

**STRUCTURAL MODELLING AND  
ANALYSIS OF THE BEHAVIOURAL  
DYNAMICS OF FOREIGN  
EXCHANGE RATE**

By

**YIP CHEE YIN**

June 2006

Thesis submitted in fulfilment of the  
Requirements for the degree of  
Doctor of Philosophy

## **ACKNOWLEDGMENTS**

I wish to thank my dissertation supervisor, Professor Quah Soon Hoe, to whom I am in great debt for his general guidance in this research and in all my graduate studies. His advice and support have been invaluable to me. I would also like to extend a big thank you to Professor James Hamilton of the University of California, San Diego, for supplying me with materials, data and advice.

Last but not least, I would like to thank the School of Mathematical Sciences, and also all the staff, lecturers and professors alike for their help one way or another.

I also would like to express my deep gratitude to my family for their unconditional love and support. Their encouragement was undoubtedly priceless. Lastly, I would like to thank my wife, Mee Yen, for her endless support at all times and in all situations. This dissertation is dedicated to her and my two children, Seng Jon and Seng Joe.

# CONTENTS

	<b>Page</b>
<b>Title Page</b>	i
<b>Acknowledgements</b>	ii
<b>Contents</b>	iii
<b>List of Figures</b>	ix
<b>List of Tables</b>	xii
<b>List of Abbreviations and Symbols</b>	xx
<b>Abstrak</b>	xxii
<b>Abstract</b>	xxv
<b>1 Introduction</b>	1
1.1 Outline	1
1.2 Objectives of the Research	6
1.3 Constraints of the Research	7
1.4 Methodology	7
1.5 The Data and Software	9
1.6 Three Different Forms of the Data	12
1.6.1 The original data	12
1.6.2 The log data	13
1.6.3 The returns series	14
1.7 Outlines of Subsequent Chapters	14
<b>2 Theoretical Background</b>	19
2.1 Introduction	19
2.2 Independent and Serially Correlated Data	19
2.3 Linear Regression	26
2.4 Parametric Models	28
2.4.1 White noise	29
2.4.2 OLS estimation and its related problems	30
2.4.3 Recursive (OLS) estimation	31
2.5 The ARMA Model and its Derivatives	33
2.6 The Autocorrelation Function and Partial Autocorrelation Function	35

2.6.1	The autocorrelation function ACF	35
2.6.2	Conditions for hypothesis testing	36
2.6.3	The partial autocorrelation function PACF	37
2.7	The $t$ , Wald, $F$ , $R^2$ Tests and Granger Causality Test	39
2.7.1	The $t$ tests	39
2.7.2	The Wald test	41
2.7.3	The $F$ tests	43
2.7.4	The $R^2$ test	44
2.7.5	The Granger causality tests	45
2.8	Random Walk, Unit Root Processes and Structural Modelling	46
2.8.1	The unit root tests	50
2.8.2	ADF unit root test	51
2.9	Structural Modelling of Time Series	54
2.9.1	Model selection and criterion for forecasting	54
2.9.2	Out-of-sample forecasting	57
2.9.3	Criterion for selecting best forecasting model	61
2.10	Structural Breaks and Outliers	64
2.10.1	Testing for structural change of unknown timing	65
2.10.2	Detection of outliers	66
2.11	Real Time or Revised Data	66
2.12	Conclusion	67
<b>3</b>	<b>Methodology</b>	<b>69</b>
3.1	Introduction	69
3.2	The ARFIMA Model	69
3.3	Estimation of the Parameter $d$	72
3.3.1	The spectral regression method	72
3.3.2	The Gaussian semiparametric method	74
3.4	Reasons for Using the Kalman Filter Method	75
3.5	Theoretical Background of the Kalman Filter Based Method	76
3.5.1	The state space representation system	77
3.5.2	Formulation of the Kalman filter	80
3.5.3	The Kalman filter with Normal Disturbances	83

3.5.4	Forecasting $y$ with the Kalman filter	88
3.6	Conclusion	90
<b>4</b>	<b>Literature Review</b>	91
4.1	Introduction	91
4.2	Jargons of Foreign Exchange Rates	92
4.2.1	Transitory and permanent components	92
4.2.2	Stationary process	93
4.2.3	Purchasing power parity (PPP)	94
4.3	Should there be a Permanent Component in Exchange Rates	95
4.3.1	Random walk	96
4.3.2	Mean reverting behaviour	97
4.3.3	Celebrated article “PPP Puzzle” by Rogoff , 1996	100
4.3.4	Reconcile PPP puzzle	100
4.4	Fractionally Integrated or Pure Unit Root Processes	103
4.5	Summary	104
<b>5</b>	<b>Mean Reversion and Persistent Behaviour of Exchange Rate</b>	105
5.1	Introduction	105
5.2	Unit Root Tests	107
5.2.1	Unit root analysis	115
5.2.2	The empirical results	116
5.3	Fractional Integrated Dynamics of Exchange Rate	118
5.4	Impulse Response Function Analysis	122
5.5	Conclusion	127
<b>6</b>	<b>Fractional Dynamics in Foreign Exchange Rate</b>	129
6.1	Introduction	129
6.2	Exploratory Evidence of Long Term Memory	131
6.3	The ARFIMA Model	145
6.4	The ARFIMA Model of Britpus	151
6.5	Stability Analysis of the YQ Specified ARFIMA Model	156
6.6	Testing the External Validity of the YQ-ARFIMA Model	163

6.6.1	Exploratory investigation of exchange rates for Thailand, Malaysia, Singapore and Hong Kong	160
6.6.2	Modelling the exchange rates of Thailand, Malaysia, Singapore and Hong Kong	166
6.7	Confirmation of the Validity of LBritpus in Prediction	178
6.8	Conclusion	180
<b>7</b>	<b>Kalman Filtering Dynamics of Foreign Exchange Rates</b>	<b>181</b>
7.1	Introduction	181
7.2	Structural Time Series Model Based on Kalman Filter	182
7.3	The Data and Empirical Analysis	185
7.4	Single Equation Dynamic Modelling of Britpus	192
7.5	The Forecast Results	193
7.6	Testing the Robustness of KFBM Model	194
7.7	Testing the External Validity of the KFBM Model	197
7.7.1	Structural modelling of the Malaysian Ringgit	199
7.7.2	Analysis of the structural modelling results	199
7.8	Conclusion	202
<b>8</b>	<b>Structural Models: YQ- ARFIMA versus KFBM</b>	<b>204</b>
8.1	Introduction	204
8.2	Short Term Memory Dynamics	205
8.3	YQ-ARFIMA Modelling of Dtchgus	206
8.4	KFBM Dynamic Modelling of Dtchgus	209
8.5	Granger Causality Test	211
8.6	Conclusion	213
<b>9</b>	<b>External Validity of the YQ-ARFIMA model</b>	<b>214</b>
9.1	Introduction	214
9.2	Exploratory Investigation	215
9.2.1	Unit root tests	219
9.3	The Standard ARFIMA Model and the YQ-ARFIMA Model	221
9.4	Comparing the Experimental Results of ARFIMA and YQ-ARFIMA	221

9.5	Conclusion	228
<b>10</b>	<b>YQ-ARFIMA Model versus Random Walk Model</b>	<b>229</b>
10.1	Introduction	229
10.2	The Simple Random Walk Model	230
10.3	The Empirical Experiment	231
10.4	YQ-ARFIMA as a Tool to Show Mean Reversion Behaviour	238
10.5	Conclusion	240
<b>11</b>	<b>The Effects of Structural Breaks and Outliers on Forecasting</b>	<b>241</b>
11.1	Introduction	241
11.2	Structural Breaks	242
11.3	The structural Break Model	243
11.3.1	The partial sum structural break model	244
11.4	Estimation of Break Points and Number of Breaks	248
11.5	The Data and Empirical Analysis of Structural Breaks	251
11.6	The Influence of the Structural Breaks on the Predictive Power of a Model	257
11.7	Outliers	261
11.7.1	Empirical estimation of outliers	262
11.8	The Data and Empirical Estimation of Outliers	264.
11.9	The Outlier Adjusted Equation Dynamic Specification Modelling	267
11.10	Conclusion	271
<b>12</b>	<b>The Relationship between Consumption Rate and Exchange Rate</b>	<b>272</b>
12.1	Introduction	272
12.2	Model for Lconsumption	273
12.3	Estimation of the Permanent Components of LQBritpus and Lcons.	278
12.4	Hypothesis Tests	281
12.4.1	The data and empirical analysis	282
12.5	The Structural Breakdowns of SeALconsumption	289
12.6	Granger Causality Test on the Validity of Lconsumption and Trend Cycles of Lconsumption	290
12.7	Conclusion	296

<b>13</b>	<b>The Explanatory Power of the Cyclical Components of Lconsumption and LQBritpus</b>	298
13.1	Introduction	298
13.2	Bivariate Structural Time Series Modelling of LQBritpus and Lconsumption	299
13.3	Single Equation Dynamic Modelling of the Cyclical Component of LQBritpus	303
13.4	Single Equation Dynamic Modelling of QBritpus with LQBritpus and Cyclical Components of Lconsumption as Regressors	312
13.5	Granger Causality Tests on the Validity of the Cyclical Components of Lconsumption and LQBritpus	315
13.6	Conclusion	319
<b>14</b>	<b>Conclusion</b>	321
14.1	Introduction	321
14.2	The GARCH Methodology	323
14.3	Direction of Future Research	324
14.4	Conclusion	324
	<b>Bibliography</b>	329
	<b>Appendix A - Proof for the Validity of the YQ-ARFIMA Model</b>	338
	<b>Appendix B - Proof for the Proposition in Section 11.3</b>	344
	<b>List of Publications</b>	349
	<b>List of Conferences attended</b>	350

## List of Figures

<b>Figure</b>	<b>Title</b>	<b>Page</b>
1.1	Graph of British pound per US dollar exchange rate	2
1.2	Exchange rate returns occur in clusters	3
1.3	Fat tails and shape peak of the density graph of Britpus	4
2.1	Variation pattern of serially correlated data- GDP in US	23
2.2	Stochastic and linear trends	48
2.3	Stochastic trend only	48
5.1	Impulse response function graphs for the exchange rates of Denmark, UK, Sweden, Europe, Switzerland, Canada, Australia and Japan	123
5.2	Impulse response graphs for South Korea, Hong Kong, Singapore and Malaysia currencies	125
5.3	Impulse response function graphs for seven third world countries	126
6.1	ACF graphs for eight exchange rate series	132
6.2	PACF graphs for eight exchange rate series	132
6.3(a)	Correlogram and structural break graphs for Austrus	133
6.3(b)	Correlogram and structural break graphs for Britpus	134
6.3(c)	Correlogram and structural break graphs for Cdndlus	134
6.3(d)	Correlogram and structural break graphs for Dtchgus	135
6.3(e)	Correlogram and structural break graphs for Frnfrus	135
6.3(f)	Correlogram and structural break graphs for Germdus	136
6.3(g)	Correlogram and structural break graphs for Japynus	136
6.3(h)	Correlogram and structural break graphs for Swisfus	137
6.4	Comparison of the eight exchange rates	147

6.5	Static long run relationship among the European exchange rates	148
6.6	Static long relationship between Britpus and the European exchange rates	148
6.7	Geographical influence on exchange rates	149
6.8	Forecasting graphs of Britpus for the best 5 specifications	156
6.9(i)	Forecasting graphs with YQ-ARFIMA model from (a) through to (d)	158
6.9(ii)	Forecast graphs with YQ-ARFIMA model from (e) through to (h)	158
6.10(i)	Forecast graphs of Britpus for different sample sizes with Britpus as a regressor for (a) through to (d)	161
6.10(ii)	Forecast graphs of Britpus for different sample sizes with Britpus as a regressor for (e) through to (h)	162
6.10(iii)	Forecast graphs of Britpus for different sample sizes with Britpus as a regressor for (i) through to (l)	162
6.11	Line graphs for exchange rates from Thailand, Malaysia, Singapore and Hong Kong	164
7.1	Outliers in the exchange rate, Britpus	186
7.2(a)	Spectrum graphs for Britpus	186
7.2(b)	Spectrum graphs for Britpus	187
7.3	The eight forecasting graphs for Britpus	195
7.4	Graphical analysis of the Ringgit behaviour	198
7.5	Part of the graphs used to estimate the periods of the 3 cycles	200
8.1	ARFIMA(1,d,0) modelling of Dtchgus	208
8.2	Forecast graph of Dtchgus with sample size 800	210
9.1	Line graphs for Euro, Philippine Pesos, Taiwan Dollar, and Sdr	215
9.2	Correlograms and density graphs for Euro	216
9.3	Correlograms and density graphs for Taiwan Dollar	217
9.4	Correlograms and density graphs for Philippine Pesos	217
9.5	Correlograms and density graphs for Sdr	218

11.1	Weekly returns, DLWeBritpus and its cubic spline	252
11.2	Structural break analysis of DLWeBritpus by using model (h) in Table 11.1	255
11.3	<i>RSS</i> graphs for confirmation of structural breaks	257
11.4	1-step residual graph for detecting outliers	264
11.5	Dissection analysis of outliers by graphical method	266
12.1	Graphical analysis of Lconsumption	274
12.2(a)	Parallel movement of LQBritpus and Lconsumption	283
12.2(a)	Parallel movement of LQBritpus and SeALconsumption	284
13.1	Cyclical component graphs for LQBritpus and Lconsumption	297
13.2	Comovement of LQBritpus and Lconsumption graphs	309
13.3(a)	Possible movement of cycle 3 of LQBritpus and Lconsumption	310
13.3(b)	Movement of the difference series of DC3LQBritpus and the difference series of DC3Lconsumption	310
13.4	Graphics for ARFIMA(2,d,4) modelling of QBritpus	314

## List of Tables

Table	Title	Page
1.1	Names of the 22 exchange rates used in the experiment	11
1.2	Names of the 6 exchange rates (Ringgit per foreign currency) used for the external validity experiment	11
3.1	Implication of the values of the parameter $d$	72
5.1(a)(i)	Test statistics of a unit root with intercept and time trend for forex in 5 small subsamples	110
5.1(a)(ii)	Test statistics of a unit root with time trend and intercept for forex in 5 small subsamples	111
5.1(b)	Test statistics of a unit root for forex in 5 large subsamples	112
5.2(a)(i)	KPSS test statistics of no unit root for forex in the 5 small subsamples	113
5.2(a)(ii)	KPSS Test statistics of no unit root for forex in the 5 small subsamples	114
5.2(b)	KPSS Test statistics of no unit root with time trend and intercept for forex in the 5 large subsamples	115
5.3(a)	Values of $d$ for large subsamples	120
5.3(b)	Values of $d$ for small subsamples	121
5.4	Half-life for the exchange rates of eight advanced countries	124
5.5	Half-life for the exchange rates of four advance developing countries	125
5.6	Half-life for the exchange rates of seven third world countries	124
6.1	Group statistics for the eight forex series	138
6.2	ADF unit root tests for 13 subsamples of each of the 8 forex series	139
6.3	Results of ADF unit root tests on the whole sample with structural breaks	140
6.4	Results of ADF unit root tests on the whole sample without structural breaks	141
6.5	Results of ADF tests on the first difference series	142

6.6	Results of ADF tests on the second difference series	142
6.7	Stationarity tests for the first difference series	143
6.8	MLE estimated maximum values of $d$	151
6.9(a)	Various specifications of ARFIMA for modelling Britpus	152
6.9(b)	The best 5 specifications of the ARFIMA model	153
6.10	Comparison of the predictive ability of the best 5 ARFMA specifications	153
6.11	Comparison of the predictive performance of the dynamic YQ-specified the ARFIMA( $p,d,q$ ) model with log series as fixed regressor and $p, q$ vary according to the series	157
6.12	Application of YQ specified ARFIMA to different sample sizes	161
6.13	Comparative study of the predictive ability of the standard ARFIMA and YQ-ARFIMA by using the loss function RMSE for the Thailand Baht. Values from YQ-ARFIMA are inside the [ ]	168
6.14	Comparative study of the predictive ability of the standard ARFIMA and YQ-ARFIMA by using the loss function MAPE for the Thailand Baht. Values from YQ-ARFIMA are inside the [ ]	169
6.15	Comparative study of the predictive ability of the standard ARFIMA and YQ-ARFIMA by using the loss function RMSE for the Malaysia Ringgit. Values from YQ-ARFIMA are inside the [ ]	170
6.16	Comparative study of the predictive ability of the standard ARFIMA and YQ-ARFIMA by using the loss function MAPE for the Malaysia Ringgit. Values from YQ-ARFIMA are inside the [ ]	171
6.17	Comparative study of the predictive ability of the standard ARFIMA and YQ-ARFIMA by using the loss function RMSE for the Singapore Dollar. Values from YQ-ARFIMA are inside the [ ]	172
6.18	Comparative study of the predictive ability of the standard ARFIMA and YQ-ARFIMA by using the loss function MAPE for the Singapore Dollar. Values from YQ-ARFIMA are inside the [ ]	173
6.19	Comparative study of the predictive ability of the standard ARFIMA and YQ-ARFIMA by using the loss function RMSE for the Hong Kong Dollar. Values from YQ-ARFIMA are inside the [ ]	174
6.20	Comparative study of the predictive ability of the standard ARFIMA and YQ-ARFIMA by using the loss function MAPE for the	

	Hong Kong Dollar. Values from YQ-ARFIMA are inside the [ ]	175
6.21	Comparing 8 period forecasting ability of YQ-ARFIMA and ARFIMA in terms of RMSE values for the case of different specifications.	176
6.22	Comparing 100 period forecasting ability of YQ-ARFIMA and ARFIMA in terms of RMSE values for the case of different specifications.	177
6.23	Granger causality test for the validity of LBritpus	179
7.1	Comparison of the four specifications of the structural model for Britpus by using KBSM modelling	188
7.2	Variances of the parameter disturbances	189
7.3	Coefficients of the parameters of state vector	190
7.4	Regressing Britpus on stochastic trend and 3 cycles components	192
7.5	Testing The Robustness Of Kalman Filter Based Model across 8 Exchange Rates	194
7.6	Testing the robustness of Kalman Filter Based Model across different sample size for the exchange rate series Britpus	196
7.7	Unit root tests on Malaysian ringgit	199
7.8	Standard deviations of the various disturbances of the components	200
7.9	Estimated coefficients of the final state vectors	201
7.10	Predictive ability of KFBM modelling of Malaysian ringgit	202
8.1	Comparing the predictive ability of structural models	204
8.2	Parameter $d$ values	206
8.3	ARFIMA( $p,d,q$ ) modelling of Dtchgus with different sample sizes with regressors, LDtchgus and constant	207
8.4	Kalman filter based method of modelling of Dtchgus with different sample sizes with regressors LDtchgus and constant	209
8.5	Comparison of the best ARFIMA model with that of the Kalman filter based model	210
8.6	Granger causality test for the validity of LDtchgus	212

9.1	Unit root tests on Euro, and Taiwan Dollar.	219
9.2	Unit root tests on Philippine Pesos and Sdr	220
9.3	Comparison of the RMSE values for the case of Euro, Taiwan Dollar, Philippine Pesos and Sdr by using the standard ARFIMA model	222
9.4	Comparison of the MAPE values for the case of Euro, Taiwan Dollar, Philippine Pesos and Sdr by using the standard ARFIMA model	223
9.5	Comparison of the RMSE values for the case of Euro, Taiwan Dollar, Philippine Pesos and Sdr by using the YQ- ARFIMA model	224
9.6	Comparison of the MAPE values for the case of Euro, Taiwan Dollar, Philippine Pesos and Sdr by using the YQ-ARFIMA model.	225
9.7	Comparison of the mean RMSE values of the standard ARFIMA and the YQ-ARFIMA	226
9.8	Comparison of the mean MAPE values of the standard ARFIMA and the YQ-ARFIMA	227
10.1(a)	Comparison of the RMSE values for the case of Euro by using the YQ-ARFIMA model and the Random Walk.	232
10.1(b)	Comparison of the MAPE values for the case of Euro by using the YQ-ARFIMA model and the Random Walk.	232
10.2(a)	Comparison of the RMSE values for the case of Canadian Dollar by using the YQ-ARFIMA model and the Random Walk.	233
10.2(b)	Comparison of the MAPE values for the case of Canadian Dollar by using the YQ-ARFIMA model and the Random Walk.	233
10.3(a)	Comparison of the RMSE values for the case of Brazilian Real by using the YQ- ARFIMA model and the Random Walk.	234
10.3(b)	Comparison of the MAPE values for the case of Brazilian Real by using the YQ- ARFIMA model and the Random Walk.	234
10.4(a)	Comparison of the RMSE values for the case of South African Rand by using the YQ- ARFIMA model and the Random Walk.	235
10.4(b)	Comparison of the MAPE values for the case of South African Rand by using the YQ- ARFIMA model and the Random Walk.	235
10.5(a)	Comparison of the RMSE values for the case of Chinese Yuan by	

	using the YQ- ARFIMA model and the Random Walk	236
10.5(b)	Comparison of the MAPE values for the case of Chinese Yuan by using the YQ- ARFIMA model and the Random Walk	236
10.6(a)	Comparison of the RMSE values for the case of Malaysia Ringgit by using the YQ- ARFIMA model and the Random Walk	237
10.6(b)	Comparison of the MAPE values for the case of Malaysia Ringgit by using the YQ- ARFIMA model and the Random Walk	237
10.7(a)	Comparison of the RMSE values for the case of British pound per unit US Dollar by using the YQ- ARFIMA model and the Random Walk.	239
10.7(b)	Comparison of the MAPE values for the case of British pound per unit US Dollar by using the YQ- ARFIMA model and the Random Walk.	239
11.1	Comparison of predictive power of model KFBM and structural breaks	254
11.2	Breakdate and the number of breaks	256
11.3	Influence of structural breaks on the predictive power of model	259
11.4	Mean values of RMSE and MAPE when structural breaks are excluded	259
11.5	Mean values of RMSE and MAPE when structural breaks are included	260
11.6	Outliers from manual grid search	267
11.7	Comparing the predictive ability with and without adjustment of outliers	270
12.1	ADF unit root test for Lconsumption	275
12.2(a)	Long run relationship between LQBritpus on Lconsumption	283
12.2(b)	Results of the ADF unit root tests	285
12.3	Long run relationship between SeALconsumption and LQBritpus	287
12.4	Regressing trendcycle 3 of LQBritpus on trendcycle 3 of SeALconsumption	287
12.5	Regressing trendcycle 2 of LQBritpus on trendcycle 2 of SeALconsumption	288
12.6	Regressing trendcycle 1 of LQBritpus on trendcycle 1 of SeALconsumption	288

12.7	Decomposition of SeALconsumption	289
12.8(a)	Granger causality test for the validity of Lconsumption	292
12.8(b)	Granger causality test for the validity of LQBritpus	292
12.9(a)	Granger causality test for the validity of trend cycle 2 of Lconsumption	293
12.9(a)	Granger causality test for the validity of trend cycle 1 of LQBritpus	293
12.10(a)	Granger causality test for the validity of trend cycle 3 of Lconsumption	294
12.10(b)	Granger causality test for the validity of trend cycle 3 of Lconsumption	294
12.11(a)	Granger causality test for the validity of SeALconsumption	295
12.11(b)	Granger causality test for the validity of LQBritpus	295
13.1	Regression of cyclic2 of LQBritpus on Cycle 2 & 3 of Lconsumption	304
13.2	Regression of cyclic3 of LQBritpus on Cycle 2 & 3 of Lconsumption	304
13.3	Long run relationship 1	305
13.4	Long run relationship 2	306
13.5(a)	ADF tests for Cycle2 of LQBritpus	307
13.5(b)	ADF tests for Cycle2 of Lconsumption	307
13.5(c)	ADF tests for residuals from regression 1	308
13.6	Regressing Cycle 2 LQBritpus on Cycle 2 Lconsumption	308
13.7	Long run relation between cycle 2 of LQBritpus on its cycle 2 component of Lconsumption	303
13.8	Failed linear regression	311
13.9	Nonlinear regression of Cycle3 LQBritpus on Cycle3 Lconsumption	311
13.10	NLS estimation of regression DC3LQBritpus on DC3Lconsumption	312

13.11	Output for checking the explanatory power of cyclic regressors	315
13.12	Granger causality test for the validity of Cycle2 of Lconsumption and LQBritpus	316
13.13	Granger causality test for the validity of Cycle3 of Lconsumption and LQBritpus	317
13.14	Granger causality test for the validity of DC3Lconsumption and DC3LQBritpus	318

## List of Abbreviations and Symbols

With respect to notations and abbreviations, we have the following rules:

- 1 Britpus denotes the daily exchange rates series for British Pound.
- 2 LBritpus denotes the log daily exchange rates series.
- 3 DLBritpus denotes the daily exchange rates returns series
- 1 WeBritpus denotes the weekly exchange rates series
- 2 LWeBritpus denotes the log weekly exchange rates series
- 3 DLWeBritpus denotes the weekly exchange rates returns series
- 4 QBritpus denotes the quarterly exchange rates series.
- 5 LQBritpus denotes log the quarterly exchange rates series
- 6 Lconsumption denotes the log quarterly consumption series.
- 7 Bold capital letter stands for matrix and small letter for vector.
- 8 Without bold means that the letter represents scalar variable.
- 9 Small letter  $t$  stands for one particular period, capital  $N$  stands for total number of periods or observations.
- 10 Lcons stands for Lconsumption.
- 11 Cy or  $C$  stands for cycle.
- 12 RMSE denotes root mean square error
- 13 MAPE denotes mean absolute prediction error
- 14 AR denotes autoregression
- 15 AR\_1 denotes autoregressive model of order 1
- 16 MA denotes moving average
- 17 MA\_1 denotes moving average model of order 1

- 18 ARMA denotes autoregressive moving average
- 19 ARIMA denotes autoregressive integrated moving average
- 20 ARFIMA denotes autoregressive fractionally integrated moving average
- 21 D denotes the first difference and DD denotes the second difference
- 22 Dm denotes Denmark
- 23 Sn denotes Sweden
- 24 Sz denotes Switzerland
- 25 Sin denotes Singapore
- 26 HK denotes Hong Kong
- 27 Frnfrus denotes French Francs
- 28 Dtchgus denotes Dutch Guilder
- 29 Austrus denotes Australian Dollar
- 30 Euro denotes European Dollar
- 31 Jappynus denotes Japanese Yen
- 32 Germdus denotes German Deutschmark
- 33 Swisfus denotes Switzerland Francs
- 34 Sgd denotes Singapore Dollar
- 35 Twd denotes Taiwan Dollar
- 36 Php denotes Philippine Pesos
- 37 Sdr denotes Currency based on the country's reserve.

# PEMODELAN STRUKTUR DAN ANALISIS TINGKAHLAKU DINAMIK BAGI KADAR PERTUKARAN WANG ASING

## ABSTRAK

Tesis ini berkaitan dengan Kadar Wang Pertukaran Asing (KWPA) yang dihasilkan oleh satu regime urusniaga bebas. Pada amnya, kita mengkaji Pemodelan Struktur dan Analisis Tingkahlaku Dinamik Kadar Pertukaran Wang Asing. Kita bermula dengan menganggap KWPA sebagai satu siri kewangan. Kita memodelkan KWPA dengan menggunakan dua jenis metodologi: kaedah ARFIMA dan kaedah Penapis Kalman (KFBM) dengan harapan ia boleh menghasilkan satu model KWPA yang terbaik. Objektif kita ialah untuk menggunakan model yang terbaik ini sebagai satu alat bagi menunjukkan keadaan stabil bagi kadar pertukaran wang asing dalam jangka masa yang panjang. Dalam proses membuat analisis, kita juga hendak mengkaji tingkahlaku dinamik KWPA.

Kita telah membina satu model YQ- ARFIMA yang dinamik and sesuai untuk pemodelan siri ingatan yang panjang dan pendek. Dinamik bermaksud bahawa parameters YQ- AFRIMA boleh diubahkan dengan mengikut ciri-ciri siri masa yang berkenaan. Perubahan yang mudah ini boleh dilakukan dengan menggunakan ujian  $t$  secara berturut-turut. Model YQ-ARFIMA ini didapati adalah lebih baik daripada model KFBM. Kita juga dapati YQ-ARFIMA menunjukkan prestasi 12 kali lebih baik daripada KFBM dalam pemodelan struktur siri masa dalam jangkamasa panjang. Dalam jangkamasa pendek, YQ-AFRIMA menunjukkan prestasi yang lebih baik lagi dengan suatu nilai RMSE, lebih kurang 40 kali dan nilai MAPE, lebih kurang 36 kali lebih tepat daripada kaeah KFBM. Sungguh pun begitu, model KFBM didapati lebih baik dalam analisis perubahan (*breaks*) struktur. Satu

sebab ialah ia lebih sesuai untuk memodelkan siri yang mempunyai komponen berkitaran (*cyclical components*). Saiz sampel yang paling baik untuk pemodelan jangka masa pendek ialah di antara 200 dan 1300 pemerhatian sementara bagi jangka masa panjang saiz sampel yang paling sesuai ialah antara 1500 and 2000 pemerhatian.

Kita juga dapati model kita adalah stabil merentasi 22 siri KWPA antara bangsa dan juga stabil merentasi perubahan saiz sampel. Kita dapati YQ-ARFIMA adalah lebih baik daripada random walk dalam ramalan yang diukur dari segi fungsi hilang RMSE dan MAPE. Dengan ketepatan ini, kita boleh menggunakan YQ-ARFIMA untuk menunjukkan tingkahlaku reversi min (*mean reversion*) bagi KWPA.

Kita menyiasat pengaruh perubahan pada model kita. Kita dapati keputusan adalah lebih teruk jika kita mengabaikan perubahan dalam analisis ramalan. Tambahan pula, kita telah membina satu kaedah pembahagian (*dissection*) untuk mengkaji cerapan luar biasa (*outliers*) apabila bilangannya adalah kurang. Kita dapati dalam pemodelan, cerapan luar biasa (*outliers*) yang teruk, lebih baik di buang untuk menambah kejituan ramalan.

Kita dapati komponen berkitaran (*cyclical components*) KWPA berubah dengan secara positif mengikut turun-naik komponen berkitaran bagi nilai keupayaan para pengguna. Ini menunjukkan terdapat satu hubungan dalam jangka masa panjang di antara KWPA dan nilai keupayaan para pengguna. Dalam lain perkataan, kedua-dua siri bergerak bersama. Dengan keputusan ini, kita sekurang-kurangnya boleh mengetahui tanda perubahan KWPA dengan meneliti nilai kupayaan para pengguna.

Dalam pemodenan satu persamaan tunggal, kita dapati dengan menambah satu komponen berkitaran sahaja daripada keupayaan para pengguna boleh memburukkan keupayaan ramalan YQ-ARFIMA. Walaupun begitu, kita dapati juga dengan menambah komponen

berkitaran daripada LQBritpus bersama-sama dengan komponen yang sama daripada Lconsumption kepada model yang bukan terbaik, boleh menambahkan keupayaan ramalan. Tetapi bagi model yang terbaik, tambahan komponen berkitaran tidak langsung mengaruh keupayaan ramalannya.

# **STRUCTURAL MODELLING AND ANALYSIS OF THE BEHAVIOURAL DYNAMICS OF FOREIGN EXCHANGE RATE**

## **ABSTRACT**

This thesis deals specifically with the foreign exchange rates that resulted from free float regimes. In general, we study the structural modelling and analysis of the behavioural dynamics of foreign exchange rates. We start with the recognition that foreign exchange rate is a financial time series. We model the foreign exchange rate by using two popular methodologies: ARFIMA and a Kalman filter based method (KFBM) with the hope that it can produce the best exchange rate model. Our objective is to use this best model to show the mean reverting behaviour of the exchange rates. In the course of carrying out the experiments and analysis, we would like to study the behavioural dynamics of the exchange rate.

We have developed a dynamic YQ-specified ARFIMA model for the long and short term memory modelling. Here 'dynamic' simply means that the model parameters can be altered according to the characteristics of the particular time series. This seemingly easy alteration is made possible by using sequential  $t$  tests. This YQ-specified ARFIMA model performs very well and much better than the Kalman filter based method. We have shown without any doubts that YQ-ARFIMA is about 12 times better in term of the RMSE values and 10.6 times better in term of the MAPE values, than the Kalman filter based method in long-term memory modelling. Moreover, for short-term memory, the YQ-ARFIMA performs even better, giving a RMSE value of 40 times and a MAPE value of 36 times

better than the Kalman filter based method. However, KFBM seems to do better in analysing structural breaks. The possible reason is that KFBM is more suitable for series with cyclical components. In addition, output results show that for short-term memory modelling, the ideal sample size is about 200 to 1300 observations while for long-memory modelling the ideal sample size is between 1500 and 2000 observations.

The output of experimental results shows that this YQ-specified ARFIMA model is robust across 22 foreign exchange markets and across sample sizes. We have found that this YQ-ARFIMA beats the random walk model soundly in out of sample forecasting in terms of the loss functions RMSE and MAPE. With this positive result, we use this YQ-ARFIMA model as a tool to show the mean reverting behaviour of the exchange rates.

We investigated the influence of the breaks on our predictive models. We found that it is costly to ignore structural breaks in forecasting. We have devised a practical method, which we refer to as the dissection method for the correction of outliers when there are not many of them in the series. However, we have found that discarding the section of the data that contains extreme outliers can improve the predictive power of the model tremendously.

We have shown that the cyclical components (stationary) of the exchange rate series are positively related with the corresponding cyclical components of the consumption series. This implies exchange rate series have a long run relationship with consumption. To put it differently, they move together. With this result, we can at least keep track of the sign of the exchange rate by examining the consumption rate.

In the single equation dynamic specification modelling, we have shown that adding only one cyclical component of consumption into the model can deteriorate the predictive ability of the YQ-ARFIMA model. However, we have found that adding the appropriate cyclical component of the LQBritpus series together with that of Lconsumption to models other than the best-fitted model can improve its predictive ability substantially. For the best fitted-model, no combination of the cyclical components can improve its predictive ability at all.

# CHAPTER 1

## INTRODUCTION

### 1.1 Outline

The currency crisis in Asia in 1997 has created havoc and disorder in the financial and economic systems of countries like Malaysia, South Korea, Indonesia and Thailand. To prevent such currency crises from recurring, it is imperative for us to study the foreign exchange rate (forex) time series. However, we have inherited an “international monetary system” that has no international standardized measurement of exchange rate. This happened since the collapse of The Gold Standard (1880-1914) system, The Gold Exchange Standard (1925 – 1936) system and The Bretton Woods System (1947 – 1970) collapsed. Over the thirty years, since the breakdown of the Bretton Woods system, countries have adopted a wide variety of regimes, ranging from dollarization and currency boards to simple pegs, basket pegs, crawling pegs, and target zones to clean and dirty floats. Figure 1.1 shows clearly that, the forex for United Kingdom during the period before the breakdown of the Bretton Woods System, 1972, is either constant or decrease sharply due to devaluation of the currency.

As a whole, all these regimes can be classified into two main types: pegged or float. In this thesis, we make mainly empirical research on floating regimes and in our case, free floats. Unlike stocks and derivatives, forex time series are not pure series generated by economic agents. Moreover, the forex trading volume is rather small compared to the volume of the stock traded in the stock exchange.

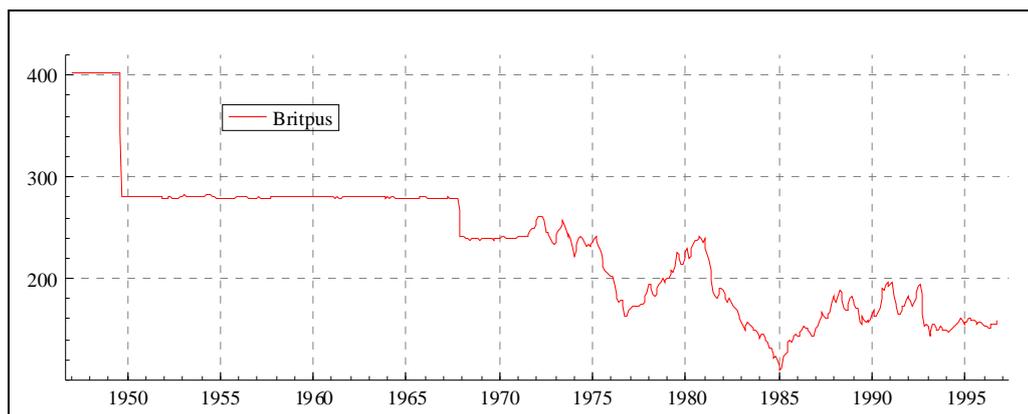


Figure 1.1 – Graph of British pound per US dollar exchange rate  
 (Note: This exchange rate Britpus is obtained from the Federal Reserve Board US)

On top of this, the time span of the exchange rate series is rather short if we consider only the period after the breakdown of the Bretton Woods System for empirical research. Perhaps, it is because of this small trading volume, that the central bank is able to intervene in its trading activity in times of needs.

Relatively speaking, there are not many research papers available on this subject. The reason could be that this intervention of the central bank makes the forex time series artificial, and artificial time series is very difficult to model or analyse. Another reason is that the time span for the exchange rate series after the recent floats is rather short for statistical inference and tests. The next question is “What are the uses of studying exchange rate?”

Exchange rate behaviour used to be described by a simple proposition, viz., the long run purchasing power parity (PPP), which states that the national price levels should be the same when expressed in a common currency. The long run PPP hypothesis has created two types of exchange rates: the nominal exchange rate, which is defined as the price of one currency in terms of another, and the real exchange rate, which is defined as the nominal exchange rate adjusted for the differences in the relative national price level. Whether the long run PPP holds or the real exchange rate is stationary have important

economic implications on a number of fronts. Among these fronts, the easier to understand is that it is used to determine the degree of misalignment of the nominal exchange rate and the appropriate policy response, the setting of exchange rate parities, and the international comparison of national income levels. Our primary interest in the long run PPP is to verify this mean reversion behaviour by using some existing tools, proposing a new tool, and constructing an effective exchange rate model for accurate forecasting.

It is by now an accepted fact that financial time series exhibit three important properties. These three important properties form the basic ingredients for further research and analysis.

- 1 Large absolute returns occur more frequently than one would expect if the data follows a normal distribution.
- 2 These large absolute returns tend to occur in clusters.
- 3 Large negative returns tend to appear more often than large positive ones in stock markets, while it may be true the other way around for foreign exchange rates (see Franses, 1998; Cochrane, 1999).

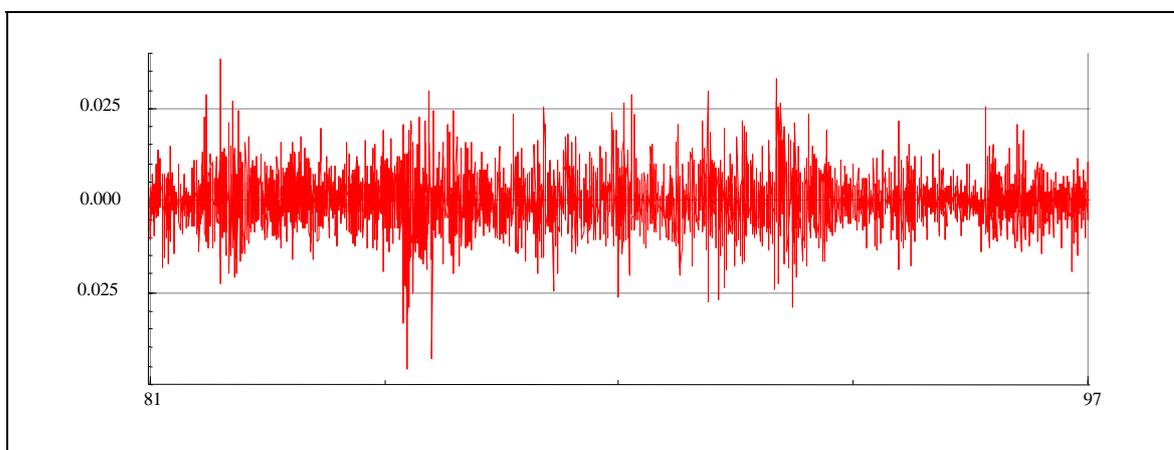


Figure 1.2 – Exchange rate returns occur in clusters  
(Note: DLBritpus denotes the returns of British pound exchange rate per US dollar)

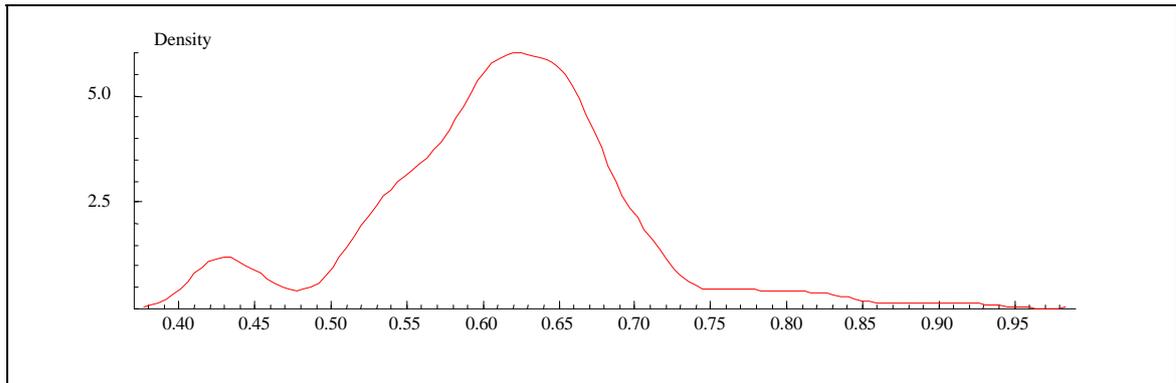


Figure 1.3 – Fat tails and shape peak of the density graph of Britpus  
(Note: Britpus denotes British pound exchange rate per US dollar)

Large absolute returns occur in clusters as in Figure 1.2, and Figure 1.3 shows the fat tails and shape peak of Britpus, which are the general characteristics of financial time series.

Forex is an example of a financial time series, and thus, is expected to exhibit the above characteristics as widely documented for the past decade. In statistical terms, fact number one is equivalent to saying that the actual kurtosis is very much larger than 3 for the actual distribution. In graphical terms, it implies that the actual density graph has a sharp peak and fat tails. Figure 1.3 shows clearly these two characteristics of Britpus. We are going to work on these fat tails. In volatility research, the contributing factor for these fat tails is that there exist additive outliers (AO), innovation outliers (IO), level change outliers (LC) and variance change outlier (VC). The existence of these outliers (aberrant observations), it can be said as the main cause for nonstationarity in the financial time series. It is accepted that these outliers cannot be modelled by the standard GARCH (see Baillie and Bollerslev (1989)) model. One possible cause for this outcome is that the GARCH model (generalized autoregressive conditional heteroskedastic) model is not able to capture these so-called outliers on the returns. We

assume that in a structural time series model for foreign exchange rate, the model cannot capture such outliers. Thus, we have to devise methods to deal with these outliers, which cause not only structural changes but also affect the forecasting performance of the relevant models. Generally, there are two methodologies to analyse a structural time series.

- 1 The Box and Jenkins methodology popularised by Box and Jenkins in 1970, and improved and refined to advanced forms such as the Autoregressive Integrated Moving Average (ARIMA) and the Autoregressive Fractional Moving Average (ARFIMA).
- 2 The Kalman filter and state space representation system used to extract unobservable variables from the observed series (hereafter referred to as KFBM).

This thesis basically tries to find an effective exchange rate structural model, which can outperform the random walk model in out of sample forecasting. If this structural model has this property, we can use it as a tool to ascertain the mean reversion behaviour of the exchange rate for any sample size. To obtain this structural model, first we have to address the question on which methodology should be preferred in structural modelling: the ARFIMA method or the Kalman filter based method (KFBM). In the course of answering this question, we will come up with the best structural model for use as a tool to determine whether an exchange rate series exhibits mean reverting behaviour. In studying exchange rate behaviour, we also propose a procedure for modelling the additive outliers in a structural time series model, analyse the structural breaks and their influence on forecasting, investigate the relationship between consumption rate and

exchange rate, and, lastly, determine whether the cyclical components of the consumption rate or the exchange rate can in any way improve the predictive ability of our best structural model, the Yip-Quah specified ARFIMA model (hereafter referred to as YQ-ARFIMA).

The layout for the rest of this chapter is as follows: In Section 1.2, we discuss the objectives of this dissertation. In Section 1.3, we present the constraints of the research and then, we discuss the methodologies used in the research in Section 1.4. In Section 1.5, we discuss our data set and the software used in the research. Then, we discuss the three common forms of a financial time series in Section 1.6. In section 1.7, we give an outline for each subsequent chapter.

## **1.2 Objectives of this Research**

Our main objective in this thesis is to find an effective exchange rate structural model, which can outperform the simple random walk model in out of sample forecasting. We can use this structural model as a new tool to verify the parity reversion and thereafter, make accurate prediction of exchange rate. We proceed as follows: first, we are going to look for a variant of the ARFIMA model that is able to produce good forecasting results. We can achieve this by making a comparative study between this variant of the ARFIMA model and the Kalman filter based method for structural modelling. We would then determine whether this variant of the ARFIMA model can be used as a tool to ascertain the mean reverting behaviour of the exchange rate, by checking whether this model can beat the random walk model or not. In addition, we will use the fractional integration technique and the impulse response function to verify the existence of parity reversion for any sample size. Only then, we will study the behaviour of exchange rate

with respect to the influence of outliers, structural breaks, consumption rate and whether the stationary components of consumption rate can in any way help to improve the predictive ability of our structural model.

### **1.3 Constraints of the Research**

There are constraints in any research. Very often, these constraints are unavoidable and cannot be got rid of completely. For research on exchange rates, our first constraint is that if the condition prevailing in the exchange rate market is not favourable to the country concerned, the central bank will, very often than not, intervene in the trading. This creates an artificial element in the market, making its series extremely difficult to model.

Secondly since there are no standardized regimes, not many free float exchange rate data sets are available for research. Many of these free float exchange rate data series have very short time span because quite a number of them switched to currency pegging regimes about a decade ago. Thirdly, we have great difficulty in obtaining the real time data especially, the most up-to-date data. This creates a difficulty when we perform out of sample forecasting. Thus, very often, we can only perform pseudo out of sample forecasting. Here “pseudo” means: pick a date near the end of the sample, estimate our forecasting model using data up to that date, and then use the estimated model to make a forecast of the data left out of the sample.

### **1.4 Methodology**

There are two main methodologies used in the research. One of them is the state space representation system of a Kalman filter. We use the state space model to generate the

residuals and the state variables for the observable foreign exchange rate data when the data is fitted with a structural dynamic time series model. Residuals and state variables in this case are unobservable and we use a Kalman filter to extract it. We use a state space representation system with a Kalman filter basically because of two reasons.

- 1 Many researchers have shown the applicability of the Kalman filter. For example, Fama and Gibbons (1982) modelled real interest rate by using a Kalman filter. The result obtained was exceptionally good. There is also the research by Halsey (2000) on stationary components of earnings and stock prices by using a Kalman filter.
- 2 A Kalman filter can extract the unobservable with excellent accuracy and moreover, it can give very good maximum likelihood estimates. With this property, it can be used to do the job of smoothing the series and to do point wise forecasting. It is a popular technique now in macroeconomic and financial research.

Besides the state space representation system using a Kalman filter, we also use the Box and Jenkins methodology (ARFIMA) for long-term memory modelling. We construct the most suitable dynamic specification for the ARFIMA modelling of the foreign exchange rates. We test the stability of the model by using 22 foreign exchange rates series, which are specially chosen from different parts of the world: Canada, Australia, Mexico, South America, Europe, Africa and Asia. This geographical variation will definitely enhance the robustness of the model after stability testing. To compare and contrast the effectiveness of each of the methodologies used in structural modelling, we perform the actual modelling on long-term memory and short-term memory series.

Next, we use a different setting and a different population, that is, Ringgit per foreign currency to repeat the experiment. This will verify the external validity of our empirical study. We also compare the robustness of each of the methodologies with regard to sample size. For comparing, we use the root mean square error (RMSE) and the mean absolute prediction error (MAPE) values.

To confirm the output, we use a sign test, which is usually called the S statistics to test whether there is any difference between the two models. Next, we use a new forecast criterion to determine whether the forecasts are consistent or not. If the forecast is consistent, the forecasted values and the actual values should be cointegrated. We use a 8-step ahead forecasting for our comparison study. Many research papers such as the one by Diebold and Mariano (1997) show that short horizon forecasting is more reliable than long horizon forecasting. Moreover a short duration forecasting is also in line with the concept of Martingale theory, used widely in financial research. However, since we are dealing with exchange rate, which most probably has the mean reverting behaviour, we also consider long horizon, 100 steps ahead forecasts.

## **1.5 The Data and Software**

There are two types of daily exchange rate data sets used in the analysis. First, we have twenty-two daily exchange rate series for our analysis. These twenty-two daily exchange rate data sets were obtained from the Federal Reserve Board of the United States of America (US). All of them are shown in Table 1.1. Among these exchange rate series, there are three series, which had been terminated in 1998 because of the adoption of Euro as a common currency. These three exchange rate series are the French Franc, the German Deutschmark and the Dutch Guilder. Many of these data sets

have quite a number of missing observations. We use the cubic spline technique to select the best values to fill in these missing observations. Bandwidths are chosen after repeated testing on whether the cubic spline fit the curves or not. Most of these data sets started from 1<sup>st</sup> January 1981 to 31<sup>st</sup> December 2004. However, quite a number of them started from 1<sup>st</sup> January 1990 to 31<sup>st</sup> December 2004, and some started from 1<sup>st</sup> January 2000 to 31<sup>st</sup> December 2004.

The second set of exchange rates is obtained from Bank Negara Malaysia. All the values of this second data set are in the number of ringgit per unit foreign currency. As we use this second set for the purpose of external validity test, only 6 series from this second set of data are used for the analysis. However, a main weakness in this second set of exchange rate series is that they all cover the period from 1<sup>st</sup> January 2000 to 31<sup>st</sup> December 2004. Thus, the time span is short even though the frequency of the observations is within reasonable limits. Table 1.2 shows these six series, which are divided into two main zones, viz., Europe and Asia.

We include Dutch guilder, German Deutschmark and French franc in experiment for the sake of obtaining the best structural exchange rate model. These exchange rates were used because they are considered as some of the best free float data sets.

In Chapters 11 through to Chapter 13, we mainly use the British pound per US dollar exchange rate data, Britpus, for our experiment. Britpus is transformed into two forms: weekly data, WeBritpus and quarterly data, QBritpus. This is because we find that weekly data is more suitable for structural breaks analysis and quarterly data is more meaningful for co-movement (move in parallel) analysis. Beside the exchange rates, we also use the UK real consumption, taken from Harvey and Scott (1994). These twenty-

two forex data series and the consumption data can be found in the CD enclosed in this thesis

Table 1.1 – Names of the 22 exchange rates used in the experiment.

America	Europe	Africa	Asia
Canada Dollar (Can)	European Dollar (Eur)	South Africa Rand (Ran)	Singapore Dollar (Sid)
Mexico Pesos (Pes)	British Pound (Britpus)		Thailand Baht (Bah)
Brazil Real (Rea)	Denmark Kroner (Kne)		Malaysia Ringgit (Rin)
Venezuela Bolivar (Bol)	Sweden Kronor (Kno)		South Korea Won (Won)
Australian Dollar (Austrus)	Switzerland Francs (Swisfus)		Chinese Yuan (Yua)
	French Francs (Frnfrus)		Indian rupees (Rup)
	German Deutschmark (Germdus)		Japanese Yen (Japynus)
	Dutch Guilder (Dtchgus)		Hong Kong Dollar (Hkd)

Table 1.2 – Names of the 6 exchange rates (Ringgit per foreign currency) used for the external validity experiment

Europe	Asia
British Pound (Britpus)	Singapore Dollar (Sid)
European Dollar (Eur)	Thailand Baht (Bah)
	Hong Kong Dollar (Hkd)
	Japanese Yen (Japynus)

In Chapters 11 through to Chapter 13, we mainly use the British pound per US dollar exchange rate data, Britpus, for our experiment. Britpus is transformed into two forms: weekly data, WeBritpus and quarterly data, QBritpus. This is because we find that weekly data is more suitable for structural breaks analysis and quarterly data is more meaningful for co-movement (move in parallel) analysis. Beside the exchange rates, we also use the UK real consumption, taken from Harvey and Scott (1994). These twenty-two forex data series and the consumption data can be found in the CD enclosed in this thesis.

With respect to software, we mainly use Eviews 4.1, PcGive10.3, Stamp 6.20 and, occasionally, we use Gauss 5.0 for certain situations only.

## **1.6 Three Different Forms of the Data**

There are three different forms of a data series, which can be used in any research. Which form is more suitable depends to a large extent on the intended outcome of the research. In this section, we discuss all the three forms of a data series: the original data series, the log data series and the returns series. We use all these three forms of data in our research.

### **1.6.1 The original data**

The raw data series can provide a better intrinsic measure of the central tendency of the series. However for cases where the distribution of the raw data series is highly skewed, the conditional mean  $E(y_t|x_t)$  where  $y_t$  denotes the dependent variable and  $x_t$  denotes the explanatory variable, may not be a useful measure of the central tendency, and estimates will be undesirably influenced by extreme observations, which we refer to as

outliers. In Chapter 1 through to Chapter 8, we use the raw data series for the experiments. This is largely because our objective is to compare and contrast the two structural modelling methodologies.

### 1.6.2 The log data

This form of the data series is widely used in the research. Basically there are four reasons for its widespread use.

- 1 Many nonlinear functions can be transformed into linear functions by using the logarithm. Thus, we have  $E[\log(y_t)|x_t]$  is roughly linear in  $x_t$  over the range of  $x_t$ , while  $E[y_t|x_t]$  is nonlinear.  $y_t$  and  $x_t$  as before denote the dependent and explanatory variables respectively. This is an advantage because linear models are easier to report and interpret. For example, we interpret the regression coefficients as percentage changes when the log series is used.
- 2 The errors in  $\varepsilon_t = \log(y_t) - E[\log(y_t)|x_t]$  may be less heteroskedastic than the errors from the linear specification when raw data is used. We can show this by using Taylor's theorem, and then apply the delta method. Let  $\mu_t = E(x_t)$  and  $Var(x_t) = k\mu_t^2$ . Then the variance of the log is approximately constant as shown below. We use Taylor's theorem which is given by:

$$\begin{aligned}
 f(x) &\approx f(\mu) + f'(\mu)(x - \mu) + \dots \\
 Var(f(x)) &\approx Var(f(\mu)) + Var(f'(\mu)(x - \mu)) + \dots \\
 Var(f(x)) &\approx [f'(\mu)]^2 Var(x)
 \end{aligned}$$

By setting  $f(x_t) = \log x_t$ , we have:

$$\text{Var}(\log x_t) \approx \left( \frac{1}{\mu_t} \right)^2 \text{Var}(x_t) = k$$

Thus:

$$\text{Var}(f(\mathbf{x})) \approx [f'(\mu)]^2 \text{Var}(\mathbf{x}) \Rightarrow \text{Var}(\log \mathbf{x}_t) \approx \left( \frac{1}{\mu_t} \right)^2 \text{Var}(\mathbf{x}_t) = k \quad (1.1)$$

However this is not consistent and the reverse may be true also.

- 3 If the distribution of  $y_t$  is highly skewed, the conditional mean  $E[y_t|x_t]$  may not be a useful measure of central tendency, and outliers will influence the estimates. In this case, the conditional mean  $E[\log(y_t)|x_t]$  may be a better measure of central tendency.
- 4 A good approximation of a returns series can be easily computed by taking the difference between two consecutive observations in the log form.

### 1.6.3 The returns series

The returns is defined as the quotient obtained by dividing the difference between the two consecutive observations by the first observation of the two consecutive observations. This form of the returns series is widely used in finance for the simple reason that we are interested in the returns for our investment. The second reason is that by using the returns series, we may not need to difference the original series again to obtain a stationary time series if the raw series is nonstationary.

## 1.7 Outlines of Subsequent Chapters

In Chapter 2, we give an overview of the theoretical background involved. This is to prepare the readers on the sort of probability and statistical theory used in the research.

In presenting the time series concepts, we stress on its difference from cross-sectional regression and how the difference is reconciled. We also present the  $F$  tests,  $t$  tests and, most important of all the so-called Hall preferential  $t$  test. We are of the opinion that, with this, we can easily guide the readers as to what contributions we have made.

We present and discuss two methodologies, ARFIMA and KFBM, which we are going to compare and contrast so as to obtain the best structural time series model for exchange rate in Chapter 3.

In Chapter 4, we present a concise literature review. From the literature review, we state categorically, the objective of each experiment, which we are going to conduct after critical analyse of the results of the relevant research papers.

In Chapter 5, we discuss the mean reverting and random walk behaviour of the exchange rate. First, we present the latest development in the study of these two behaviours with respect to the controversies or rather puzzle encountered. With that, we present our experiment. We use fractional dynamics, and impulse response function in addition to unit root tests, to ascertain mean reverting behaviour.

Subsequently, in Chapter 6, we first make an exploratory investigation of eight foreign exchange rate series by using graphical analysis and hypothesis testing. We develop the most suitable dynamic specification for the ARFIMA modelling of long-term memory. We test the stability of the specification by fitting model to the 8 foreign exchange rates series. We also test its robustness with regard to variations in sample size. To test the external validity of the model, we fit the model to 6 more exchange rates specially chosen from different parts of the globe.

In Chapter 7, we use the series Britpus for the experiment. We generate the component series for Britpus. Next, we regress Britpus on all its three components in a single equation dynamic modelling. We perform pseudo out of sample forecasting by assuming the case of uncertain variances. We assume that its effectiveness in forecasting depends only on the values of RMSE and MAPE. We test its robustness by fitting it to the eight-forex series, and then across different sample sizes.

We state the conclusions obtained from the results obtained from Chapter 6 and Chapter 7 in Chapter 8 . Next, we use each of the two structural models for modelling short-term memory series. We vary the sample size and for each variation, we perform the modelling. We compare and contrast each of the output of the experiments. In this way, we can make decision on which model or, rather, methodology is the best, and under what conditions, it is the best.

In Chapter 9, we repeat the experiment in Chapter 6 in a new setting, that is, foreign exchange rate in Ringgit per unit of foreign currency. We perform this additional experiment in order to verify whether our best structural model can still be workable in a new setting and population.

We perform empirical analysis in Chapter 10, where we devise and execute experiments to compare and contrast the forecasting ability of three models, the standard ARFIMA, the YQ-ARFIMA and the random walk model in the context of exchange rate. We use two loss functions, RMSE and MAPE, for the comparison. To further confirm the results, we perform a forecast consistent procedure for the experiment.

In Chapter 11, we present a theoretical framework for a structural break model based on the partial sum concept. We use recursive least squares and recursive residual sums of

squares to investigate the structural breaks and outliers in the DLWeBritpus series. Next, we investigate the influence of the structural breaks on the predictive performance of our KFBM model. With respect to outliers, we use the simple ARFIMA model to model the mild outliers, and for shape outliers, we use two approaches: we scale them down to normal size, and then, we exclude them altogether. We then perform structural modelling again on DLWeBritpus, but this time with the corrections of the outliers. The output from this last experiment is intended to serve as a check on the validity of the forecasting ability of the two models developed earlier, and also to determine the usefulness of outlier correction in general.

In Chapter 12, we construct a structural model for Lconsumption. We use the Kalman filter to extract the cyclic components of the consumption (Lconsumption) and LQBritpus series. We regress LQBritpus on Lconsumption, and, after this, we regress Lconsumption on its three important components. Next, we regress the trend cyclic components of LQBritpus on the trend cyclic components of Lconsumption in order to obtain their relationship.

In Chapter 13, we perform a bivariate structural time series modelling of LQBritpus and Lconsumption with the intention of obtaining more robust and precise estimates of their cyclic components. Next, we perform a single equation dynamic modelling of the cyclic components of LQBritpus on Lconsumption. We want to determine and confirm whether the exchange rate and the consumption move in parallel. We also want to investigate whether the inclusion of the cyclic components as regressors can improve significantly the predictive power of our YQ-specified ARFIMA model. Finally in Chapter 14, we conclude the thesis. We present the empirical results and make

inferences based on the experiments and discuss their implications. We list our contributions in the research. Last, but not least, we suggest future areas of research.



# CHAPTER 2

## THEORETICAL BACKGROUND

### 2.1 Introduction

The objective of this chapter is to give a brief overview of all the basic financial econometric theory as well as statistical theory involved in the analysis. As for the methodologies used, we will introduce them briefly in this chapter and discuss them in more details in Chapter 3. The financial econometric theory and statistical theory used in the research are: independent and serially correlated data, long memory, short memory, white noise, linear and nonlinear regression, autocorrelation function, partial autocorrelation function, Akaike, Schwarz and Hannan-Quinn information criteria, theory of ARMA model,  $t$ -tests, Wald tests,  $F$  tests,  $R^2$  tests, unit roots and its use in testing, Granger causality tests, structural modelling, stochastic trends, forecasting and selection criterion for the best forecasting model, structural breaks, and real time data versus transformed data.

### 2.2 Independent and Serially Correlated Data

In our research, we deal mainly with time series data. In general, there are two types of data available for research; time series data and cross-sectional data. Regression techniques and OLS estimators are developed specifically for cross-sectional data. Can we apply these techniques without alterations to time series data? This section deals mainly with the answer to this question. In the course of answering this question, we will point out the important role played by the characteristics “independent and serially

correlated” of the data. We will provide a formal definition for independent and serially correlated data after the explanation.

There is a basic difference between time series data and cross-sectional data. The former can only have a single realization generated by the data generating process (DGP), and the latter can be generated by multiple realizations of the DGP. This major difference eliminates altogether the possibility of using random sampling distributional techniques and sampling statistical inferences for time series analysis. However, if we impose the restriction of ergodic stationarity on the time series, the time average over the elements of the time series will be consistent for the ensemble mean of the cross-sectional data series. Normally, in practice, any time series we encounter would have more than 100 data. With this length of data, we would be able to use large sample theory (asymptotic theory) for making statistical inferences (see Stock and Watson, 2003). Thus, we can use asymptotic properties of the time series data to construct confidence levels for hypothesis testing and make statistical inferences.

The major tool used to derive statistics and construct confidence levels for studying cross-sectional data is the Central Limit Theorem (CLT) initiated by Lindeberg-Levy. However, this CLT is valid only for independent and identical distributional observations (i.i.d) as is usually assumed in regression analysis. As most of the time series data are serially correlated, we cannot apply the CLT wholesale without generalization and alteration. This generalization and alteration of the CLT were done by Gordin (1969) and subsequently restated by White (1984). We call this final version of the CLT as Gordin’s CLT for zero-mean ergodic stationary process (see Hayashi, 2000). Thus, we have to use Gordin’s version of CLT to derive statistics and construct

confidence intervals for time series analysis. We shall now explain the term independent, serially correlated, stationarity and ergodicity.

The observations in most financial time series are serially correlated and seldom are independent. Serially correlated observations and how to test its existence play an important role in structural analysis because the output results will be very different if we take serially correlated observations as independent observations. It is an accepted fact, that independent will imply no serial correlation but the reverse is not true in general. However, for normal random variables, no serial correlation can imply independent. In time series analysis, one important assumption is that the time interval between two consecutive observations is uniformly the same, and we refer to such property of the time series as stationarity. Stationarity is a restriction imposed on the time series so that the number of parameters needs to be estimated is minimum. To estimate the parameters of a time series, the time series must exhibit stationarity behaviour and ergodicity as well. Stationarity simply means that the joint probability distribution of any set of  $k$  observations in the sequence of observations is the same regardless of the origin,  $t$ , in the time scale. Ergodicity means that events separated far enough in time are “asymptotically independent”. Put differently, the observations are not too persistent, and every observation contains some information, which are not available in other observations. However, recent research found and confirmed that most time series are nonstationary and very often cointegrated. We shall discuss this new development in Section 2.8.

**Definition:** If  $\{y_t\}_{t=1}^{t=N}$  represents a time series of data for the sample period from  $t = 1$  to  $t = N$ , then the time interval between any two consecutive time must be the same.

If the observations are independent, we have the following relations:

$$\text{Cov}[y_{t-k}y_t] = 0 \text{ and } E[y_{t-k}y_t] = 0 \quad (2.1)$$

for any  $t$  and  $k$  with  $k < t$

However, if the observations were serially correlated instead, we would have the following relations:

$$\text{Cov}[y_{t-k}y_t] \neq 0 \text{ and } E[y_{t-k}y_t] \neq 0 \quad (2.2)$$

for any  $t$  and  $k$  with  $k < t$

**Definition:** A first order serial correlated linear regression model is defined as:

$$y_t = a + bx_t + u_t \quad \text{where } u_t = \rho u_{t-1} + \varepsilon_t, \text{ and } |\rho| \leq 1 \quad (2.3)$$

with the  $x$ 's being non random:

$$E(u_t) = 0, \text{ Var}(u_t) = C, \text{ and } \forall t, \text{ Cov}(u_t, u_k) \neq 0 \quad (2.4)$$

where  $C$  denotes a constant.

The above model can be represented in a more compact way as shown below:

$$y_t = \mu + (1 - \rho L)^{-1} \varepsilon_t \quad (2.5)$$

where  $\mu = E(y_t) = a + bx_t$ .

Notice that Equation (2.5) is in the moving average form. A moving average process will generate serially correlated data. When Ordinary Least Squares (OLS) or Generalized Least Squares (GLS) is used to estimate the parameters in the above-reduced equation, we would find that the OLS or GLS is no longer unbiased or consistent. We refer to such problem as simultaneous equation bias and we use instrumental variable regression to overcome the problem. Thus, very often, we avoid any lagged values in the dependent variables when doing OLS regression. However, we can also overcome this problem by testing the validity of each parameter of the output equation after doing the regression by using the  $t$  tests, which is robust, consistent and pivotal. This can be done easily by using econometric software package, for examples, Eviews and PcGive.

As for the analysis, we can use a graphical method or hypothesis testing to determine the correlation of a set of data. Figure 2.1 shows graphically what serially correlated data set means. Notice how the variation within one period can be used to indicate the likely variation in the next period. The data set of GDP in US is obtained from “Econometric Analysis” by William H. Greene (2003).

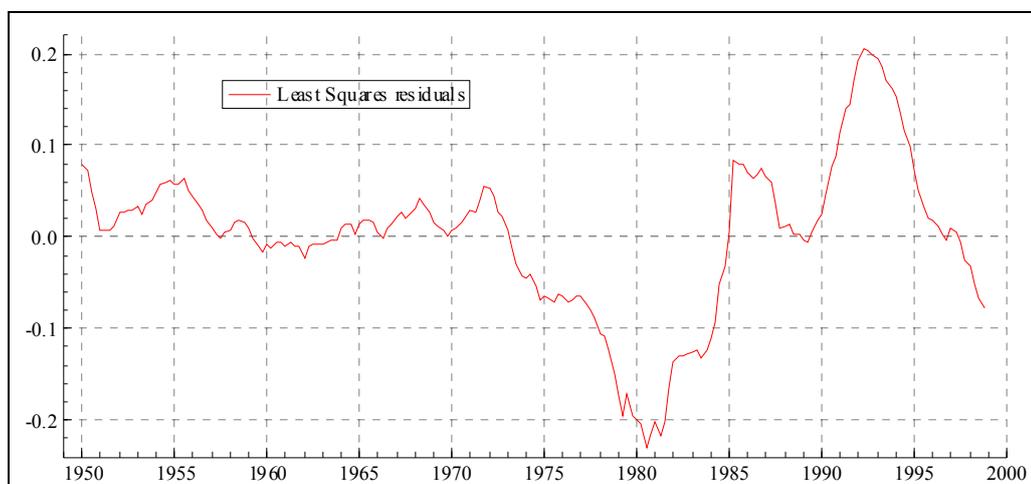


Figure 2.1 – Variation pattern of serially correlated data- GDP in US

As for additional graphical analysis, we shall discuss it under the heading of the autocorrelation function ACF and the partial autocorrelation function PACF in Section 2.5. In this aspect, scatter plots can also be used to detect the correlation of the data. In practice, we divide the scatter plot into four quadrants. Equal number of points in each quadrant simply means lack of correlation. Excess points in any quadrant indicate correlation. There are two common tests for the null hypothesis testing of no serial correlation. They are the Durbin Watson Test and the Ljung-Box Test.

### **The Durbin-Watson Test**

Durbin Watson test is a test for the existence of serially correlated random disturbances of the dependent variable. It is a classical test. The model has to have an intercept and the regressor has to be non-random. The test statistic is given by:

$$d = \frac{\sum_{t=2}^N (\hat{u}_t - \hat{u}_{t-1})^2}{\sum_{t=1}^N (\hat{u}_t)^2} \quad (2.6)$$

For testing the null hypothesis of no serial correlation against serial correlation, we have the following decision rule:

Reject if:  $d < d_{\text{lower}}$

Accept if:  $d > d_{\text{upper}}$

No conclusion if  $d \in (d_{\text{lower}}, d_{\text{upper}})$