UNIVERSITI SAINS MALAYSIA

Peperiksaan Kursus Semasa Cuti Panjang
Sidang Akademik 2006/2007

Jun 2007

MGM 501 – Analysis
[Analisis]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of FIVE pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi LIMA muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions: Answer **all eleven** [11] questions.

*[**Arahan:** Jawab **semua sebelas** [11] soalan.]*

...2/-

1. Let S be a nonempty bounded subset of $\mathbb{R}$, and a a real number. Define the set $a+S := \{a+s : s \in S\}$. Show that $\sup(a+S) = a + \sup S$.

[8 marks]

2. Construct a sequence (x_n) of rational numbers such that (x_n) converges to $\sqrt{5}$.

[8 marks]

3. Give (with proof) an example of a divergent sequence (x_n) for which the sequence $(x_{n+1} - x_n)_{n\in\mathbb{N}}$ is Cauchy.

[10 marks]

4. Prove that $\lim_{n\to\infty}(n/2^n) = 0$.

[8 marks]

5. Give, if possible, an example of a bounded sequence (x_n) which does not have a convergent subsequence. If it is not possible, then explain your reason.

[5 marks]

6. Let $f(x) = x^2 + 4x$. Find $\delta > 0$ such that $|f(x) - 5| < 1/10$ for all x where $0 < |x-1| < \delta$.

[5 marks]

7. Investigate the continuity of $f(x) = \frac{x}{1+|x|}$ on $\mathbb{R}$.

[8 marks]

8. Suppose $f : \mathbb{R} \to \mathbb{R}$ is defined by

$$f(x) = \begin{cases} 1 & \text{for } x \in \mathbb{Q}, \\ 0 & \text{for } x \notin \mathbb{Q}. \end{cases}$$

(a) Show that $\lim_{x\to 0} f(x)$ does not exist.

(b) Show that $\lim_{x\to 0} xf(x) = 0$.

[10 marks]

9. Suppose $f : \mathbb{R} \to \mathbb{R}$ is defined by

$$f(x) = \begin{cases} x & \text{for } x \geq 0, \\ 2x & \text{for } x < 0. \end{cases}$$

(a) Is f continuous on $\mathbb{R}$?

(b) Is f uniformly continuous on $\mathbb{R}$?

Justify your answers.

[10 marks]

...3/-

1. *Biar S subset terbatas tak kosong bagi* $\mathbb{R}$, *dan a nombor nyata. Takrifkan set* $a+S := \{a+s : s \in S\}$. *Tunjukkan bahawa* $\sup(a+S) = a + \sup S$.

[8 markah]

2. *Bina satu jujukan* (x_n) *nombor nisbah sedemikian* (x_n) *menumpu kepada* $\sqrt{5}$.

[8 markah]

3. *Beri (dengan bukti) satu contoh jujukan mencapah* (x_n) *sedemikian jujukan* $(x_{n+1} - x_n)_{n\in\mathbb{N}}$ *adalah Cauchy.*

[10 markah]

4. *Buktikan bahawa* $\lim_{n\to\infty}(n/2^n) = 0$.

[8 markah]

5. *Beri, jika boleh, satu contoh jujukan* (x_n) *yang tidak mempunyai subjujukan menumpu. Jika tidak boleh berbuat demikian, berikan alasan anda.*

[5 markah]

6. *Biar* $f(x) = x^2 + 4x$. *Cari* $\delta > 0$ *supaya* $|f(x) - 5| < 1/10$ *untuk semua x yang memenuhi* $0 < |x-1| < \delta$.

[5 markah]

7. *Selidik keselanjaran untuk* $f(x) = \frac{x}{1+|x|}$ *pada* $\mathbb{R}$.

[8 markah]

8. *Andaikan* $f : \mathbb{R} \to \mathbb{R}$ *ditakrif sebagai*

$$f(x) = \begin{cases} 1 & \text{untuk } x \in \mathbb{Q}, \\ 0 & \text{untuk } x \notin \mathbb{Q}. \end{cases}$$

(a) *Tunjukkan bahawa* $\lim_{x\to 0} f(x)$ *tidak wujud.*

(b) *Tunjukkan bahawa* $\lim_{x\to 0} xf(x) = 0$.

[10 markah]

9. *Andaikan* $f : \mathbb{R} \to \mathbb{R}$ *ditakrif sebagai*

$$f(x) = \begin{cases} x & \text{untuk } x \geq 0, \\ 2x & \text{untuk } x < 0. \end{cases}$$

(a) *Adakah f selanjar pada* $\mathbb{R}$?

(b) *Adakah f selanjar secara seragam pada* $\mathbb{R}$?

Berikan alasan untuk jawapan anda.

[10 markah]

...4/-