UNIVERSITI SAINS MALAYSIA

Second Semester Examination
Academic Session 2006/2007

April 2007

MAT 516 – Curve and Surface Methods for CAGD
[Kaedah Lengkung dan Permukaan untuk RGBK]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of FIVE pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi LIMA muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions: Answer **all three** [3] questions.

*[**Arahan:** Jawab **semua tiga** [3] soalan.]*

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1. The normalized B-spline basis functions, $N_{i,k}(t)$ of order k are defined recursively by

$$N_{i,1}(t) = \begin{cases} 1 & , \text{ if } t \in [t_i, t_{i+1}) \\ 0 & , \text{ otherwise} \end{cases}$$

and $N_{i,k}(t) = \dfrac{t - t_i}{t_{i+k-1} - t_i} N_{i,k-1}(t) + \dfrac{t_{i+k} - t}{t_{i+k} - t_{i+1}} N_{i+1,k-1}(t)$, where $0 \le i \le n$ and $T = \{t_0, t_1, \cdots, t_{n+k}\}$ is a knot vector.

(a) Calculate and plot the basis functions, $N_{i,3}(t)$, $i = 0, 1, 2, 3,$ when $T = \{0,1,2,3,4,5,6,7\}$.

(b) A B-spline curve of order 3 is defined as $P(t) = \sum_{i=0}^{2} V_i N_{i,3}(t)$ where V_i are B-spline control points. Show that

$$P(u) = \frac{1}{2}\begin{bmatrix} u^2 & u & 1 \end{bmatrix} \begin{bmatrix} 1 & -2 & 1 \\ -2 & 2 & 0 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix} \text{ where } u = t - 2 \in [0,1].$$

(c) Show that two quadratic B-spline curves, $P(t) = \sum_{i=0}^{2} V_i N_{i,3}(t)$ and $Q(t) = \sum_{i=1}^{3} V_i N_{i,3}(t)$ are connected with C^1 continuity.

(d) Show that when $T = \{0,0,0,0,1,1,1,1\}$, $N_{i,4}(t) = B_{i,3}(t)$, for $t \in [0,1]$, $i = 0,1,2,3,$ where $B_{i,3}(t) = \dfrac{3!}{(3-i)!i!} t^i (1-t)^{3-i}$ are Bernstein polynomials of degree 3.

[100 marks]

2. (a) The algebraic form of a parametric cubic curve, $\boldsymbol{p}$, defined on the interval [0,1] can be written as $\boldsymbol{p}(t) = \boldsymbol{a}t^3 + \boldsymbol{b}t^2 + \boldsymbol{c}t + \boldsymbol{d}$, where $\boldsymbol{p}(t)$ is the position vector of any point on the curve and $\boldsymbol{a}$, $\boldsymbol{b}$, $\boldsymbol{c}$ and $\boldsymbol{d}$ are the vector equivalents of the scalar algebraic coefficients.

(i) Calculate the endpoints $\boldsymbol{p}(0)$ and $\boldsymbol{p}(1)$ and the corresponding tangent vectors $p'(0)$ and $p'(1)$. Give your answers in term of **a**, **b**, **c** and **d**.

(ii) Write **a**, **b**, **c** and **d** respectively in term of $p(0)$, $p(1)$, $p'(0)$ and $p'(1)$. Then, calculate the Hermite basis functions $F_1(t)$, $F_2(t)$, $F_3(t)$ and $F_4(t)$, such that $p(t) = p(0)F_1(t) + p'(0)F_2(t) + p'(1)F_3(t) + p(1)F_4(t)$.

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1. *Fungsi asas splin-B ternormal, $N_{i,k}(t)$, peringkat k ditakrif secara rekursi oleh*

$$N_{i,1}(t)=\begin{cases}1 \ , & jika\ t\in[t_i,t_{i+1}) \\ 0 \ , & selainnya\end{cases}$$

dan $N_{i,k}(t)=\dfrac{t-t_i}{t_{i+k-1}}N_{i,k-1}(t)+\dfrac{t_{i+k}-t}{t_{i+k}-t_{i+1}}N_{i+1,k-1}(t)$ *dengan* $0\le i\le n$ *dan* $T=\{t_0,t_1,\cdots,t_{n+k}\}$ *adalah vektor knot (simpulan).*

(a) *Kira dan lakar fungsi asas, $N_{i,3}(t)$, $i = 0, 1, 2, 3$, apabila* $T=\{0,1,2,3,4,5,6,7\}$.

(b) *Suatu lengkung splin-B peringkat 3 ditakrif oleh* $P(t)=\sum_{i=0}^{2}V_iN_{i,3}(t)$ *dengan V_i adalah titik kawalan splin-B. Tunjukkan bahawa*

$$P(u)=\frac{1}{2}\begin{bmatrix}u^2 & u & 1\end{bmatrix}\begin{bmatrix}1 & -2 & 1\\ -2 & 2 & 0\\ 1 & 1 & 0\end{bmatrix}\begin{bmatrix}V_0\\ V_1\\ V_2\end{bmatrix}\ \text{dengan}\ u=t-2\in[0,1].$$

(c) *Tunjukkan yang dua lengkung splin-B kuadratik,* $P(t)=\sum_{i=0}^{2}V_iN_{i,3}(t)$ *dan* $Q(t)=\sum_{i=1}^{3}V_iN_{i,3}(t)$ *adalah tersambung dengan keselanjaran C^1.*

(d) *Tunjukkan bahawa apabila* $T=\{0,0,0,0,1,1,1,1\}$, $N_{i,4}(t) = B_{i,3}(t)$, *bagi* $t\in[0,1], i = 0,1,2,3,$ *dengan* $B_{i,3}(t)=\dfrac{3!}{(3-i)!\,i!}t^i(1-t)^{3-i}$ *adalah polinomial Bernstein darjah 3.*

[100 markah]

2. (a) *Bentuk algebra suatu lengkung kubik berparameter, **p**, yang ditakrif pada selang [0,1] boleh ditulis sebagai* $\mathbf{p}(t)=\mathbf{a}t^3+\mathbf{b}t^2+\mathbf{c}t+\mathbf{d}$, *dengan **p**(t) sebagai vektor kedudukan sebarang titik di atas lengkung dan **a**, **b**, **c** dan **d** adalah kesetaraan vektor bagi pekali algebra skalar.*

(i) *Kira titik hujung, **p**(0) dan **p**(1) dan vektor tangen sepadan **p**'(0) dan **p**'(1). Nyatakan jawapan anda dalam sebutan **a**, **b**, **c** dan **d**.*

(ii) *Tulis **a**, **b**, **c** dan **d** masing-masingnya dalam sebutan **p**(0), **p**(1), **p**'(0) dan **p**'(1). Seterusnya, kira fungsi asas Hermite* $F_1(t)$, $F_2(t)$, $F_3(t)$ dan $F_4(t)$, supaya

$$p(t)=p(0)F_1(t)+p'(0)F_2(t)+p'(1)F_3(t)+p(1)F_4(t).$$

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(b) Show that $B^n_{i,j}(u,v) = uB^{n-1}_{i-1,j}(u,v) + vB^{n-1}_{i,j-1}(u,v) + (1-u-v)B^{n-1}_{i,j}(u,v)$

where $B^n_{i,j}(u,v) = \dfrac{n!}{i!\,j!(n-i-j)!}u^i v^i (1-u-v)^{n-i-j}$.

[100 marks]

3. (a) The bicubic B-spline patch is defined by 16 control points, $V_{ij}, i = 0,1,2,3; j = 0,1,2,3$ and is given by $P(u,v) = (\frac{1}{6})^2 UMCM^T V^T$

where $U = \begin{bmatrix} u^3 & u^2 & u & 1 \end{bmatrix}$, $V = \begin{bmatrix} v^3 & v^2 & v & 1 \end{bmatrix}$,

$$M = \begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 0 & 3 & 0 \\ 1 & 4 & 1 & 0 \end{bmatrix} \text{ and } C = \begin{bmatrix} V_{00} & V_{01} & V_{02} & V_{03} \\ V_{10} & V_{11} & V_{12} & V_{13} \\ V_{20} & V_{21} & V_{22} & V_{23} \\ V_{30} & V_{31} & V_{32} & V_{33} \end{bmatrix}.$$

Write its four corners $P(0,0), P(0,1), P(1,0)$ and $P(1,1)$ as a barycentric sum of nine control points.

(b) Given the four corner points *P(0,0) = (-1,-1,0), P(0,1) = (-1,1,0), P(1,0) = (1,-1,0)* and *P(1,1) = (1,1,1)* of a linear Coons surface *P(u,v),* calculate the four boundary curves:

(i) *P(0,v)*: the straight line from *P(0,0)* to *P(0,1)*,

(ii) *P(u,0)*: the uniform quadratic Lagrange polynomial with $\left(t_0 = 0, t_1 = \frac{1}{2}, t_2 = 1\right)$ 'determined by the interpolating points *P(0,0), P(1,0)* and *(0,-1,-0.5)*,

(iii) *P(u,1)*: the quadratic Bezier polynomial determined by the control points *P(0,1), P(1,1)* and *(0,1,0.5)*,

(iv) *P(1,v)*: the cubic Bezier polynomial determined by the control points *P(1,0), P(1,1), (1, -0.5, 0.5)* and *(1,0.5,-0.5)*.

Then, discuss the interpolation method to generate a linear Coons surface *P(u,v)* with these four boundary curves, for $0 \le u, v \le 1$.

[100 marks]

(b) *Tunjukkan bahawa* $B^n_{i,j}(u,v) = uB^{n-1}_{i-1,j}(u,v) + vB^{n-1}_{i,j-1}(u,v) + (1-u-v)B^{n-1}_{i,j}(u,v)$ *dengan* $B^n_{i,j}(u,v) = \dfrac{n!}{i!\,j!(n-i-j)!}u^i v^j (1-u-v)^{n-i-j}$.

[100 markah]

3. (a) *Tampalan splin-B bikubik ditakrifkan oleh 16 titik kawalan,* $V_{ij}, i = 0,1,2,3; j = 0,1,2,3$ *dan diberikan sebagai* $P(u,v) = (\frac{1}{6})^2 UMCM^T V^T$ *dengan* $U = \begin{bmatrix} u^3 & u^2 & u & 1 \end{bmatrix}$, $V = \begin{bmatrix} v^3 & v^2 & v & 1 \end{bmatrix}$,

$$M = \begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 0 & 3 & 0 \\ 1 & 4 & 1 & 0 \end{bmatrix} \text{ dan } C = \begin{bmatrix} V_{00} & V_{01} & V_{02} & V_{03} \\ V_{10} & V_{11} & V_{12} & V_{13} \\ V_{20} & V_{21} & V_{22} & V_{23} \\ V_{30} & V_{31} & V_{32} & V_{33} \end{bmatrix}.$$

Tulis empat bucunya $P(0,0), P(0,1), P(1,0)$ *dan* $P(1,1)$ *sebagai hasil tambah baripusat sembilan titik kawalan.*

(b) *Diberi empat titik penjuru P(0,0) = (-1,-1,0), P(0,1) = (-1,1,0), P(1,0) = (1,-1,0) dan P(1,1) = (1,1,1) suatu permukaan Coons linear P(u,v), kira empat lengkung sempadan:*

(i) *P(0,v): garis lurus dari titik P(0,0) ke titik P(0,1),*

(ii) *P(u,0): polinomial Lagrange kuadratik seragam dengan* $\left(t_0 = 0, t_1 = \frac{1}{2}, t_2 = 1\right)$ *yang ditentukan oleh titik-titik interpolasi P(0,0), P(1,0) dan(0,-1, -0.5),*

(iii) *P(u,1): polinomial Bezier kuadratik yang ditentukan oleh titik-titik kawalan P(0,1), P(1,1) dan (0,1,0.5),*

(iv) *P(1,v): polinomial Bezier kubik yang ditentukan oleh titik- titik kawalan P(1,0), P(1,1), (1, -0.5, 0.5) dan (1,0.5,-0.5).*

Seterusnya, bincang kaedah interpolasi untuk menjana permukaan Coons linear P(u,v) dengan empat lengkung sempadan tersebut, bagi $0 \le u, v \le 1$.

[100 markah]

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