UNIVERSITI SAINS MALAYSIA

Second Semester Examination
Academic Session 2006/2007

April 2007

MAT 122 – Differential Equations I
[Persamaan Pembezaan I]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of NINETEEN pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi SEMBILAN BELAS muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions: Answer all **TWENTY** (20) questions in Section I using the objective answer paper (OMR answer paper) provided. For this section, answers should be written in 2B pencil only. **The OMR answer paper together with the question paper of Section I** will be collected 1½ hours after the examination starts.

Answer all **TWO** (2) questions in Section II. All answers in this section must be written on the answer script papers provided.

*[**Arahan**: Jawab semua **DUA PULUH** (20) soalan dalam Bahagian I dengan menggunakan kertas jawapan soalan objektif (kertas jawapan OMR) yang disediakan. Bagi bahagian ini, jawapan perlu dituliskan dengan pensel 2B sahaja. **Kertas jawapan OMR ini berserta kertas soalan Bahagian I** akan dikutip 1½ jam setelah peperiksaan bermula.*

*Jawab semua **DUA** (2) soalan dalam Bahagian II. Semua jawapan dalam bahagian ini mestilah dituliskan pada kertas skrip jawapan yang disediakan.]*

...2/-

Section 1: Answer ALL 20 questions. Each correct answer will be given 2 ½ marks [50/100].

1. Choose the **non linear** differential equation from the following:

   (a) $x\dfrac{dy}{dx} = x^2 y + \cos x$ (b) $\dfrac{dy}{dx} = x\sin y$

   (c) $\dfrac{d^2y}{dx^2} + \dfrac{dy}{dx} = x^2$ (d) $y' = xy$

   (e) none of above.

2. Choose the **false** statement regarding the following differential equation:

$$\left(\frac{d^3x}{dt^3}\right)^4 + \left(\frac{d^2x}{dt^2}\right)^2 + \frac{dx}{dt} + x = 1$$

   (a) is an ordinary differential equation
   (b) x is dependent variable
   (c) a 4th degree differential equation
   (d) a 4th order differential equation
   (e) t is the independent variable

3. The solution of the initial value problem $y' + y = 1,\ y(0) = 2$ is

   (a) $y = e^{-x} + 1$ (b) $y = e^{-x} - 1$

   (c) $y = e^{x} + 1$ (d) $y = e^{x} - 1$

   (e) $y = e^{2x} + 1$

4. The general solution of the differential equation $\left(e^{2y} - y\cos xy\right)dx + \left(2xe^{2y} - x\cos xy + 2y\right)dy = 0$ is

   (a) $xe^{y} - \sin xy + y^2 + c = 0$

   (b) $xe^{y} - \cos xy + y^2 + c = 0$

   (c) $xe^{2y} - \sin xy + y^2 + c = 0$

   (d) $xe^{y} - \sin xy + y + c = 0$

   (e) $e^{y} - \sin xy + y^2 + c = 0$

5. An integrating factor for the equation $x\dfrac{dy}{dx} + y = x^2y^2$ is

   (a) x^2 (b) $\log x$

   (c) x^{-4} (d) $1/x$

   (e) x

...3/-

Bahagian 1: Jawab SEMUA 20 soalan. Setiap jawapan betul diberi 2 ½ markah [50/100].

1. *Pilih persamaan pembezaan yang **tak linear** daripada yang berikut:*

   (a) $x\dfrac{dy}{dx} = x^2 y + \cos x$ (b) $\dfrac{dy}{dx} = x \sin y$

   (c) $\dfrac{d^2y}{dx^2} + \dfrac{dy}{dx} = x^2$ (d) $y' = xy$

   (e) *bukan semua yang di atas*

2. *Pilih pernyataan yang **tidak benar** berkenaan persamaan pembezaan berikut:*

$$\left(\frac{d^3x}{dt^3}\right)^4 + \left(\frac{d^2x}{dt^2}\right)^2 + \frac{dx}{dt} + x = 1$$

   (a) *ialah suatu persamaan pembezaan biasa*
   (b) *x ialah pembolehubah bersandar*
   (c) *ialah suatu persamaan pembezaan berdarjah 4*
   (d) *ialah suatu persamaan pembezaan berperingkat 4*
   (e) *t ialah pembolehubah tak bersandar*

3. *Penyelesaian bagi persamaan pembezaan* $y' + y = 1, y(0) = 2$ *ialah*

   (a) $y = e^{-x} + 1$ (b) $y = e^{-x} - 1$

   (c) $y = e^{x} + 1$ (d) $y = e^{x} - 1$

   (e) $y = e^{2x} + 1$

4. *Penyelesaian am bagi persamaan pembezaan*
   $\left(e^{2y} - y\cos xy\right)dx + \left(2xe^{2y} - x\cos xy + 2y\right)dy = 0$ *ialah*

   (a) $xe^{y} - \sin xy + y^2 + c = 0$
   (b) $xe^{y} - \cos xy + y^2 + c = 0$
   (c) $xe^{2y} - \sin xy + y^2 + c = 0$
   (d) $xe^{y} - \sin xy + y + c = 0$
   (e) $e^{y} - \sin xy + y^2 + c = 0$

5. *Suatu faktor pengamir bagi persamaan* $x\dfrac{dy}{dx} + y = x^2y^2$ *ialah*

   (a) x^2 (b) $\log x$
   (c) x^{-4} (d) $1/x$
   (e) x

...4/-

6. The solution of the equation $\left(x^2+y^2\right)dx+\left(x^2-xy\right)dy=0$ is

(a) $\left(x+y\right)^2=cxe^{y/x}$
(b) $\left(x+y\right)^2=cxe^{-y/x}$
(c) $x+y=cxe^{y/x}$
(d) $\left(x+y\right)^2=cx^2e^{y/x}$
(e) $\left(x+y\right)^2=xe^{y/x}$

7. A particular solution of the differential equation $\dfrac{dy}{dx}=\dfrac{xy^2-\cos x\sin x}{y(1-x^2)}$, $y(0)=2$, is

(a) $y^2(1-x^2)-\cos x=3$
(b) $y^2(1-x^2)-\cos^2 x=3$
(c) $y^2(1-x^2)-\sin x=3$
(d) $y^2(1-x^2)-\sin^2 x=3$
(e) $y(1-x^2)-\sin^2 x=3$

8. The set of orthogonal trajectories of the family of circles $x^2+y^2=r^2$ is

(a) $x^2-y^2=k^2$
(b) $x+y=k$
(c) $y=cx^2$
(d) $y=kx$
(e) $y=-cx$

9. Choose the particular solution of the equation $y''+4y=0, y(0)=1, y'(0)=-1$ from the following:

(a) $y(x)=\cos 2x-\dfrac{1}{2}\sin 2x$
(b) $(b)y(x)=\cos 2x+\dfrac{1}{2}\sin 2x$
(c) $(c)y(x)=\cos 2x-\sin 2x$
(d) $y(x)=\dfrac{1}{2}\cos 2x-\sin 2x$
(e) None of the above

10. Using the method of undetermined coefficients, a particular solution for $y''-3y'-4y=e^{-x}$ is

(a) $y_p(x)=\dfrac{1}{5}xe^{-x}$
(b) $y_p(x)=-\dfrac{1}{5}xe^{-x}$
(c) $y_p(x)=-\dfrac{1}{5}xe^{x}$
(d) $y_p(x)=-\dfrac{1}{5}e^{-x}$
(e) $y_p(x)=-\dfrac{1}{5}x^2e^{-x}$

...5/-

6. *Suatu penyelesaian bagi persamaan* $\left(x^2+y^2\right)dx+\left(x^2-xy\right)dy=0$ *ialah*

(a) $(x+y)^2 = cxe^{y/x}$

(b) $(x+y)^2 = cxe^{-y/x}$

(c) $x+y = cxe^{y/x}$

(d) $(x+y)^2 = cx^2e^{y/x}$

(e) $(x+y)^2 = xe^{y/x}$

7. *Suatu penyelesaian khusus bagi persamaan pembezaan*

$\dfrac{dy}{dx}=\dfrac{xy^2-\cos x\sin x}{y(1-x^2)}$, $y(0)=2$, *ialah*

(a) $y^2(1-x^2)-\cos x=3$

(b) $y^2(1-x^2)-\cos^2 x=3$

(c) $y^2(1-x^2)-\sin x=3$

(d) $y^2(1-x^2)-\sin^2 x=3$

(e) $y(1-x^2)-\sin^2 x=3$

8. *Set trajektori ortogon bagi keluarga bulatan* $x^2+y^2=r^2$ *ialah*

(a) $x^2-y^2=k^2$

(b) $x+y=k$

(c) $y=cx^2$

(d) $y=kx$

(e) $y=-cx$

9. *Pilih suatu penyelesaian khusus bagi persamaan* $y''+4y=0, y(0)=1, y'(0)=-1$ *daripada yang berikut:*

(a) $y(x)=\cos 2x-\dfrac{1}{2}\sin 2x$

(b) $(b)y(x)=\cos 2x+\dfrac{1}{2}\sin 2x$

(c) $(c)y(x)=\cos 2x-\sin 2x$

(d) $y(x)=\dfrac{1}{2}\cos 2x-\sin 2x$

(e) *bukan semua yang di atas*

10. *Dengan menggunakan kaedah pekali belum tentu, suatu penyelesaian khusus bagi* $y''-3y'-4y=e^{-x}$ *ialah*

(a) $y_p(x)=\dfrac{1}{5}xe^{-x}$

(b) $y_p(x)=-\dfrac{1}{5}xe^{-x}$

(c) $y_p(x)=-\dfrac{1}{5}xe^{x}$

(d) $y_p(x)=-\dfrac{1}{5}e^{-x}$

(e) $y_p(x)=-\dfrac{1}{5}x^2e^{-x}$,

...6/-

INSTRUCTION: Answer questions (11), (12), (13) and (14) referring to the following information:

Given the Hermite equation

$$y'' - 2xy' + \lambda y = 0, \quad -\infty < x < \infty \qquad \text{(I)}$$

where λ is a constant.

11. By substituting $y = \sum_{n=0}^{\infty} a_n x^n$ in the differential equation (I), choose the equivalent equation for equation (I) from the following:

(a) $\sum_{n=1}^{\infty}[(n+2)(n+1)a_{n+2} - (2n-\lambda)a_n]x^n = 0$

(b) $\sum_{n=0}^{\infty}[(n+2)(n+1)a_{n+2}]x^n + \sum_{n=1}^{\infty}(\lambda - 2n)a_n x^n = 0$

(c) $\sum_{n=1}^{\infty}[(n+2)(n+1)a_{n+2} - 2na_n]x^n + \sum_{n=0}^{\infty}\lambda a_n x^n = 0$

(d) $(2a_2 + \lambda a_0) + \sum_{n=1}^{\infty}[(n+2)(n+1)a_{n+2} - 2na_n + \lambda a_n]x^n = 0$

(e) $\sum_{n=1}^{\infty}[n(n+1)a_{n+2} - 2na_n]x^n + \sum_{n=0}^{\infty}\lambda a_n x^n = 0$

12. The recurrence relation for a_n is given by

(a) $a_2 = 0, \quad a_{n+2} = \dfrac{(2n-\lambda)}{(n+2)(n+1)}a_n \quad \text{for } n \geq 1$

(b) $a_2 = -\lambda a_0/2, \quad a_{n+2} = \dfrac{(2n-\lambda)}{(n+2)(n+1)}a_n \quad \text{for } n \geq 1$

(c) $a_2 = 0, \quad a_{n+2} = \dfrac{(2n-\lambda)}{(n+2)(n-1)}a_n \quad \text{for } n \geq 1$

(d) $a_0 = 0, \quad a_{n+2} = \dfrac{(2n-\lambda)}{(n+2)(n+1)}a_n \quad \text{for } n \geq 0$

(e) $a_0 = 0, \quad a_{n+2} = \dfrac{(2n-\lambda)}{(n+3)(n+2)}a_n \quad \text{for } n \geq 0$

ARAHAN: Jawab soalan-soalan (11), (12), (13) dan (14) berpandukan maklumat berikut:

Diberikan persamaan Hermite

$$y'' - 2xy' + \lambda y = 0, \quad -\infty < x < \infty \qquad (I)$$

di mana λ ialah suatu pemalar.

11. Dengan menggantikan $y = \sum_{n=0}^{\infty} a_n x^n$ di dalam persamaan pembezaan (I), pilih persamaan yang setara untuk persamaan (I) daripada yang berikut:

(a) $\sum_{n=1}^{\infty} [(n+2)(n+1)a_{n+2} - (2n-\lambda)a_n]x^n = 0$

(b) $\sum_{n=0}^{\infty} [(n+2)(n+1)a_{n+2}]x^n + \sum_{n=1}^{\infty} (\lambda - 2n)a_n x^n = 0$

(c) $\sum_{n=1}^{\infty} [(n+2)(n+1)a_{n+2} - 2na_n]x^n + \sum_{n=0}^{\infty} \lambda a_n x^n = 0$

(d) $(2a_2 + \lambda a_0) + \sum_{n=1}^{\infty} [(n+2)(n+1)a_{n+2} - 2na_n + \lambda a_n]x^n = 0$

(e) $\sum_{n=1}^{\infty} [n(n+1)a_{n+2} - 2na_n]x^n + \sum_{n=0}^{\infty} \lambda a_n x^n = 0$

12. Hubungan jadi semula bagi a_n diberikan oleh

(a) $a_2 = 0, \quad a_{n+2} = \dfrac{(2n-\lambda)}{(n+2)(n+1)} a_n \quad \text{for } n \geq 1$

(b) $a_2 = -\lambda a_0/2, \quad a_{n+2} = \dfrac{(2n-\lambda)}{(n+2)(n+1)} a_n \quad \text{for } n \geq 1$

(c) $a_2 = 0, \quad a_{n+2} = \dfrac{(2n-\lambda)}{(n+2)(n-1)} a_n \quad \text{for } n \geq 1$

(d) $a_0 = 0, \quad a_{n+2} = \dfrac{(2n-\lambda)}{(n+2)(n+1)} a_n \quad \text{for } n \geq 0$

(e) $a_0 = 0, \quad a_{n+2} = \dfrac{(2n-\lambda)}{(n+3)(n+2)} a_n \quad \text{for } n \geq 0$

13. The series solution for differential equation (I) is given by

(a) $y = a_0\left[1 - \frac{(4-\lambda)}{4!}\lambda x^4 - \frac{(8-\lambda)(4-\lambda)}{6!}\lambda x^6 - \cdots\right] + a_2 x^2$
$+ a_1\left[x + \frac{(2-\lambda)}{3!}x^3 + \frac{(6-\lambda)(2-\lambda)}{5!}x^5 + \frac{(10-\lambda)(6-\lambda)(2-\lambda)}{7!}x^7\right] + \cdots$

(b) $y = a_0\left[1 - (4-\lambda)\lambda x^4 - (8-\lambda)(4-\lambda)\lambda x^6 - \cdots\right]$
$+ a_1\left[x + \frac{(2-\lambda)}{3!}x^3 + \frac{(6-\lambda)(2-\lambda)}{5!}x^5 + \frac{(10-\lambda)(6-\lambda)(2-\lambda)}{7!}x^7\right] + \cdots$

(c) $y = a_0\left[1 - \frac{\lambda}{2!}x^2 - \frac{(4-\lambda)}{4!}\lambda x^4 - \frac{(8-\lambda)(4-\lambda)}{6!}\lambda x^6 - \cdots\right]$
$+ a_1\left[x + \frac{(2-\lambda)}{3!}x^3 + \frac{(6-\lambda)(2-\lambda)}{5!}x^5 + \frac{(10-\lambda)(6-\lambda)(2-\lambda)}{7!}x^7 + \cdots\right]$

(d) $y = a_1\left[x + \frac{(2-\lambda)}{3!}x^3 + \frac{(6-\lambda)(2-\lambda)}{5!}x^5 + \frac{(10-\lambda)(6-\lambda)(2-\lambda)}{7!}x^7\right] + \cdots$

(e) $y = a_1\left[x + \frac{(-2\lambda)}{4!}x^3 + \frac{(6-\lambda)(-2\lambda)}{5!}x^5 + \frac{(10-\lambda)(6-\lambda)(-2\lambda)}{8!}x^7\right] + \cdots$

14. The radius of convergence, ρ, for the series solution of the differential equation (I) is

(a) $\rho = \infty$
(b) $\rho = 1$
(c) $\rho = 3$
(d) $\rho = 2$
(e) none of the above

15. The singular point/points of the equation

$$(1 + x^2)y'' + 2xy' + 4x^2 y = 0$$

is/are

(a) 0
(b) 2
(c) $\pm i$
(d) ± 1
(e) none of the above

13. *Penyelesaian siri bagi persamaan pembezaan (I) diberikan oleh*

(a) $$y = a_0\left[1-\frac{(4-\lambda)}{4!}\lambda x^4 - \frac{(8-\lambda)(4-\lambda)}{6!}\lambda x^6 - \cdots\right] + a_2x^2$$
$$+a_1\left[x+\frac{(2-\lambda)}{3!}x^3+\frac{(6-\lambda)(2-\lambda)}{5!}x^5+\frac{(10-\lambda)(6-\lambda)(2-\lambda)}{7!}x^7\right]+\cdots$$

(b) $$y = a_0\left[1-(4-\lambda)\lambda x^4-(8-\lambda)(4-\lambda)\lambda x^6-\cdots\right]$$
$$+a_1\left[x+\frac{(2-\lambda)}{3!}x^3+\frac{(6-\lambda)(2-\lambda)}{5!}x^5+\frac{(10-\lambda)(6-\lambda)(2-\lambda)}{7!}x^7\right]+\cdots$$

(c) $$y = a_0\left[1-\frac{\lambda}{2!}x^2-\frac{(4-\lambda)}{4!}\lambda x^4-\frac{(8-\lambda)(4-\lambda)}{6!}\lambda x^6-\cdots\right]$$
$$+a_1\left[x+\frac{(2-\lambda)}{3!}x^3+\frac{(6-\lambda)(2-\lambda)}{5!}x^5+\frac{(10-\lambda)(6-\lambda)(2-\lambda)}{7!}x^7+\cdots\right]$$

(d) $$y = a_1\left[x+\frac{(2-\lambda)}{3!}x^3+\frac{(6-\lambda)(2-\lambda)}{5!}x^5+\frac{(10-\lambda)(6-\lambda)(2-\lambda)}{7!}x^7\right]+\cdots$$

(e) $$y = a_1\left[x+\frac{(-2\lambda)}{4!}x^3+\frac{(6-\lambda)(-2\lambda)}{5!}x^5+\frac{(10-\lambda)(6-\lambda)(-2\lambda)}{8!}x^7\right]+\cdots$$

14. *Jejari penumpuan,* ρ, *bagi penyelesaian siri kuasa persamaan pembezaan (I) ialah*

(a) $\rho = \infty$
(b) $\rho = 1$
(c) $\rho = 3$
(d) $\rho = 2$
(e) *bukan semua yang di atas*

15. *Titik (-titik) singular bagi persamaan pembezaan*

$$(1+x^2)y'' + 2xy' + 4x^2y = 0$$

ialah

(a) 0
(b) 2
(c) $\pm i$
(d) ± 1
(e) *bukan semua yang di atas*

...10/-

16. Suppose the given vectors below are solutions of the system $\mathbf{X}' = \mathbf{AX}$. Choose the set of vectors that form a fundamental set on $(-\infty, \infty)$ from the following:

(i) $\mathbf{X}_1 = \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{-2t}, \quad \mathbf{X}_2 = \begin{pmatrix} 3 \\ 5 \end{pmatrix} e^{6t}$

(ii) $\mathbf{X}_1 = \begin{pmatrix} 1 \\ 2 \end{pmatrix} e^{3t}, \quad \mathbf{X}_2 = \begin{pmatrix} 1 \\ -2 \end{pmatrix} e^{-t}$

(iii) $\mathbf{X}_1 = \begin{pmatrix} 1 \\ \sqrt{2} \end{pmatrix} e^{-t}, \quad \mathbf{X}_2 = \begin{pmatrix} -\sqrt{2} \\ 1 \end{pmatrix} e^{-4t}$

(iv) $\mathbf{X}_1 = \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{t}, \quad \mathbf{X}_2 = \begin{pmatrix} 2 \\ 6 \end{pmatrix} e^{t} + \begin{pmatrix} 8 \\ -8 \end{pmatrix} te^{t}$

(a) (i) only
(b) (ii) only
(c) (i) and (ii) only
(d) (i), (ii) and (iii) only
(e) (i), (ii), (iii) and (iv)

Let the system of differential equations

$$\frac{dx}{dt} = -x + y, \quad \frac{dy}{dt} = 2x + y + 2z, \quad \frac{dz}{dt} = -x + y + 2z$$

be written in the matrix form $\mathbf{X}'(t) = \mathbf{M}\,\mathbf{X}(t)$, where $\mathbf{X}(t) = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ and $\mathbf{M}$ is the constant 3×3 matrix. Choose the correct answers for Questions 17, 18 and 19.

17. $\mathbf{M} =$

(a) $\begin{pmatrix} -1 & 2 & -1 \\ 1 & 1 & 1 \\ 0 & 2 & 2 \end{pmatrix}$

(b) $\begin{pmatrix} -1 & 1 & 0 \\ 2 & 1 & 2 \\ -1 & 1 & 2 \end{pmatrix}$

(c) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

(d) $\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$

(e) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$

...11/-

16. *Katakan vektor-vektor yang diberikan di bawah merupakan penyelesaian bagi sistem* **X′ = AX**. *Pilih set vektor yang membentuk suatu set asas pada* $(-\infty,\infty)$ *daripada yang berikut:*

(i) $\mathbf{X}_1 = \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{-2t}, \quad \mathbf{X}_2 = \begin{pmatrix} 3 \\ 5 \end{pmatrix} e^{6t}$

(ii) $\mathbf{X}_1 = \begin{pmatrix} 1 \\ 2 \end{pmatrix} e^{3t}, \quad \mathbf{X}_2 = \begin{pmatrix} 1 \\ -2 \end{pmatrix} e^{-t}$

(iii) $\mathbf{X}_1 = \begin{pmatrix} 1 \\ \sqrt{2} \end{pmatrix} e^{-t}, \quad \mathbf{X}_2 = \begin{pmatrix} -\sqrt{2} \\ 1 \end{pmatrix} e^{-4t}$

(iv) $\mathbf{X}_1 = \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{t}, \quad \mathbf{X}_2 = \begin{pmatrix} 2 \\ 6 \end{pmatrix} e^{t} + \begin{pmatrix} 8 \\ -8 \end{pmatrix} te^{t}$

(a) (i) *sahaja*
(b) (ii) *sahaja*
(c) (i) *dan* (ii) *sahaja*
(d) (i), (ii) *dan* (iii) *sahaja*
(e) (i), (ii), (iii) *dan* (iv)

Katakan sistem persamaan pembezaan

$$\frac{dx}{dt} = -x + y, \quad \frac{dy}{dt} = 2x + y + 2z, \quad \frac{dz}{dt} = -x + y + 2z$$

boleh ditulis dalam bentuk matriks $\mathbf{X}'(t) = \mathbf{M}\,\mathbf{X}(t)$, *di mana* $\mathbf{X}(t) = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ *dan* $\mathbf{M}$ *ialah matriks pemalar* 3×3. *Pilih jawapan yang betul bagi Soalan 17, 18 and 19.*

17. $\mathbf{M} =$

(a) $\begin{pmatrix} -1 & 2 & -1 \\ 1 & 1 & 1 \\ 0 & 2 & 2 \end{pmatrix}$

(b) $\begin{pmatrix} -1 & 1 & 0 \\ 2 & 1 & 2 \\ -1 & 1 & 2 \end{pmatrix}$

(c) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

(d) $\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$

(e) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$

...12/-

Section II: Answer both questions [50/100].

1. (a) Show that $y = c_1 y_1(x) + c_2 y_2(x)$ is a solution of $a_0(x)y'' + a_1(x)y' + a_2(x)y = 0$, where y_1 and y_2 are the fundamental set of solutions of the given equation and c_1 and c_2 are arbitrary constants. Is it a general solution? Give reasons.

   (b) A body of temperature 80° F is placed in a room of constant temperature 50° F at time $t = 0$; and at the end of 5 minutes, the body has cooled to a temperature of 70 ° F. Determine the temperature of the body as a function of time t, for $t > 0$.

   (c) Define complementary function and particular integral. Using the method of variation of parameters find a particular integral of the equation

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = (x+1)e^{2x}.$$

2. (a) Given the in initial value problem

$$y' = \frac{1}{2} - x + 2y, \qquad y(0) = 1 \qquad \text{(II)}$$

   (i) By using Euler's formula, show that

$$y_k = (1+2h)y_{k-1} + h(\frac{1}{2} - x_{k-1})$$

   (ii) Show that $y_1 = (1+2h) + x_1/2$ and subsequently,

$$y_n = (1+2h)^n + x_n/2$$

   (iii) Obtain the local formula error e_{n+1}, in terms of x and the exact solution ϕ if the Euler's method is used for the initial value problem (II) above.

   (b) Consider the two interconnected tanks shown in Figure 1 below.

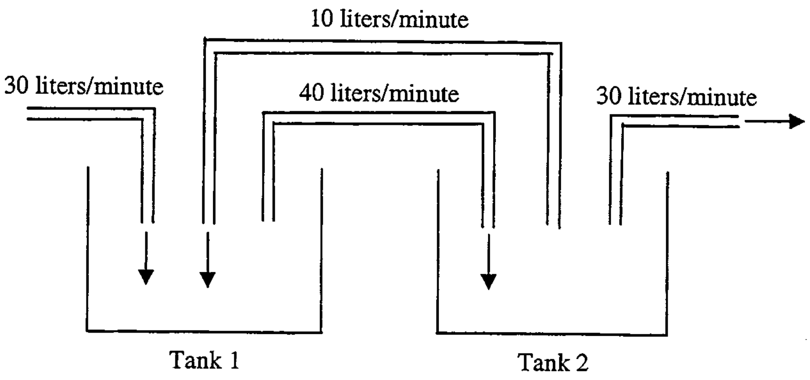


Figure 1

...17/-

Bahagian II: Jawab kedua-dua soalan [50/100].

1. (a) *Tunjukkan bahawa* $y = c_1 y_1(x) + c_2 y_2(x)$ *ialah suatu penyelesaian bagi* $a_0(x)y'' + a_1(x)y' + a_2(x)y = 0$, *di mana* y_1 *dan* y_2 *ialah set asas penyelesaian bagi persamaan tersebut dan* c_1 *dan* c_2 *ialah pemalar sebarangan. Adakah ia suatu penyelesaian am? Berikan sebab-sebab.*

   (b) *Suatu benda yang mempunyai suhu 80° F, diletakkan di dalam suatu bilik yang mempunyai suhu malar 50° F pada masa t = 0; dan pada akhir 5 minit, benda tersebut telah kurang suhunya kepada 70 ° F. Tentukan suhu benda tersebut sebagai suatu fungsi masa, t, bagi t > 0.*

   (c) *Takrifkan fungsi pelengkap dan kamiran khusus. Dengan menggunakan kaedah ubahan parameter, dapatkan suatu kamiran khusus bagi persamaan*

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = (x+1)e^{2x}.$$

2. (a) *Diberikan masalah nilai awal*

$$y' = \frac{1}{2} - x + 2y, \quad y(0) = 1 \qquad (II)$$

   (i) *Tunjukkan dengan mengguna rumus Euler bahawa*

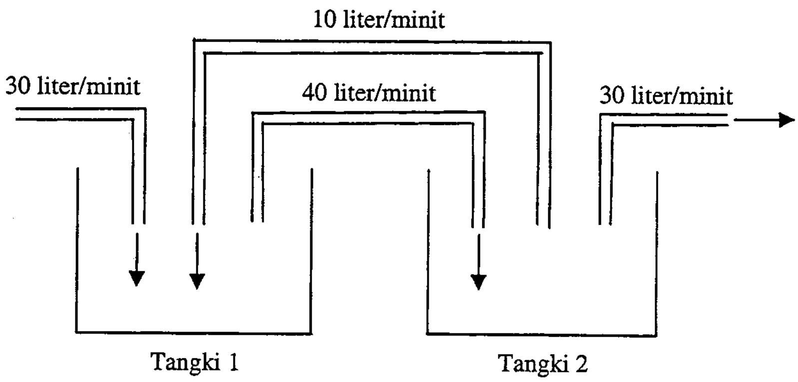
$$y_k = (1+2h)y_{k-1} + h(\frac{1}{2} - x_{k-1})$$

   (ii) *Tunjukkan bahawa* $y_1 = (1+2h) + x_1/2$ *dan seterusnya,*

$$y_n = (1+2h)^n + x_n/2$$

   (iii) *Dapatkan satu rumus bagi ralat rumus setempat* e_{n+1} *dalam sebutan x dan penyelesaian tepat* ϕ *jika kaedah Euler digunakan bagi masalah nilai awal (II) di atas.*

   (b) *Pertimbangkan dua tangki yang di sambung seperti yang ditunjukkan dalam Rajah 1 di bawah.*



Rajah 1

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Tank 1 initially contains 500 litres of salt water with 500 gm of dissolved salt, and Tank 2 initially contains 500 litres of water. Water flows into Tank 1 at a rate of 30 liters/min and the mixture flows from Tank 1 to Tank 2 at a rate of 40 litres/min. The mixture drains from Tank 2 and some flows back into Tank 1 at a rate of 10 litres/min , while the remainder leaves the system at a rate of 30 litres/min. Let x and y be the amount of salt in Tank 1 and Tank 2 at time t, respectively. Then, the rate of change of the amount of salt in each tank can be formulated as follows:

$$\frac{dx}{dt} = -\frac{40}{500}x + \frac{10}{500}y,$$
$$\frac{dy}{dt} = \frac{40}{500}x - \left(\frac{10}{500} + \frac{30}{500}\right)y.$$

(i) Show that

$$625\frac{d^2y}{dt^2} + 100\frac{dy}{dt} + 3y = 0.$$

Hence, find y.

(ii) Find the maximum amount of salt in Tank 2. When does this maximum concentration occur?

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Tangki 1 pada awalnya mengandungi 500 liter air garam dengan 500 gm garam telah dilarutkan, sedangkan Tangki 2 pada awalnya mengandungi 500 liter air. Air mengalir ke dalam Tangki 1 dengan kadar 30 liter/min dan campuran itu mengalir dari Tangki 1 ke Tangki 2 dengan kadar 40 liter/min. Campuran tersebut dikeluarkan dari Tangki 2 dan sebahagiannya kembali mengalir ke Tangki 1 pada kadar 10 litres/min , sementara yang baki keluar dari sistem pada kadar of 30 liter/min. Katakan x dan y mewakili amaun garam dalam Tangki 1 dan Tangki 2 pada masa t , masing-masing Maka kadar perubahan amaun garam di dalam setiap tangki dapat dirumuskan seperti berikut:

$$\frac{dx}{dt} = -\frac{40}{500}x + \frac{10}{500}y,$$
$$\frac{dy}{dt} = \frac{40}{500}x - \left(\frac{10}{500} + \frac{30}{500}\right)y.$$

(i) *Tunjukkan bahawa*

$$625\frac{d^2y}{dt^2} + 100\frac{dy}{dt} + 3y = 0.$$

Seterusnya, dapatkan y.

(ii) *Cari amaun garam yang maksimum di dalam Tangki 2. Bilakah kepekatan maksimum ini berlaku?*

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