UNIVERSITI SAINS MALAYSIA

Peperiksaan Kursus Semasa Cuti Panjang
Sidang Akademik 2006/2007

Jun 2007

**MAT 202 – Introduction To Analysis**
***[Pengantar Analisis]***

Duration : 3 hours
*[Masa : 3 jam]*

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Please check that this examination paper consists of FIVE pages of printed material before you begin the examination.

*[Sila pastikan bahawa kertas peperiksaan ini mengandungi LIMA muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]*

**Instructions** : Answer **all three** [3] questions.

*[**Arahan** : Jawab **semua tiga** [3] soalan.]*

...2/-

1. (a) (i) Prove that if $S \subset \mathbb{R}$ has an infimum, then infimum $S$ is unique.

(ii) Find the supremum and infimum for $S = \left\{ \frac{n}{n+1} : n \in \mathbb{N} \right\}$.

[ $\mathbb{R}$ = set of real numbers, $\mathbb{N}$ = set of natural numbers]

(b) Prove that between any two distinct real numbers, there exists an irrational number. Next, given $u > 0$ and $x < y$, show that there exist an irrational number $a$ such that $x < au < y$.

(c) Let $S$ and $T$ be non-empty subsets of $\mathbb{R}$, where $T$ is a bounded set and $S \subset T$. Prove that $\sup S \leq \sup T$ and $\inf T \leq \inf S$.

(d) State the Archimedian Principle. Prove this principle.

(e) Define a countable set. If $A$ and $B$ are countable sets, then show that $A \times B$ is countable.

[100 marks]

2. (a) Use the definition to determine whether the sequence $\left\{ \frac{2^{n-1}}{2^n} \right\}$ is Canchy.

(b) For each $n \in \mathbb{N}$, let $I_n = [a_n, b_n]$ be a closed niterval on $\mathbb{R}$. If $I_n \supset I_{n+1}, \forall n \in \mathbb{N}$, prove that $\bigcap_{n=1}^{\infty} I_n \neq \phi$.

(c) Given $A$ is a subset of $\mathbb{R}$ and $A$ is compact. Show that $A$ is closed.

(d) Given a set $A = \left\{ \frac{n}{2n+1} : n \in \mathbb{N},\ n \leq 10^6 \right\}$. Find the interior points, accumulation/limit points and isslated points. Furthermore, determine whether $A$ is open or closed.

(e) Define compactness in terms of open covering. Give an example of a set with an open covering which has no finite sub-covering. Justify your answer.

[100 marks]

...3/-

1. (a) (i) *Buktikan bahawa jika $S \subset \mathbb{R}$ mempunyai infimum, maka infimum $S$ adalah unik.*

(ii) *Tentukan supremum dan infimum bagi* $S = \left\{ \frac{n}{n+1} : n \in \mathbb{N} \right\}$.

*[$\mathbb{R}$ = set nombor nyata, $\mathbb{N}$ = set nombor asli]*

(b) *Buktikan di antara dua nombor nyata yang berbeza, wujud suatu nombor tak nisbah. Diberi $u > 0$ dan $x < y$, tunjukkan bahawa wujudnya nombor $a$ supaya $x < au < y$.*

(c) *Biarkan $S$ dan $T$ adalah subset tak korong pada $\mathbb{R}$, dimana $T$ ialah set terbatas dan $S \subset T$. Buktikan* $\sup S \leq \sup T$ *dan* $\inf T \leq \inf S$.

(d) *Nyatakan Prinsip Archimedes. Buktikan prinsip ini.*

(e) *Takrifkan set terbilangkan. Jika $A$ dan $B$ adalah set terbilangkan, tunjukkan bahawa $A \times B$ terbilangkan.*

*[100 markah]*

2. (a) *Menggunakan takrifan, tentu bahawa jujukan* $\left\{ \frac{2^{n-1}}{2^n} \right\}$ *adalah Canchy.*

(b) *Untuk setiap $n \in \mathbb{N}$, biarkan $I_n = [a_n, b_n]$ adalah selang tertutup pada $\mathbb{R}$.*

*Jika $I_n \supset I_{n+1}, \forall n \in \mathbb{N}$, buktikan bahawa* $\bigcap_{n=1}^{\infty} I_n \neq \phi$.

(c) *Diberikan $A$ subset bagi $\mathbb{R}$ dan $A$ padat. Tunjukkan bahawa $A$ adalah tertutup.*

(d) *Diberi* $A = \left\{ \frac{n}{2n+1} : n \in \mathbb{N},\ n \leq 10^6 \right\}$. *Cari titik pedalaman, titik had dan titik terpencil bagi $A$. Seterusnya, tentukan samada $A$ terbuka atau tertutup.*

(e) *Takrifkan kepadatan set dalam sebutan tudung terbuka. Beri satu contoh tudung terbuka yang tidak mempunyai subtudung terhingga. Jelaskan jawapan anda.*

*[100 markah]*