
UNIVERSITI SAINS MALAYSIA

Peperiksaan Kursus Semasa Cuti Panjang
Sidang Akademik 2006/2007

Jun 2007

MAT 122 – Differential Equations I
[Persamaan Pembezaan I]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of FIVE pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi LIMA muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions : Answer **all four** [4] questions.

Arahan : Jawab **semua empat** [4] soalan.]

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1. (a) For each of the following differential equations, determine whether it is linear or nonlinear. If the equation is linear, decide whether it is homogeneous or nonhomogeneous.
- (i) $y' = ty^2$
- (ii) $y' = t^2y$
- (iii) $(\cos t)y' + e^t y = \sin t$
- (iv) $\frac{y'}{y} + t^3 = \sin t$
- (b) Find the general solution and then solve the initial value problem $y' + 2ty = 4t, y(0) = 5$.
- (c) State and prove the necessary condition for the exactness of the differential equation $M(x, y)dx + N(x, y)dy = 0$ and then solve $2xy dx + (x^2 - 1) dy = 0$.
[25 marks]
2. (a) A 12-volt battery is connected to a series circuit in which the inductance is $\frac{1}{2}$ henry and the resistance is 10 ohms. Determine the current i if the initial current is zero.
- (b) Consider the homogeneous linear differential equation $y'' + 8y' + 15y = 0$
- (i) Find all values of m such that $y(t) = e^{mt}$ is a solution of the differential equation.
- (ii) Do the functions found in part (i) form a fundamental set of solutions for the differential equation? If so, what is the general solution of the differential equation?
- (c) Use the method of undetermined coefficients to find a particular solution of the following equation: $y'' - y' - 2y = 4e^{-t}$
[25 marks]
3. (a) Consider the initial value problem $y' = 1 - x + y, y(x_0) = y_0$ (I)
- (i) By using Euler's formula, show that $y_k = (1 + h)y_{k-1} + h - hx_{k-1}, k=1,2,\dots$
- (ii) Show that $y_1 = (1 + h)(y_0 - x_0) + x_1$ and subsequently, $y_n = (1 + h)^n(y_0 - x_0) + x_n$
- (iii) Obtain the local formula error e_{n+1} , in terms of x and the exact solution ϕ if the Euler's method is used for the initial value problem (I)
- (b) Solve $y'' + 2x^2y = 0$ in terms of the powers of x . Determine the radius of convergence for the series solution obtained.
[25 marks]

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1. (a) Bagi setiap persamaan pembezaan berikut, tentukan sama ada ia linear atau tak linear. Sekiranya linear, tentukan sama ada ia homogen atau tak homogen.
- (i) $y' = ty^2$
- (ii) $y' = t^2y$
- (iii) $(\cos t)y' + e^t y = \sin t$
- (iv) $\frac{y'}{y} + t^3 = \sin t$
- (b) Dapatkan penyelesaian am dan seterusnya selesaikan masalah nilai awal berikut: $y' + 2ty = 4t, y(0) = 5$.
- (c) Nyatakan syarat perlu dan seterusnya buktikan supaya persamaan pembezaan $M(x, y)dx + N(x, y)dy = 0$ tepat.
Seterusnya selesaikan $2xy dx + (x^2 - 1)dy = 0$.

[25 markah]

2. (a) Suatu bateri 12-volt disambung ke atas litar elektrik bersiri di mana induktans ialah $\frac{1}{2}$ henry dan rintangan ialah 10 ohms. Tentukan arus elektrik i jika arus awal ialah sifar.
- (b) Pertimbangkan persamaan pembezaan linear dan homogen berikut:
 $y'' + 8y' + 15y = 0$
- (i) Dapatkan semua nilai m supaya $y(t) = e^{mt}$ ialah suatu penyelesaian bagi persamaan pembezaan tersebut.
- (ii) Adakah fungsi-fungsi yang diperolehi dalam bahagian (i) membentuk suatu set penyelesaian asasi bagi persamaan pembezaan tersebut? Sekiranya ya, apakah penyelesaian am bagi persamaan pembezaan tersebut?
- (c) Gunakan kaedah pekali belum tentu untuk mendapatkan suatu penyelesaian khusus bagi persamaan pembezaan berikut:
 $y'' - y' - 2y = 4e^{-t}$

[25 markah]

3. (a) Pertimbangkan masalah nilai awal
 $y' = 1 - x + y, y(x_0) = y_0$ (I)
- (i) Tunjukkan dengan mengguna rumus Euler bahawa
 $y_k = (1+h)y_{k-1} + h - hx_{k-1}, k=1,2,\dots$
- (ii) Tunjukkan bahawa $y_1 = (1+h)(y_0 - x_0) + x_1$ dan seterusnya,
 $y_n = (1+h)^n(y_0 - x_0) + x_n$
- (iii) Dapatkan satu rumus bagi ralat rumus setempat e_{n+1} dalam sebutan x dan penyelesaian tepat ϕ jika kaedah Euler digunakan bagi masalah nilai awal (I).
- (b) Selesaikan $y'' + 2x^2y = 0$ dalam kuasa x . Tentukan jejari penumpuan penyelesaian siri kuasa ini.

[25 markah]

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