2 & a_3 & 1 \end{bmatrix} \quad \text{and} \quad U = \begin{bmatrix} b_1 & b_2 & b_3 \\ 0 & b_4 & b_5 \\ 0 & 0 & b_6 \end{bmatrix}.$$

(i) Find A given $A = LU$.

(ii) Find $\underset{\sim}{b}$ such that $\underset{\sim}{x} = (3,-4,2)$ is a solution for $A\underset{\sim}{x} = \underset{\sim}{b}$. Discuss why this solution must be unique.

(iii) Solve the equation $A\underset{\sim}{x} = \underset{\sim}{b}$ by using the Gauss-Jordan method.

[30 marks]

3. Pilih sebarang nilai-nilai tak-sifar pilihan anda, yang berbeza, a_i dan b_i supaya

$$L = \begin{bmatrix} 1 & 0 & 0 \\ a_1 & 1 & 0 \\ a_2 & a_3 & 1 \end{bmatrix} \quad \textit{dan} \quad U = \begin{bmatrix} b_1 & b_2 & b_3 \\ 0 & b_4 & b_5 \\ 0 & 0 & b_6 \end{bmatrix}.$$

(i) Cari A diberikan $A = LU$.

(ii) Cari $\underset{\sim}{b}$ supaya $\underset{\sim}{x} = (3,-4,2)$ merupakan suatu penyelesaian bagi $A\underset{\sim}{x} = \underset{\sim}{b}$. Bincangkan kenapa penyelesaian ini unik.

(iii) Selesaikan persamaan $A\underset{\sim}{x} = \underset{\sim}{b}$ menggunakan kaedah Gauss-Jordan.

[30 markah]

4. Let $W = \{(1,-2,5,-3),(2,3,1,-4),(3,8,-3,-5)\}$.

(i) Show that span (W) is a subspace of $\mathbb{R}^4$.

(ii) Find a basis and the dimension of span (W).

(iii) Extend your basis in (ii) to form a basis of $\mathbb{R}^4$.

(iv) Given $U = \{(1,5,-4,-1),(3,1,2,0)\}$, find a basis and the dimension of each of the vector spaces $\text{span}(U) + \text{span}(W)$ and $\text{span}(U) \cap \text{span}(W)$.

[40 marks]

4. *Katakan* $W = \{(1,-2,5,-3),(2,3,1,-4),(3,8,-3,-5)\}$.

(i) *Tunjukkan bahawa* span (W) *merupakan suatu subruang bagi* $\mathbb{R}^4$.

(ii) *Cari suatu asas dan dimensi bagi* span (W).

(iii) *Panjangkan asas anda di (ii) untuk membentuk suatu asas bagi* $\mathbb{R}^4$.

(iv) *Diberikan* $U = \{(1,5,-4,-1),(3,1,2,0)\}$, *cari suatu asas dan dimensi bagi setiap ruang vektor* $\operatorname{span}(U)+\operatorname{span}(W)$ *dan* $\operatorname{span}(U)\cap\operatorname{span}(W)$.

[40 markah]

5. (a) Define linear mappings on vector spaces.

(b) One of the following three mappings are linear; use your definition above to verify which is the one. Then find a basis and the dimension of the image and the kernel of that mapping. Also discuss the possibility of existence of a basis for the image and the kernel of both the non-linear mappings:

(i) The transformation $F:\mathbb{R}^3 \to \mathbb{R}^3$ defined by $F(x,y,z)=(x+|y|,\ y+|z|,\ z+|x|)$.

(ii) The transformation $F:\mathbb{R}^3 \to \mathbb{R}^2$ defined by $F(x,y,z)=(xy,z)$.

(iii) The matrix mapping $A:\mathbb{R}^4 \to \mathbb{R}^3$ where $A=\begin{bmatrix}1 & 3 & 8 & 5\\ 1 & 2 & -1 & 2\\ 2 & 7 & 25 & 13\end{bmatrix}$.

[60 marks]

5. *(a)* *Takrifkan pemetaan linear atas ruang-ruang vektor.*

(b) *Satu daripada pemetaan berikut adalah linear; gunakan takrif anda di atas untuk menentukan yang mana satu. Seterusnya cari suatu asas dan dimensi bagi set imej serta inti bagi pemetaan itu. Bincangkan juga mengenai kemungkinan kewujudan asas bagi set imej serta inti bagi kedua-dua pemetaan yang tak linear:*

(i) *Transformasi* $F:\mathbb{R}^3 \to \mathbb{R}^3$ *yang ditakrifkan sebagai* $F(x,y,z)=(x+|y|,\ y+|z|,\ z+|x|)$.

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(ii) *Transformasi* $F:\mathbb{R}^3 \to \mathbb{R}^2$ *yang ditakrifkan sebagai* $F(x,y,z)=(xy,z)$.

(iii) *Pemetaan matriks* $A:\mathbb{R}^4 \to \mathbb{R}^3$ *di mana* $A=\begin{bmatrix} 1 & 3 & 8 & 5 \\ 1 & 2 & -1 & 2 \\ 2 & 7 & 25 & 13 \end{bmatrix}$.

[60 markah]

6. Let S and S' be the bases of a finite dimensional vector space V, and let 1_v be the identity mapping on V. Show that the matrix representing 1_v relative to the bases S and S' is the inverse of the change-of-basis matrix from S to S'.

[40 marks]

6. Katakan S dan S' merupakan asas-asas bagi suatu ruang vektor berdimensi terhingga V, dan katakan 1_v adalah pemetaan identiti atas V. Tunjukkan bahawa matriks perwakilan bagi 1_v relatif kepada S dan S' merupakan matriks songsangan bagi matriks penukaran asas dari S ke S'.

[40 markah]

7. Let $a,b \in \mathbb{R}$ and $C[a,b]$ denote the vector space of all continuous real functions on the closed interval $[a,b]$.

(a) Define
   (i) the norm, $\|f\|$, of a function $f(t)$ in $C[a,b]$,
   (ii) the usual inner product, $\langle f,g\rangle$, for the functions $f(t)$ and $g(t)$ in $C[a,b]$, and
   (iii) orthogonality in $C[a,b]$.

(b) Given $f(t)=3t-5$ and $g(t)=t^2-t$ in $C[0,1]$.

   (i) Find $\|f\|$, $\|g\|$ and $\langle f,g\rangle$.
   (ii) Are f and g orthogonal in $C[0,1]$?
   (iii) Find a (non-zero) function in $C[0,1]$ that is orthogonal to $f+g$.

[60 marks]

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7. *Katakan* $a,b \in \mathbb{R}$ *dan* $C[a,b]$ *mewakili ruang vektor yang mengandungi semua fungsi nyata yang selanjar pada selang tertutup* $[a,b]$.

(a) *Takrifkan*

(i) *norma,* $\|f\|$, *bagi suatu fungsi* $f(t)$ *dalam* $C[a,b]$,

(ii) *hasil darab terkedalam biasa,* $\langle f,g\rangle$, *bagi fungsi-fungsi* $f(t)$ *dan* $g(t)$ *dalam* $C[a,b]$, *dan*

(iii) *keortogonan dalam* $C[a,b]$.

(b) *Diberikan* $f(t)=3t-5$ *dan* $g(t)=t^2-t$ *di dalam* $C[0,1]$.

(i) *Cari* $\|f\|, \|g\|$ *dan* $\langle f,g\rangle$.

(ii) *Adakah* f *dan* g *berortogon dalam* $C[0,1]$?

(iii) *Cari suatu fungsi (tak-sifar) dalam* $C[0,1]$ *yang berortogon dengan* $f+g$.

[60 markah]

8. Diagonalize the matrix

$$\begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

if possible.

[50 marks]

8. *Pepenjurukan matriks*

$$\begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

jika mungkin.

[50 markah]

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