UNIVERSITI SAINS MALAYSIA

Second Semester Examination
Academic Session 2005/2006

April/May 2006

MAT 122E – Differential Equations I
[Persamaan Pembezaan I]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of SIXTEEN pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi ENAM BELAS muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions: Answer all **TWENTY** (20) questions in Section I using the objective answer paper (OMR answer paper) provided. For this section, answers should be written in 2B pencil only. **The OMR answer paper together with the question paper of Section I** will be collected 1½ hours after the examination starts.

Answer all **TWO** (2) questions in Section II. All answers in this section must be written on the answer script papers provided.

*[**Arahan**: Jawab semua **DUA PULUH** (20) soalan dalam Bahagian I dengan menggunakan kertas jawapan soalan objektif (kertas jawapan OMR) yang disediakan. Bagi bahagian ini, jawapan perlu dituliskan dengan pensel 2B sahaja. **Kertas jawapan OMR ini berserta kertas soalan Bahagian I** akan dikutip 1½ jam setelah peperiksaan bermula.*

*Jawab semua **DUA** (2) soalan dalam Bahagian II. Semua jawapan dalam bahagian ini mestilah dituliskan pada kertas skrip jawapan yang disediakan.]*

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Section 1: Answer ALL 20 questions. Each correct answer will be given 2 ½ marks [50/100].

Bahagian 1: Jawab SEMUA 20 soalan. Setiap jawapan betul diberi 2 ½ markah [50/100].

1. Choose the linear differential equation from the following:

(a) $\frac{dy}{dx} = x^2y^2 + \cos x$ (b) $\frac{dy}{dx} = x\sin y$

(c) $y\frac{d^2y}{dx^2} + \frac{dy}{dx} = x^2$ (d) $yy' = xy$

(e) none of the above

1. Pilih persamaan pembezaan yang linear daripada yang berikut:

(a) $\frac{dy}{dx} = x^2y^2 + \cos x$ *(b)* $\frac{dy}{dx} = x\sin y$

(c) $y\frac{d^2y}{dx^2} + \frac{dy}{dx} = x^2$ *(d)* $yy' = xy$

(e) bukan semua yang di atas

2. Given the following differential equation

$$\left(\frac{d^4y}{dx^4}\right)^3 + \left(\frac{d^3y}{dx^3}\right)^4 + \left(\frac{d^2y}{dx^2}\right)^2 + \frac{dy}{dx} + y = 1, \quad (1)$$

choose the **false** statement:

(a) (1) is an ordinary differential equation
(b) y is the dependent variable in (1)
(c) (1) is a 4th degree differential equation
(d) (1) is a 4th order differential equation
(e) x is the independent variable in (1)

2. Diberikan persamaan pembezaan berikut:

$$\left(\frac{d^4y}{dx^4}\right)^3 + \left(\frac{d^3y}{dx^3}\right)^4 + \left(\frac{d^2y}{dx^2}\right)^2 + \frac{dy}{dx} + y = 1, \quad (1)$$

*pilih pernyataan yang **tidak benar**:*

(a) (1) *ialah suatu persamaan pembezaan biasa*
(b) *y ialah pembolehubah bersandar dalam* (1)
(c) (1) *ialah suatu persamaan pembezaan berdarjah 4*
(d) (1) *ialah suatu persamaan pembezaan berperingkat 4*
(e) *x ialah pembolehubah tak bersandar dalam* (1)

3. The differential equation from the relation $x = a\cos t + b\sin t$ is

(a) $x = t\dfrac{d^2x}{dt^2}$ (b) $x + \dfrac{d^2x}{dt^2} = 0$

(c) $x = \dfrac{dx}{dt}$ (d) $x = t^2\dfrac{dx}{dt}$

(e) $x^2 = t^2\dfrac{dx}{dt}$

3. *Persamaan pembezaan bagi hubungan* $x = a\cos t + b\sin t$ *ialah*

(a) $x = t\dfrac{d^2x}{dt^2}$ *(b)* $x + \dfrac{d^2x}{dt^2} = 0$

(c) $x = \dfrac{dx}{dt}$ *(d)* $x = t^2\dfrac{dx}{dt}$

(e) $x^2 = t^2\dfrac{dx}{dt}$

4. The general solution of the differential equation $(1 + y^2)\,dx + (1 + x^2)dy = 0$ is

(a) $x + y = c(1 - xy)$ (b) $x + y^2 = \text{c}$

(c) $x^2 + 4xy + 4y^2 + 6x - 2y = \text{c}$ (d) $y = x^2 + \text{c}$

(e) $x + y = c(1 + xy)$

4. *Penyelesaian am bagi persamaan pembezaan* $(1 + y^2)\,dx + (1 + x^2)dy = 0$ *ialah*

(a) $x + y = c(1 - xy)$ *(b)* $x + y^2 = \text{c}$

(c) $x^2 + 4xy + 4y^2 + 6x - 2y = \text{c}$ *(d)* $y = x^2 + \text{c}$

(e) $x + y = c(1 + xy)$

5. An integrating factor for the equation $\left(x^3+y^3\right)dx - xy^2dy = 0$ is

(a) x^2 (b) $\log x$
(c) x^{-4} (d) $1/x$
(e) x^{-3}

5. *Suatu faktor pengamir bagi persamaan* $\left(x^3+y^3\right)dx - xy^2dy = 0$ *ialah*

(a) x^2 *(b)* $\log x$
(c) x^{-4} *(d)* $1/x$
(e) x^{-3}

6. The solution of the initial value problem $\frac{dy}{dx} + y = 0, y(3) = 2$ is

(a) $y = 2e^{3+x}$ (b) $y = 2e^{3-x}$
(c) $y = -2e^{3+x}$ (d) $y = -2e^{3-x}$
(e) $y = -e^{3-x}$

6. *Suatu penyelesaian bagi masalah nilai awal* $\frac{dy}{dx} + y = 0, y(3) = 2$ *ialah*

(a) $y = 2e^{3+x}$ *(b)* $y = 2e^{3-x}$
(c) $y = -2e^{3+x}$ *(d)* $y = -2e^{3-x}$
(e) $y = -e^{3-x}$

7. The transformation $y = v\,x$ will reduce the equation $\left(x^2+y^2\right)dx - 2xydy = 0$ to $\frac{dx}{x} = f(v)dv$. Choose the correct $f(v)$ from the following:

(a) $\frac{2v}{v^2-1}$ (b) $\frac{2v}{1-v^2}$

(c) $\frac{v}{v^2-1}$ (d) $\frac{v^2}{v^2-1}$

(e) $\frac{2v^2}{v^2-1}$

.../5-

7. *Transformasi* $y = v\,x$ *akan menukarkan persamaan* $\left(x^2+y^2\right)dx - 2xydy = 0$ *kepada* $\frac{dx}{x} = f(v)dv$. *Pilih* $f(v)$ *yang betul daripada yang berikut:*

(a) $\frac{2v}{v^2-1}$ (b) $\frac{2v}{1-v^2}$

(c) $\frac{v}{v^2-1}$ (d) $\frac{v^2}{v^2-1}$

(e) $\frac{2v^2}{v^2-1}$

8. The set of orthogonal trajectories of the family of parabolas $y = cx^2$ is

(a) $x^2+2y^2=k^2$ (b) $x^2+y^2=k$

(c) $y=cx$ (d) $y^2=kx^2$

(e) $x^2-y^2=k$

8. *Set trajektori ortogon bagi keluarga parabola* $y = cx^2$ *ialah:*

(a) $x^2+2y^2=k^2$ (b) $x^2+y^2=k$

(c) $y=cx$ (d) $y^2=kx^2$

(e) $x^2-y^2=k$

9. Choose the general solution of the equation $(2x^2+y)dx+(x^2y-x)dy=0$ from the following:

(a) $y=c-2x+\frac{y}{x}$ (b) $y=-x^2+cx$

(c) $2x+\frac{y^2}{2}-\frac{y}{x}=c$ (d) $y=\frac{1}{-x^2+cx}$

(e) $y=x^2-cx$

.../6-

9. *Pilih penyelesaian am bagi persamaan* $(2x^2 + y)dx + (x^2 y - x)dy = 0$ *daripada yang berikut:*

(a) $y = c - 2x + \frac{y}{x}$ *(b)* $y = -x^2 + cx$

(c) $2x + \frac{y^2}{2} - \frac{y}{x} = c$ *(d)* $y = \frac{1}{-x^2 + cx}$

(e) $y = x^2 - cx$

10. The general solution of the equation $\frac{dy}{dx} + y = xy^3$ is

(a) $y^2 = x + \frac{1}{2} + ce^{2x}$ (b) $\frac{1}{y^2} = x + \frac{1}{2} + ce^{2x}$

(c) $y = x + \frac{c}{x}e^{-x}$ (d) $y = -x^2 + cx$

(e) none of above

10. *Penyelesaian am bagi persamaan* $\frac{dy}{dx} + y = xy^3$ *ialah*

(a) $y^2 = x + \frac{1}{2} + ce^{2x}$ *(b)* $\frac{1}{y^2} = x + \frac{1}{2} + ce^{2x}$

(c) $y = x + \frac{c}{x}e^{-x}$ *(d)* $y = -x^2 + cx$

(e) *bukan semua yang di atas*

11. The differential equation $y'' - 6y' + 9y = 6x^2 + 2 - 12e^{3x}$ has the complementary solution $y_c = c_1 e^{3x} + c_2 x e^{3x}$. A suitable choice for the particular solution, y_p, is given by

(a) $y_p = Ax^2 + Bx + C + Ee^{3x}$

(b) $y_p = Ax^2 + Bx + C + Exe^{3x}$

(c) $y_p = Ax^2 + Bx + C + Ex^2 e^{3x}$

(d) $y_p = x(Ax^2 + Bx + C) + Ee^{3x}$

(e) $y_p = x^2(Ax^2 + Bx + C) + Ee^{3x}$

11. *Persamaan pembezaan* $y'' - 6y' + 9y = 6x^2 + 2 - 12e^{3x}$ *mempunyai penyelesaian pelengkap* $y_c = c_1e^{3x} + c_2xe^{3x}$. *Suatu pilihan sesuai untuk penyelesaian khusus,* y_p, *ialah*

(a) $y_p = Ax^2 + Bx + C + Ee^{3x}$

(b) $y_p = Ax^2 + Bx + C + Exe^{3x}$

(c) $y_p = Ax^2 + Bx + C + Ex^2e^{3x}$

(d) $y_p = x(Ax^2 + Bx + C) + Ee^{3x}$

(e) $y_p = x^2(Ax^2 + Bx + C) + Ee^{3x}$

12. A general solution for the differential equation $y'' - 6y' + 9y = 6x^2 + 2 - 12e^{3x}$ is

(a) $y = c_1e^{3x} + c_2xe^{3x} + \frac{2}{3}x^2 + \frac{8}{9}x + \frac{2}{3} - 6x^2e^{3x}$

(b) $y = c_1e^{3x} + c_2xe^{3x} + \frac{2}{3}x^2 + \frac{8}{9}x + \frac{2}{3} - 6xe^{3x}$

(c) $y = c_1e^{3x} + c_2xe^{3x} + \frac{2}{3}x^2 + \frac{8}{9}x + \frac{2}{3} - 6e^{3x}$

(d) $y = c_1e^{3x} + c_2xe^{3x} + x(\frac{2}{3}x^2 + \frac{8}{9}x + \frac{2}{3}) - 6x^2e^{3x}$

(e) $y = c_1e^{3x} + c_2xe^{3x} + x^2(\frac{2}{3}x^2 + \frac{8}{9}x + \frac{2}{3}) - 6x^2e^{3x}$

12. *Suatu penyelesaian am bagi persamaan pembezaan* $y'' - 6y' + 9y = 6x^2 + 2 - 12e^{3x}$ *ialah*

(a) $y = c_1e^{3x} + c_2xe^{3x} + \frac{2}{3}x^2 + \frac{8}{9}x + \frac{2}{3} - 6x^2e^{3x}$

(b) $y = c_1e^{3x} + c_2xe^{3x} + \frac{2}{3}x^2 + \frac{8}{9}x + \frac{2}{3} - 6xe^{3x}$

(c) $y = c_1e^{3x} + c_2xe^{3x} + \frac{2}{3}x^2 + \frac{8}{9}x + \frac{2}{3} - 6e^{3x}$

(d) $y = c_1e^{3x} + c_2xe^{3x} + x(\frac{2}{3}x^2 + \frac{8}{9}x + \frac{2}{3}) - 6x^2e^{3x}$

(e) $y = c_1e^{3x} + c_2xe^{3x} + x^2(\frac{2}{3}x^2 + \frac{8}{9}x + \frac{2}{3}) - 6x^2e^{3x}$

13. A non homogeneous second order linear differential equation $y'' - 4y' + 4y = (x+1)e^{2x}$ has the complementary solution $y_c = c_1 y_1(x) + c_2 y_2(x)$ where $y_1(x) = e^{2x}$ and $y_2(x) = xe^{2x}$. Using the method of variation of parameters, we seek a particular solution of the form $y_p = u_1(x)e^{2x} + u_2(x)xe^{2x}$ where u_1' and u_2' are given by

(a) $u_1' = \frac{(x+1)xe^{4x}}{e^{4x}}, \quad u_2' = \frac{(x+1)e^{4x}}{e^{4x}}$

(b) $u_1' = -\frac{(x+1)xe^{4x}}{e^{4x}}, \quad u_2' = \frac{(x+1)e^{4x}}{e^{4x}}$

(c) $u_1' = -\frac{(x+1)xe^{4x}}{e^{4x}}, \quad u_2' = -\frac{(x+1)e^{4x}}{e^{4x}}$

(d) $u_1' = \frac{(x+1)xe^{4x}}{e^{4x}}, \quad u_2' = -\frac{(x+1)e^{4x}}{e^{4x}}$

(e) $u_1' = -(x+1)xe^{4x}, \quad u_2' = (x+1)e^{4x}$

13. Suatu persamaan pembezaan linear peringkat kedua tak homogen $y'' - 4y' + 4y = (x+1)e^{2x}$ *mempunyai penyelesaian pelengkap* $y_c = c_1 y_1(x) + c_2 y_2(x)$ *di mana* $y_1(x) = e^{2x}$ and $y_2(x) = xe^{2x}$. *Menggunakan kaedah perubahan parameter, kita mencari suatu penyelesaian khusus dalam bentuk* $y_p = u_1(x)e^{2x} + u_2(x)xe^{2x}$ *di mana* u_1' *dan* u_2' *diberikan oleh*

(a) $u_1' = \frac{(x+1)xe^{4x}}{e^{4x}}, \quad u_2' = \frac{(x+1)e^{4x}}{e^{4x}}$

(b) $u_1' = -\frac{(x+1)xe^{4x}}{e^{4x}}, \quad u_2' = \frac{(x+1)e^{4x}}{e^{4x}}$

(c) $u_1' = -\frac{(x+1)xe^{4x}}{e^{4x}}, \quad u_2' = -\frac{(x+1)e^{4x}}{e^{4x}}$

(d) $u_1' = \frac{(x+1)xe^{4x}}{e^{4x}}, \quad u_2' = -\frac{(x+1)e^{4x}}{e^{4x}}$

(e) $u_1' = -(x+1)xe^{4x}, \quad u_2' = (x+1)e^{4x}$

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14. A general solution for the differential equation $y''-4y'+4y=(x+1)e^{2x}$ is given by

(a) $y=c_1e^{2x}+c_2xe^{2x}+(-x^2-x)e^{2x}+(x+1)e^{2x}$

(b) $y=c_1e^{2x}+c_2xe^{2x}+(-x^3-\frac{1}{2}x^2)e^{2x}+(\frac{1}{2}x^2+x)e^{2x}$

(c) $y=c_1e^{2x}+c_2xe^{2x}+(-\frac{1}{3}x^3-x^2)e^{2x}+(\frac{1}{2}x^2+x)e^{2x}$

(d) $y=c_1e^{2x}+c_2xe^{2x}+\frac{1}{6}x^3e^{2x}+\frac{1}{2}x^2e^{2x}$

(e) $y=c_1e^{2x}+c_2xe^{2x}+\frac{1}{6}x^2e^{2x}+\frac{1}{2}xe^{2x}$

14. Suatu penyelesaian am bagi persamaan pembezaan $y''-4y'+4y=(x+1)e^{2x}$ *diberikan oleh*

(a) $y=c_1e^{2x}+c_2xe^{2x}+(-x^2-x)e^{2x}+(x+1)e^{2x}$

(b) $y=c_1e^{2x}+c_2xe^{2x}+(-x^3-\frac{1}{2}x^2)e^{2x}+(\frac{1}{2}x^2+x)e^{2x}$

(c) $y=c_1e^{2x}+c_2xe^{2x}+(-\frac{1}{3}x^3-x^2)e^{2x}+(\frac{1}{2}x^2+x)e^{2x}$

(d) $y=c_1e^{2x}+c_2xe^{2x}+\frac{1}{6}x^3e^{2x}+\frac{1}{2}x^2e^{2x}$

(e) $y=c_1e^{2x}+c_2xe^{2x}+\frac{1}{6}x^2e^{2x}+\frac{1}{2}xe^{2x}$

15. From $y'=y+2x,\ y(0)=1$ with $h=0.2$, the approximate value of $y(0.2)$ through Euler's method is

(a) 1.2642 (b) 1.2174
(c) 1.2000 (d) 1.2624
(e) insufficient information to compute $y(0.2)$

15. Daripada $y'=y+2x,\ y(0)=1$ *dengan menggunakan* $h=0.2$, *nilai anggaran bagi* $y(0.2)$ *melalui kaedah Euler ialah*

(a) 1.2642 *(b)* 1.2174
(c) 1.2000 *(d)* 1.2624
(e) *maklumat tidak mencukupi untuk menilai* $y(0.2)$

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16. From $y' = y \sin x$ with $h = 0.2$, the approximate value of $y(0.2)$ through Euler's method is

(a) 3.2879 (b) 3.2129
(c) 3.2147 (d) 3.2140
(e) insufficient information to compute $y(0.2)$

16. Daripada $y' = y \sin x$ dengan menggunakan $h = 0.2$, nilai anggaran bagi $y(0.2)$ melalui Kaedah Euler ialah

(a) 3.2879 *(b)* 3.2129
(c) 3.2147 *(d)* 3.2140
(e) maklumat tidak mencukupi untuk menilai $y(0.2)$

Let the system of differential equations

$$\frac{dx}{dt} = 4x - y$$

$$\frac{dy}{dt} = x + 2y$$

be written in the matrix form $\bar{x}\,'(t) = M\,\bar{x}(t)$, where $\bar{x}(t) = \begin{pmatrix} x \\ y \end{pmatrix}$ and M is the constant 2×2 matrix. Choose the correct answers for Questions 17, 18 and 19.

17. $M =$

(a) $\begin{pmatrix} 4 & 1 \\ -1 & 2 \end{pmatrix}$ (b) $\begin{pmatrix} -4 & 1 \\ -1 & -2 \end{pmatrix}$

(c) $\begin{pmatrix} 1 & 2 \\ 4 & -1 \end{pmatrix}$ (d) $\begin{pmatrix} 4 & -1 \\ 1 & 2 \end{pmatrix}$

(e) $\begin{pmatrix} 4 & 1 \\ 1 & -2 \end{pmatrix}$

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Katakan sistem persamaan pembezaan

$$\frac{dx}{dt} = 4x - y$$

$$\frac{dy}{dt} = x + 2y$$

ditulis dalam bentuk matriks $\bar{x}\,'(t) = M\,\bar{x}(t)$, *di mana* $\bar{x}(t) = \begin{pmatrix} x \\ y \end{pmatrix}$ *dan* M *ialah matriks pemalar* 2×2. *Pilih jawapan betul bagi soalan* 17, 18 *dan* 19.

17. $M =$

(a) $\begin{pmatrix} 4 & 1 \\ -1 & 2 \end{pmatrix}$ *(b)* $\begin{pmatrix} -4 & 1 \\ -1 & -2 \end{pmatrix}$

(c) $\begin{pmatrix} 1 & 2 \\ 4 & -1 \end{pmatrix}$ *(d)* $\begin{pmatrix} 4 & -1 \\ 1 & 2 \end{pmatrix}$

(e) $\begin{pmatrix} 4 & 1 \\ 1 & -2 \end{pmatrix}$

18. The characteristic values of M are

(a) 3, 3
(b) -3, 2
(c) 2, 2
(d) 2, 3, 5
(e) -1,2,7

18. *Nilai-nilai cirian* M *ialah*

(a) 3, 3 *(b)* -3, 2
(c) 2, 2 *(d)* 2, 3, 5
(e) -1,2,7

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19. The general solution of the system is

(a) $x = c_1e^{2t} + c_2e^{3t}$
$y = c_1e^{2t} + 3c_2e^{3t}$

(b) $x = c_1e^{3t} + c_2e^{3t}$
$y = c_1e^{3t} + 2c_2e^{3t}$

(c) $x = c_1e^{3t} + c_2(t+1)e^{3t}$
$y = c_1e^{3t} + c_2t\,e^{3t}$

(d) $x = c_1e^{3t} + c_2(t-2)e^{3t}$
$y = c_1e^{3t} - 2c_2\,t\,e^{3t}$

(e) $x = c_1e^{3t} + c_2(t-1)e^{3t}$
$y = c_1e^{3t} - c_2t\,e^{3t}$

19. Penyelesaian am bagi sistem tersebut ialah

(a) $x = c_1e^{2t} + c_2e^{3t}$
$y = c_1e^{2t} + 3c_2e^{3t}$

(b) $x = c_1e^{3t} + c_2e^{3t}$
$y = c_1e^{3t} + 2c_2e^{3t}$

(c) $x = c_1e^{3t} + c_2(t+1)e^{3t}$
$y = c_1e^{3t} + c_2t\,e^{3t}$

(d) $x = c_1e^{3t} + c_2(t-2)e^{3t}$
$y = c_1e^{3t} - 2c_2\,t\,e^{3t}$

(e) $x = c_1e^{3t} + c_2(t-1)e^{3t}$
$y = c_1e^{3t} - c_2t\,e^{3t}$

20. Given the initial value problem $(1+x^3)dy - x^2y\,dx = 0,\ \ y(2) = -1,$ choose the statement/statements which are **true**.

(i) Both the functions $y_1(x) = 1-x, x \geq 2$ and $y_2(x) = -x^2/4$ satisfy the above initial value problem.
(ii) The Existence and Uniqueness Theorem **does not** guarantee a unique solution through the points that lie on the curve $y_2(x) = -x^2/4$.
(iii) The Existence and Uniqueness Theorem **does not** guarantee a unique solution through the point (2,-1).

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(iv) Through any given point (x_0, y_0) in the xy-plane there passes one and only one solution of the above differential equation.

(a) (i) and (ii) only
(b) (i), (ii) and (iii) only
(c) (i) and (iii) only
(d) (iv) only
(e) (i) and (iv) only

20. *Diberikan masalah milai awal* $(1+x^3)dy - x^2 y dx = 0,\ y(2) = -1,$ *pilih pernyataan /pernyataan-pernyataan yang* ***benar***

(i) *Kedua-dua fungsi* $y_1(x) = 1 - x, x \geq 2$ *dan* $y_2(x) = -x^2/4$ *memenuhi masalah nilai awal di atas.*
(ii) *Teoram Kewujudan dan Keunikan* ***tidak*** *menjamin suatu penyelesaian unik wujud melalui titik-titik yang terletak pada lengkung* $y_2(x) = -x^2/4$.
(iii) *Teoram Kewujudan dan Keunikan* ***tidak*** *menjamin suatu penyelesaian unik wujud melalui titik* (2,-1).
(iv) *Melalui sebarang titik* (x_0, y_0) *yang terletak pada satah-* xy, *wujud satu dan hanya satu penyelesaian bagi persamaan pembezaan di atas.*

(a) *(i) dan (ii) sahaja*
(b) *(i), (ii) dan(iii) sahaja*
(c) *(i) dan (iii) sahaja*
(d) *(iv) sahaja*
(e) *(i) dan (iv) sahaja*

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Section II: Answer both questions [50/100].
Bahagian II: Jawab kedua-dua soalan [50/100].

1. (a) A 12-volt battery is connected to a series circuit in which the inductance is ½ henry and the resistance is 10 ohms. Determine the current i if the initial current is zero.

(b) (i) What is the radius of convergence of the Taylor series for $\left(x^2-2x+2\right)^{-1}$ about x = 1 ?

(ii) Determine a lower bound for the radius of convergence of series solutions of the differential equation $(1+x^2)y''+2xy'+4x^2y=0$ about the point $x = 0$.

(iii) Find a power series solution of $y''+y=0, (-\infty<x<\infty)$.

(c) Let $x(t)$ and $y(t)$ be the populations of two species at time t. Suppose that the species coexist peacefully, that is, each population increases proportionately to the other population and decreases proportionately to its own population. Then, the two populations satisfy the following equations

$$x' = -4x + 3y$$
$$y' = 8x - 6y$$

(i) If the initial population is $x(0)=y(0)=3000$, determine the populations $x(t)$ and $y(t)$ at time t.

(ii) Graph the two populations $x(t)$ and $y(t)$.

(iii) Find $x(t)$ and $y(t)$ when $t \to \infty$.

1. (a) Suatu bateri 12-volt disambung ke suatu litar elektrik bersiri di mana induktans ialah ½ henry dan rintangan ialah 10 ohms. Tentukan arus elektrik i jika arus awal ialah sifar.

(b) (i) Apakah jejari penumpuan siri Taylor bagi $\left(x^2-2x+2\right)^{-1}$ sekitar $x = 1$?

(ii) Tentukan batas bawah jejari penumpuan penyelesaian siri bagi persamaan pembezaan $(1+x^2)y''+2xy'+4x^2y=0$ sekitar titik $x = 0$.

(iii) Dapatkan penyelesaian siri kuasa bagi $y''+y=0, (-\infty<x<\infty)$.

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(c) *Katakan x(t) dan y(t) ialah populasi dua spesis pada masa t. Katakan spesis-spesis ini wujud secara aman, iaitu, setiap spesis bertambah berkadaran dengan populasi yang satu lagi dan berkurang berkadaran dengan populasinya sendiri. Maka, kedua-dua populasi memenuhi persamaan-persamaan berikut:*

$$x' = -4x + 3y$$
$$y' = 8x - 6y$$

(i) *Jika populasi awal ialah* $x(0)=y(0)=3000$, *tentukan populasi x(t) dan y(t) pada masa t.*

(ii) *Grafkan kedua-dua populasi x(t) dan y(t).*

(iii) *Dapatkan x(t) dan y(t) apabila* $t \to \infty$.

2. (a) Let the vector functions $\bar{\varphi}_1(t), \bar{\varphi}_2(t), \bar{\varphi}_3(t), \cdots, \bar{\varphi}_n(t)$ defined by $\bar{\varphi}_i(t) = \left[\varphi_{1i}(t), \varphi_{2i}(t), \cdots, \varphi_{ni}(t)\right]^T$ be n solutions of the homogeneous linear vector differential equation $\bar{x}\,' = A(t)\bar{x}$ where

$$\bar{x}\,' = \left[x_1'(t), x_2'(t), \cdots, x_n'(t)\right]^T,$$
$$\bar{x} = \left[x_1(t), x_2(t), \cdots, x_n(t)\right]^T,$$
$$A(t) = \left[a_{ij}(t)\right] \text{ of order } n \times n.$$

If the n vectors $\bar{\varphi}_i(t)$ are linearly independent on $a \le t \le b$, then prove that the Wronskian of these vectors, $W(\bar{\varphi}_1, \bar{\varphi}_2, \cdots, \bar{\varphi}_n)(t) \ne 0$ for all $t \in [a,b]$.

(b) Show that, the solutions

$$\bar{\varphi}_1 = \begin{pmatrix} e^t \\ -e^t \\ -e^t \end{pmatrix}, \quad \bar{\varphi}_2 = \begin{pmatrix} e^{3t} \\ -2e^{3t} \\ -e^{3t} \end{pmatrix} \text{ and } \bar{\varphi}_3 = \begin{pmatrix} 3e^{5t} \\ -6e^{5t} \\ -2e^{5t} \end{pmatrix}$$

are the linearly independent solutions of the system of equations

$$x' = 7x - y + 6z$$
$$y' = -10x + 4y - 12z$$
$$z' = -2x + y - z$$

and hence find the general solution.

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2. (a) *Katakan fungsi-fungsi vektor* $\bar{\varphi}_1(t), \bar{\varphi}_2(t), \bar{\varphi}_3(t), \cdots, \bar{\varphi}_n(t)$ *yang ditakrifkan sebagai* $\bar{\varphi}_i(t) = [\varphi_{1i}(t), \varphi_{2i}(t), \cdots, \varphi_{ni}(t)]^T$ *merupakan* n *penyelesaian bagi persamaan pembezaan vektor linear tak homogen* $\bar{x}\,' = A(t)\bar{x}$ *di mana*

$$\bar{x}\,' = \left[x_1'(t), x_2'(t), \cdots, x_n'(t) \right]^T,$$
$$\bar{x} = \left[x_1(t), x_2(t), \cdots, x_n(t) \right]^T,$$
$$A(t) = \left[a_{ij}(t) \right] \text{ berperingkat } n \times n.$$

Jika n *vektor* $\bar{\varphi}_i(t)$ *adalah tak bersandar linear pada* $a \le t \le b$, *maka buktikan bahawa Wronskian vektor-vektor ini,* $W(\bar{\varphi}_1, \bar{\varphi}_2, \cdots, \bar{\varphi}_n)(t) \neq 0$ *bagi semua* $t \in [a,b]$.

(b) *Tunjukkan bahawa, penyelesaian*

$$\bar{\varphi}_1 = \begin{pmatrix} e^t \\ -e^t \\ -e^t \end{pmatrix}, \quad \bar{\varphi}_2 = \begin{pmatrix} e^{3t} \\ -2e^{3t} \\ -e^{3t} \end{pmatrix} \text{ dan } \bar{\varphi}_3 = \begin{pmatrix} 3e^{5t} \\ -6e^{5t} \\ -2e^{5t} \end{pmatrix}$$

adalah penyelesaian tak bersandar secara linear bagi sistem persamaan

$$x' = 7x - y + 6z$$
$$y' = -10x + 4y - 12z$$
$$z' = -2x + y - z$$

dan seterusnya, dapatkan penyelesaian amnya.

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