UNIVERSITI SAINS MALAYSIA

Peperiksaan Kursus Semasa Cuti Panjang
Sidang Akademik 2005/2006

Jun 2006

MAT 122E – Differential Equations I
[Persamaan Pembezaan I]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of FIVE pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi LIMA muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions : Answer **all four** [4] questions.

*[Arahan : Jawab **semua empat** [4] soalan.]*

1. (a) State the necessary condition for the exactness of the differential equation $M(x,y)\,dx + N(x,y)\,dy = 0$, and solve $\left(2x^2 + y^2 + x\right)dx + xy\,dy = 0$.

(b) Explain the method of solving the linear first order differential equation $\frac{dy}{dx} = Q - Py$, where P and Q are functions of x. Solve $\cos x \frac{dy}{dx} + y \sin x = 1$.

(c) Solve the initial value problem $\frac{dy}{dx} = -\frac{x}{y}$, $y(4) = -3$.

(d) A 10-volt battery is connected to a series circuit in which the inductance is $\frac{1}{2}$ henry and the resistance is $10\,ohms$. Determine the current i if the initial current is zero.

[100 marks]

1. (a) Nyatakan syarat perlu supaya persamaan pembezaan $M(x,y)\,dx + N(x,y)\,dy = 0$ tepat, dan selesaikan $\left(2x^2 + y^2 + x\right)dx + xy\,dy = 0$.

(b) Terangkan kaedah menyelesaikan persamaan pembezaan linear peringkat pertama $\frac{dy}{dx} = Q - Py$, di mana P dan Q adalah fungsi-fungsi x. Selesaikan $\cos x \frac{dy}{dx} + y \sin x = 1$.

(c) Selesaikan masalah nilai awal $\frac{dy}{dx} = \frac{-x}{y}$, $y(4) = -3$.

(d) Suatu bateri 10-volt disambung ke suatu litar elektrik bersiri dimana induktans ialah $\frac{1}{2}$ henry dan rintangan ialah $10\,ohms$. Tentukan arus elektrik i jika arus awal ialah sifar.

[100 markah]

2. (a) Write down the Euler formula for solving the initial value problem

$$y' = f(x,y),\, y(x_0) = y_0$$

Hence derive the Heun formula (improved Euler formula) for solving this initial value problem.

(b) Use Heun formula to obtain the approximate values of $y(2\cdot1), y(2\cdot2)$ from the initial value problem

$$y' = xy - y, \quad y(2) = 1$$

by choosing $h = 0.1$.

(c) Find the general solution of the following homogeneous system of equations

$$x' = 6x - 3y$$
$$y' = 2x + y$$

[100 marks]

2. (a) *Tuliskan rumus Euler untuk menyelesaikan masalah nilai awal*

$$y' = f(x, y), y(x_0) = y_0$$

Seterusnya, terbitkan rumus Heun (rumus Euler diperbaiki) untuk menyelesaikan masalah nilai awal tersebut.

(b) *Gunakan rumus Heun untuk memperolehi nilai anggaran bagi* $y(2\cdot 1), y(2\cdot 2)$ *daripada masalah nilai awal*

$$y' = xy - y, \quad y(2) = 1$$

dengan memilih $h = 0.1$.

(c) *Dapatkan penyelesaian am bagi sistem am bagi sistem persamaan homogen :*

$$x' = 6x - 3y$$
$$y' = 2x + y$$

[100 markah]

3. (a) Solve $y'' - 3y' = 8e^{3x} + 4\sin x$

(b) Solve $y'' - 4y' + 4y = (x+1)e^{2x}$

(c) Find a power series solution for $y'' + xy = 0$. Determine a lower bound for the radius of convergence of the series solution.

[100 marks]

3. (a) *Selesaikan* $y'' - 3y' = 8e^{3x} + 4\sin x$

(b) *Selesaikan* $y'' - 4y' + 4y = (x+1)e^{2x}$

(c) *Dapatkan penyelesaian siri kuasa bagi* $y'' + xy = 0$. *Tentukan suatu batas bawah bagi jejari penumpuan penyelesaian siri tersebut.*

[100 markah]

4. (a) Suppose that the vector functions

$$\bar{u} = \begin{pmatrix} e^{2t} \\ -e^{2t} \\ 2e^{2t} \end{pmatrix}, \qquad \bar{v} = \begin{pmatrix} -e^{3t} \\ 2e^{3t} \\ 3e^{3t} \end{pmatrix}, \qquad \bar{w} = \begin{pmatrix} 2e^{5t} \\ -3e^{5t} \\ e^{5t} \end{pmatrix}$$

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are the solutions of a system of homogeneous differential equations. Show that the solutions are linearly independent for $t \in (-\infty, \infty)$. Hence write the general solution.

(b) A model for the competition between two species with population densities x and y leads to the differential equation

$$\frac{dx}{dt} = ax - by, \quad \frac{dy}{dt} = -cx + dy$$

where a, b, c and d are positive constants.

(i) Show that x satisfies

$$\frac{d^2x}{dt^2} - (a+d)\frac{dx}{dt} + (ad - bc)x = 0$$

(ii) Show that x has a solution of the form

$$x = Ae^{\alpha_1 t} + Be^{\alpha_2 t}$$

with at least one α_i positive.

(iii) Find the solution for y.

(iv) Using the values $a = d = 4, b = 1, c = 4$ and $x(0) = 700, y(0) = 3400$, determine the time t when one of the species become extinct.

[100 marks]

4. *(a)* *Katakan fungsi-fungsi vektor*

$$\bar{u} = \begin{pmatrix} e^{2t} \\ -e^{2t} \\ 2e^{2t} \end{pmatrix}, \quad \bar{v} = \begin{pmatrix} -e^{3t} \\ 2e^{3t} \\ 3e^{3t} \end{pmatrix}, \quad \bar{w} = \begin{pmatrix} 2e^{5t} \\ -3e^{5t} \\ e^{5t} \end{pmatrix}$$

merupakan penyelesaian bagi suatu sistem pesamaan pembezaan homogen. Tunjukkan bahawa penyelesaian-penyelesaian tersebut adalah tak bersandar linear bagi $t \in (-\infty, \infty)$. *Seterusnya, tuliskan penyelesaian amnya.*

(b) *Perkembangan bagi dua species yang bersaing untuk suatu sumber makanan diwakili oleh dua persamaan*

$$\frac{dx}{dt} = ax - by, \quad \frac{dy}{dt} = -cx + dy$$

dengan x, y *sebagai populasi kedua-dua spesies dan* a, b, c, d *ialah pemalar positif.*

(i) *Tunjukkan bahawa x memenuhi*

$$\frac{d^2x}{dt^2}-(a+d)\frac{dx}{dt}+(ad-bc)x=0$$

(ii) *Tunjukkan bahawa x mempunyai penyelesaian di dalam bentuk*

$$x=Ae^{\alpha_1 t}+Be^{\alpha_2 t}$$

dengan sekurang-kurangnya satu α_i *positif.*

(iii) *Dapatkan penyelesaian bagi* y.

(iv) *Dengan menggunakan nilai* $a=d=4, b=1, c=4$ *dan* $x(0)=700, y(0)=3400,$ *tentukan bilakah salah satu spesies menjadi pupus.*

[100 markah]

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