
UNIVERSITI SAINS MALAYSIA

Second Semester Examination
Academic Session 2006/2007

April 2007

MGM 501 – Analysis
[Analisis]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of FIVE pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi LIMA muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions: Answer **all eleven** [11] questions.

Arahan: Jawab **semua sebelas** [11] soalan.]

...2/-

1. Suppose S is a nonempty bounded subset of \mathbb{R} , and let B be the set of all lower bounds for S . Show that

(a) the set B is bounded above.

(b) $\sup B = \inf S$.

[8 marks]

2. (a) Give an example of a nonempty bounded subset A of \mathbb{Q} such that $\sup A \in \mathbb{Q}$. Verify/Prove your answer.

(b) Give an example of a nonempty bounded subset B of \mathbb{Q} such that $\sup B \notin \mathbb{Q}$. Verify/Prove your answer.

[10 marks]

3. Suppose (x_n) is a sequence.

(a) Write the definition of $\lim_{n \rightarrow \infty} x_n = x$.

(b) Let assume that $x_n \neq 0$ for all $n \in \mathbb{N}$, and that (x_n) converges to $x \neq 0$. Prove that the sequence $(1/x_n)$ converges to $1/x$.

[2+5=7 marks]

4. Is the sequence $(x_n) = \left(\sqrt{2}, \sqrt{2\sqrt{2}}, \sqrt{2\sqrt{2\sqrt{2}}}, \dots \right)$ convergent? Justify your answer. If you say yes, then find the limit.

[10 marks]

5. Prove that a sequence (x_n) is Cauchy if and only if it is convergent.

[8 marks]

6. Prove/Disprove: The sequence (x_n) given by $x_n = \sqrt{n+1} - \sqrt{n}$ for all n is a Cauchy sequence.

[10 marks]

.../3-

1. Andaikan S subset terbatas tak kosong bagi \mathbb{R} , dan biarkan B sebagai set bagi semua batas bawah untuk S . Tunjukkan bahawa
- (a) set B adalah terbatas dari atas.
- (b) $\sup B = \inf S$.
- [8 markah]
2. (a) Beri satu contoh subset terbatas tak kosong A bagi \mathbb{Q} sedemikian $\sup A \in \mathbb{Q}$. Jelaskan/Buktikan jawapan anda.
- (b) Beri satu contoh subset terbatas tak kosong B bagi \mathbb{Q} sedemikian $\sup B \in \mathbb{Q}$. Jelaskan/Buktikan jawapan anda.
- [10 markah]
3. Andaikan (x_n) ialah suatu jujukan.
- (a) Tuliskan takrif bagi $\lim_{n \rightarrow \infty} x_n = x$.
- (b) Andaikan $x_n \neq 0$ bagi semua $n \in \mathbb{N}$, dan (x_n) menumpu ke $x \neq 0$. Buktikan bahawa jujukan $(1/x_n)$ menumpu ke $1/x$.
- [2+5=7 markah]
4. Adakah jujukan $(x_n) = (\sqrt{2}, \sqrt{2\sqrt{2}}, \sqrt{2\sqrt{2\sqrt{2}}}, \dots)$ menumpu? Berikan alasan/bukti bagi jawapan anda. Jika ia adalah benar, cari hadnya.
- [10 markah]
5. Buktikan bahawa suatu jujukan (x_n) adalah Cauchy jika dan hanya jika ia menumpu.
- [8 markah]
6. Bukti/Sangkal: Jujukan (x_n) dengan $x_n = \sqrt{n+1} - \sqrt{n}$ bagi semua n adalah jujukan Cauchy.
- [10 markah]

7. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a function defined by

$$f(x) = \begin{cases} 0 & \text{if } x \in \mathbb{Q}, \\ x^2 & \text{otherwise.} \end{cases}$$

Is f continuous at 0? Give your answer with a justification/proof.

[7 marks]

8. Show that if $f: \mathbb{R} \rightarrow \mathbb{R}$ is a function such that $|f(x)| \leq |x|$ for all $x \in \mathbb{R}$, then f must be continuous at 0.

[5 marks]

9. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a function defined by $f(x) = 2x$ for x rational, and $f(x) = x + 3$ for x irrational. Find all points at which f is continuous. Prove your answer.

[8 marks]

10. Consider the function $f(x) = 1/x^2$ for $x \neq 0$. Answer each of the following questions and justify/prove your answer:

(a) Is f continuous uniformly on $(0,1)$?

(b) Is f continuous uniformly on $[1,10]$?

(c) Is f continuous uniformly on $[1,\infty)$?

[6+3+6=15 marks]

11. Let the function $f: (0,1] \rightarrow \mathbb{R}$ be uniformly continuous on $(0,1]$. Prove that $\lim_{x \rightarrow 0} f(x)$ exists.

[10 marks]

.../5-

7. Biarkan $f: \mathbb{R} \rightarrow \mathbb{R}$ suatu fungsi yang diberikan oleh

$$f(x) = \begin{cases} 0 & \text{if } x \in \mathbb{Q}, \\ x^2 & \text{selainnya.} \end{cases}$$

Adakah f selanjat pada 0? Berikan jawapan anda dengan alasan/bukti.

[7 markah]

8. Tunjukkan bahawa jika $f: \mathbb{R} \rightarrow \mathbb{R}$ ialah suatu fungsi sedemikian $|f(x)| \leq |x|$ bagi semua $x \in \mathbb{R}$, maka f mesti selanjat pada 0.

[5 markah]

9. Biarkan $f: \mathbb{R} \rightarrow \mathbb{R}$ suatu fungsi yang diberikan oleh $f(x) = 2x$ bagi x nisbah, dan $f(x) = x + 3$ bagi x tak nisbah. Cari semua titik di mana f adalah selanjat. Beri alasan/bukti untuk jawapan anda.

[8 markah]

10. Timbangkan fungsi $f(x) = 1/x^2$ bagi $x \neq 0$. Jawab setiap soalan yang berikut and beri penjelasan/bukti bagi jawapan anda:

(a) Adakah f selanjat secara seragam pada $(0,1)$?

(b) Adakah f selanjat secara seragam pada $[1,10]$?

(c) Adakah f selanjat secara seragam pada $[1,\infty)$?

[6+3+6=15 markah]

11. Biarkan fungsi $f: (0,1] \rightarrow \mathbb{R}$ selanjat secara seragam pada $(0,1]$. Buktikan bahawa $\lim_{x \rightarrow 0} f(x)$ wujud.

[10 markah]

- 000 O 000 -