UNIVERSITI SAINS MALAYSIA

Second Semester Examination
Academic Session 2005/2006

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MAT 516 – Curve and Surface Methods For CAGD
[Kaedah Lengkung dan Permukaan Untuk RGBK]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of FIVE pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi LIMA muka surat yang bercetak sebelum anda memulakan peperiksaan ini].

Instructions: Answer **all three** [3] questions.

[Arahan: *Jawab* ***semua tiga*** *[3] soalan].*

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1. The nth degree Bezier curve is defined as $P(t) = \sum_{i=0}^{n} V_i B_i^n(t), \quad 0 \le t \le 1$ with

$$B_i^n(t) = \frac{n!t^i(1-t)^{n-i}}{(n-i)!i!}$$ and V_i its Bezier control point.

(a) Show that

(i) $B_i^n(t) = \frac{n+1-i}{n+1} B_i^{n+1}(t) + \frac{i+1}{n+1} B_{i+1}^{n+1}(t).$

(ii) $(1-t)B_i^{n-1}(t) + tB_{i-1}^{n-1}(t) = B_i^n(t).$

(iii) $\frac{dB_i^n(t)}{dt} = n(B_{i-1}^{n-1}(t) - B_i^{n-1}(t)).$

(b) (i) Write an equation which represents a cubic Bezier curve in a matrix product form **TMV** where $\mathbf{T} = [t^3 \ t^2 \ t \ 1]$ and $\mathbf{V}^t = [V_0 \ V_1 \ V_2 \ V_3]$.

(ii) Discuss the parametric and geometric continuity of Bezier curves. Write the respective equations and sketch the position of the control points of two adjacent cubic Bezier curves which meet at the common control point with C^1, C^2, G^1 and G^2 continuity.

(iii) If a cubic Hermite curve is determined by the geometric coefficients $[V_0 \ V_1 \ V_0' \ V_1']$, write the equivalent Bezier curve control points. Sketch and show this relation in the same figure.

[100 marks]

1. Lengkung Bezier berdarjah n ditakrifkan oleh $P(t) = \sum_{i=0}^{n} V_i B_i^n(t), \quad 0 \le t \le 1$

dengan $B_i^n(t) = \frac{n!t^i(1-t)^{n-i}}{(n-i)!i!}$ *dan* V_i *sebagai titik kawalan Bezier.*

(a) Tunjukkan bahawa

(i) $B_i^n(t) = \frac{n+1-i}{n+1} B_i^{n+1}(t) + \frac{i+1}{n+1} B_{i+1}^{n+1}(t).$

(ii) $(1-t)B_i^{n-1}(t) + tB_{i-1}^{n-1}(t) = B_i^n(t).$

(iii) $\frac{dB_i^n(t)}{dt} = n(B_{i-1}^{n-1}(t) - B_i^{n-1}(t)).$

(b) (i) Tulis persamaan yang mewakili lengkung Bezier kubik dalam bentuk hasil darab matrik ***TMV*** *dengan* $\boldsymbol{T} = [t^3 \ t^2 \ t \ 1]$ *dan* $\boldsymbol{V}^t = [V_0 \ V_1 \ V_2 \ V_3]$.

.../3-

(ii) *Bincang keselanjaran berparameter dan keselanjaran geometri lengkung Bezier. Tulis persamaan dan lakarkan kedudukan titik-titik kawalan dua lengkung Bezier kubik bersebelahan yang masing-masingnya memenuhi keselanjaran C^1, C^2, G^1 and G^2 pada titik kawalan sepunya.*

(iii) *Jika lengkung Hermite kubik ditentukan oleh pekali geometri $[V_0\ V_1\ V_0'\ V_1']$, nyatakan hubungan titik kawalan lengkung Bezier yang setara. Lakar dan tunjukkan hubungan tersebut pada rajah yang sama.*

[100 markah]

2. (a) Write the Aitken algorithm to construct a polynomial $\mathbf{p}$ which satisfies the interpolatory constraints $\mathbf{p}(t_i) = \mathbf{p}_i$; $i = 0,\cdots,n$ starting from data points $\mathbf{p}_i = p_i^0$ with corresponding parameter values t_i. Arrange the points p_i^r; $r = 1,\cdots,n$; $i = 0,\cdots,n-r$ in a systolic array for the case $n = 3$.

(b) A quadratic rational Bezier curve is defined as $r(t) = \dfrac{\sum_{i=0}^{2} w_i p_i B_i^2(t)}{\sum_{i=0}^{2} w_i B_i^2(t)}$, with the control points, p_i and weights w_i, $i = 0,1,2$.

(i) Show that when $w_i = 1$, the curve reduces to just a quadratic Bezier curve.

(ii) Let the set of weights be $\{w_0 = 1, w_1, w_2 = 1\}$. Show that when $w_1 = 0$, the curve reduces to the straight line between p_0 and p_1. Subsequently, show that a point S = $r(0.5)$, which is called the shoulder point of the curve, moves along a straight line from $\dfrac{p_0 + p_2}{2}$ to p_1 when w_1 varies from 0 to ∞ (or equivalently when $\dfrac{w_1}{1+w_1}$ varies from 0 to 1).

(iii) Given the three control points p_0 = (1,0), p_1 = (0,0) and p_2 = (0,1), calculate the quadratic rational curve segment defined by these points whose shape is a circular arc spanning 90°. Sketch the curve together with the control polygon and control points in the same figure.

[100 marks]

2. *(a) Tulis algoritma Aitken bagi membina satu polinomial $\mathbf{p}$ yang memenuhi kekangan interpolasi $\mathbf{p}(t_i) = \mathbf{p}_i$; $i = 0,\cdots,n$ bermula dari titik data $\mathbf{p}_i = p_i^0$ dengan nilai parameter sepadan t_i. Susun titik-titik p_i^r; $r = 1,\cdots,n$; $i = 0,\cdots,n-r$ tersebut dalam tatasusunan sistol bagi kes $n = 3$.*

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(b) Lengkung Bezier nisbah kuadratik ditakrifkan sebagai $r(t) = \dfrac{\sum_{i=0}^{2} w_i p_i B_i^2(t)}{\sum_{i=0}^{2} w_i B_i^2(t)}$,

dengan p_i, *titik kawalan dan* w_i, *pemberat untuk* $i = 0,1,2$.

(i) Tunjukkan bahawa apabila $w_i = 1$, *lengkung tersebut terturun kepada hanya lengkung Bezier kuadratik.*

(ii) Andaikan set pemberat adalah $\{w_0 = 1, w_1, w_2 = 1\}$. *Tunjukkan bahawa apabila* $w_1 = 0$, *lengkung terturun kepada garis lurus di antara* p_0 *dan* p_1. *Seterusnya, tunjukkan bahawa titik* $S = r(0.5)$, *yang dipanggil titik bahu lengkung, bergerak sepanjang garis lurus dari* $\dfrac{p_0 + p_2}{2}$ *ke* p_1 *apabila* w_1 *berubah dari 0 ke* ∞ *(atau secara setara apabila* $\dfrac{w_1}{1+w_1}$ *berubah dari 0 ke 1).*

(iii) Diberi 3 titik kawalan $p_0 = (1,0)$, $p_1 = (0,0)$ *dan* $p_2 = (0,1)$, *kira tembereng lengkung nisbah kuadratik yang bentuknya adalah lengkok bulatan yang merentang* 90° *yang ditakrifkan oleh titik-titik tersebut. Lakar lengkung bersama-sama poligon dan titik kawalan pada rajah yang sama.*

[100 markah]

3. (a) Assume that a Bezier patch of degree (m x n) is defined by $\sum_{i=0}^{m}\sum_{j=0}^{n} V_{ij} B_i^m(u) B_j^n(v)$ where $0 \le u, v \le 1$ and V_{ij} are its Bezier control points.

(i) Show that this surface satisfies the convex hull property with respect to its control points.

(ii) State the entries of matrices M and W respectively, when the biquadratic Bezier patch (m = n = 2) is written as the matrix-product $UMWM^TV^T$, where $U = (1\ u\ u^2)$ and $V = (1\ v\ v^2)$.

(iii) When the degree of Bezier patch is increased to (m + 1) in the u direction, i.e. $\sum_{i=0}^{m+1}\sum_{j=0}^{n} \hat{V}_{ij} B_i^{m+1}(u) B_j^n(v) = \sum_{i=0}^{m}\sum_{j=0}^{n} V_{ij} B_i^m(u) B_j^n(v)$, show that $\hat{V}_{ij} = \dfrac{i}{m+1} V_{(i-1)j} + (1 - \dfrac{i}{m+1}) V_{ij}, i = 0,..,m+1; j = 0,..,n$. Then, state the relation between the new control points and V_{ij} when the degree is raised to (n + 1) in the v direction. As such, write in matrix form the relation between the new and the old control points if the patch of degree (m + 1) x (n + 1) is obtained from the previous patch of degree (m x n).

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(b) Discuss the interpolation method to generate a linear Coons surface with four boundary curves, $P_{u0} = (u, 0, \sin\pi u)$, $P_{u1} = (u, 1, \sin\pi u)$, $P_{0v} = (0, v, \sin\pi v)$ dan $P_{1v} = (1, v, \sin\pi v)$, for $0 \le u, v \le 1$.

[100 marks]

3. *(a) Andaikan tampalan Bezier darjah (m x n) ditakrifkan oleh*

$\sum_{i=0}^{m}\sum_{j=0}^{n} V_{ij} B_i^m(u) B_j^n(v)$ *dengan* $0 \le u, v \le 1$ *dan* V_{ij} *adalah titik kawalan Bezier.*

(i) Tunjukkan bahawa permukaan ini memenuhi sifat hul cembung terhadap titik-titik kawalannya.

(ii) Beri pemasukkan matriks M dan W masing-masingnya, apabila tampalan Bezier bikuadratik (m = n = 2) ditulis sebagai hasil darab matriks $UMWM^TV^T$, *dengan* $U = (1\ u\ u^2)$ *dan* $V = (1\ v\ v^2)$.

(iii) Apabila darjah tampalan Bezier ditingkatkan kepada (m + 1) dalam arah u, iaitu, $\sum_{i=0}^{m+1}\sum_{j=0}^{n} \hat{V}_{ij} B_i^{m+1}(u) B_j^n(v) = \sum_{i=0}^{m}\sum_{j=0}^{n} V_{ij} B_i^m(u) B_j^n(v)$, *tunjukkan bahawa* $\hat{V}_{ij} = \frac{i}{m+1} V_{(i-1)j} + (1 - \frac{i}{m+1}) V_{ij}, i = 0,..,m+1; j = 0,..,n.$

Seterusnya, nyatakan hubungan di antara titik kawalan baru dan V_{ij} *apabila darjah ditingkatkan kepada (n + 1) dalam arah v. Dengan demikian, tulis dalam bentuk matriks hubungan di antara titik kawalan baru dan lama apabila tampalan berdarjah (m + 1) x (n + 1) diperoleh daripada tampalan asal berdarjah (m x n).*

(b) Bincang kaedah interpolasi Coons untuk menjana permukaan linear Coons dengan 4 lengkung sempadan, $P_{u0} = (u, 0, \sin\pi u)$, $P_{u1} = (u, 1, \sin\pi u)$, $P_{0v} = (0, v, \sin\pi v)$ *dan* $P_{1v} = (1, v, \sin\pi v)$, *bagi* $0 \le u, v \le 1$.

[100 markah]

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