UNIVERSITI SAINS MALAYSIA

Peperiksaan Kursus Semasa Cuti Panjang
Sidang Akademik 2006/2007

Jun 2007

MAT 111 – Linear Algebra
[Aljabar Linear]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of <u>ELEVEN</u> pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi <u>SEBELAS</u> muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

<u>Instructions</u> : Answer **<u>all four</u>** [4] questions.

*[<u>**Arahan**</u> : Jawab <u>**semua empat**</u> [4] soalan.]*

...2/-

1. (a) Consider the points $A(1,2,-3)$, $B(-2,1,1)$ and $C(0,2,1)$.

   (i) Find the point D such that $ABCD$ is a parallelogram.

   (ii) Let P be the mid-point of AC. Let O be the origin. Find a vector expression for $\overline{OP}$ in terms of $\overline{OA}$ and $\overline{OC}$.

   (iii) Show that $\overline{BP} = \overline{PD}$ and hence that P bisects BD as expected.

(b) A plane $\prod$ through the line of intersection of the planes $x+3y+5z=-1$ and $2x+y+4z=-1$ is perpendicular to the line $\dfrac{x-2}{1-s}=\dfrac{y}{1+s}=\dfrac{z-1}{s}$.

Determine

   (i) the value of s,

   (ii) the equation of plane $\prod$,

   (iii) the coordinates of the point where the plane and the line meet.

(c) Given that $u_1=(1,\ 0,\ 1)$ and $u_2=(1,\ 0,\ 0)$.

   (i) Show that $(2,\ 3,\ 4)$ cannot be written as a linear combination of u_1 and u_2.

   (ii) From your observation, explain why $\{u_1,u_2\}$ cannot generate $\mathbb{R}^3$.

   (iii) Let $u_3=(0,\ 1,\ 1)$. Show that $\{u_1,u_2,u_3\}$ is linearly independent.

(d) Given that $B=\{(3,\ -1),\ (2,\ 1)\}$ is a basis of $\mathbb{R}^2$. Find the coordinates of $(7,\ 1)$ in $\mathbb{R}^2$ with respect to B. Sketch a diagram to illustrate this.

[100 marks]

1. (a) *Pertimbangkan titik A(1,2, -3), B(-2,1,1) and C(0, 2,1).*

   (i) *Cari titik D supaya ABCD adalah suatu segiempat selari.*

   (ii) *Biar P sebagai titik tengah AC. Biar O sebagai titik asalan. Cari ungkapan vector bagi* $\overrightarrow{OP}$ *dalam sebutan* $\overrightarrow{OA}$ *dan* $\overrightarrow{OC}$.

   (iii) *Tunjukkan bahawa* $\overrightarrow{BP} = \overrightarrow{PD}$ *maka P membahagi dua sama BD seperti yang dijangkakan.*

(b) *Satah* Π *yang melalui garis persilangan antara satah* $x+3y+5z=-1$ *dan* $2x+y+4z=-1$ *adalah berseranjang dengan garislurus* $\dfrac{x-2}{1-s}=\dfrac{y}{1+s}=\dfrac{z-1}{s}$.

Tentukan

   (i) *nilai s,*

   (ii) *persamaan satah* Π,

   (iii) *kordinat titik di mana satah dan garislurus tersebut bertemu.*

(c) *Diberi* $u_1 = (1,\ 0,\ 1)$ *dan* $u_2 = (1,\ 0,\ 0)$.

   (i) *Tunjukkan bahawa* $(2,\ 3,\ 4)$ *tidak boleh ditulis sebagai gabungan linear* u_1 dan u_2.

   (ii) *Dari pemerhatian anda, terangkan mengapa* $\{u_1, u_2\}$ *tidak boleh merentang* $\mathbb{R}^3$.

   (iii) *Biar* $u_3 = (0,\ 1,\ 1)$. *Tunjukkan bahawa* $\{u_1, u_2, u_3\}$ *adalah tak bersandar linear.*

(d) *Diberi* $B = \{(3,\ -1),\ (2,\ 1)\}$ *adalah asas* $\mathbb{R}^2$. *Cari kordinat bagi* $(7,\ 1)$ *dalam* $\mathbb{R}^2$ *yang berkaitan dengan B. Lakarkan gambarajah untuk mengilustrasikannya.*

[100 markah]

...4/-

2. (a) (i) Write out the 4×4 matrix $M=\left[m_{ij}\right]$ whose entries are given by

$$m_{ij}=\begin{cases} i, & \text{if } i+j \text{ is odd} \\ 0, & \text{otherwise} \end{cases}$$

(Show your working).

(ii) Given the matrices $A=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$ and $B=\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}$. Solve the matrix equation $2(X+A)=X-B$.

(b) (i) Show that the linear transformation $T:\mathbb{R}^3\to\mathbb{R}^4$ defined by

$$(x,y,z)T=(x+y,3x-y,x+y+z,x-2z)$$

is a linear transformation.

(ii) Show that the linear transformation $T:\mathbb{R}^2\to\mathbb{R}^4$ defined by

$$(x,y)T=(x,y,x^2+y^2)$$

is not a linear transformation.

(c) Consider the basis $S = \{v_1, v_2, v_3\}$ for $\mathbb{R}^3$ where $v_1=(1,2,1)$, $v_2=(2,9,0)$ and $v_3=(3,3,4)$ .Let T be the linear transformation $T:\mathbb{R}^3\to\mathbb{R}^2$ such that

$$v_1T=(1,0) \quad , \quad v_2T=(-1,1) \quad , \quad v_3T=(0,1)$$

(i) Find the definition for T.

(ii) From (i), find the standard matrix A such that $v\,T = vA$.

(d) Given two matrices $A=\begin{bmatrix}1&2\\3&4\end{bmatrix}$ and $B=\begin{bmatrix}3&1\\0&5\end{bmatrix}$, show that $AB\neq BA$ using the concept of linear transformation.

[100 marks]

...5/-

2. (a) (i) *Tuliskan matriks* 4×4 $M=\left[m_{ij}\right]$ *yang mempunyai pemasukan*

$$m_{ij}=\begin{cases} i, & \textit{jika } i+j \textit{ adalah ganjil} \\ 0, & \textit{selain dari itu} \end{cases}.$$

(Tunjukkan jalankerja anda).

(ii) *Diberi matriks* $A=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$ *dan* $B=\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}$. *Selesaikan persamaan matriks* $2(X+A)=X-B$.

(b) (i) *Tunjukkan bahawa transformasi linear* $T:\mathbb{R}^3\to\mathbb{R}^4$ *dengan takrif*

$$(x,y,z)T=(x+y,3x-y,x+y+z,x-2z)$$

adalah suatu transformasi linear.

(ii) *Tunjukkan bahawa transformasi linear* $T:\mathbb{R}^2\to\mathbb{R}^4$ *dengan takrif*

$$(x,y)T=\left(x,y,x^2+y^2\right)$$

adalah suatu transformasi linear.

(c) *Pertimbangkan asas* $S=\{v_1, v_2, v_3\}$ *bagi* $\mathbb{R}^3$ *dengan* $v_1=(1,2,1)$, $v_2=(2,9,0)$ *dan* $v_3=(3,3,4)$. *Biar T sebagai transformasi linear* $T:\mathbb{R}^3\to\mathbb{R}^2$ *sedemikian hingga*

$$v_1T=(1,0) \quad , \quad v_2T=(-1,1) \quad , \quad v_3T=(0,1)$$

(i) *Cari takrif bagi T.*

(ii) *Dari (i), cari matriks piawai A sedemikian hingga* $v\,T=vA$.

(d) *Diberi dua matriks* $A=\begin{bmatrix}1&2\\3&4\end{bmatrix}$ *dan* $B=\begin{bmatrix}3&1\\0&5\end{bmatrix}$, *tunjukkan bahawa* $AB\neq BA$ *menggunakan konsep transformasi linear.*

[100 markah]

...6/-

4. (a) The following system of equations is given.

$$\begin{array}{rcrcrcrcrcr} x & + & y & - & 2z & + & w & + & 3u & = & 1 \\ 2x & - & y & + & 2z & + & 2w & + & 6u & = & 2 \\ 3x & + & 2y & - & 4z & - & 3w & - & 9u & = & 3 \end{array}$$

(i) Write out the augmented matrix representing the system and use the Gauss-Jordan method to obtain its row-reduced echelon form(rref).

(ii) Write out the equation obtained from the rref in (i). Determine the leading and free variables and then solve the system to get the solution in the form

$$(x,y,z,w,u)=(a,b,c,d,e)+\alpha(a',b',c',d',e')+\beta(a'',b'',c'',d'',e'').$$

(iii) Write out the matrix A where $(x,y,z,w,u)A=(1,2,3)$. Check the solution obtained in (ii) by showing that (a,b,c,d,e) is a particular solution to the system and $\alpha(a',b',c',d',e')+\beta(a'',b'',c'',d'',e'')\in Ker\,A$.

(b) Given that $S=\{(1,0,1,0),(2,0,0,1),(0,0,2,3)\}$ is a linearly independent set in $\mathbb{R}^3$. Construct an orthonormal set from S using the Gram-Schmidt orthogonalization process.

(c) For a vector space V, we define its orthogonal complement as

$$V^{\perp}=\{w \mid w\cdot v=0, \forall v\in V\}$$

which is also a vector space.

Show that

(i) $(V^{\perp})^{\perp}=V$,

(ii) $U\subseteq V \Rightarrow V^{\perp}\subseteq U^{\perp}$.

....9/-

4. (a) *Diberi sistem persamaan linear berikut.*

$$\begin{array}{ccccccccccc} x & + & y & - & 2z & + & w & + & 3u & = & 1 \\ 2x & - & y & + & 2z & + & 2w & + & 6u & = & 2 \\ 3x & + & 2y & - & 4z & - & 3w & - & 9u & = & 3 \end{array}$$

(i) *Tuliskan matriks imbuhan yang mewakili sistem tersebut dan gunakan kaedah Gauss-Jordan untuk mendapatkan bentuk baris terturunnya(bebt).*

(ii) *Tuliskan persamaan-persamaan yang diperoleh daripada bebt dalam (i). Tentukan pembolehubah utama dan bebas dan selesaikan sistem tersebut untuk mendapatkan penyelesaian dalam bentuk*

$$(x,y,z,w,u)=(a,b,c,d,e)+\alpha(a',b',c',d',e')+\beta(a'',b'',c'',d'',e'').$$

(iii) *Tuliskan matriks A yang mana* $(x,y,z,w,u)A=(1,2,3)$. *Semak penyelesaian yang diperoleh dalam (ii) dengan menunjukkan bahawa* (a,b,c,d,e) *adalah penyelesaian khusus terhadap sistem ini dan* $\alpha(a',b',c',d',e')+\beta(a'',b'',c'',d'',e'')\in Ker\ A$.

(b) *Diberi* $S=\{(1,0,1,0),(2,0,0,1),(0,0,2,3)\}$ *adalah suatu set tak bersandar linear dalam* $\mathbb{R}^3$. *Bina suatu set ortonormal dari S mengggunakan proses pengortogonan Gram-Schmidt.*

(c) *Bagi suatu ruang vektor V, kita takrifkan pelengkap ortogonnya sebagai*

$$V^{\perp}=\{w\,|\,w\cdot v=0,\forall v\in V\}$$

Tunjukkan bahawa

(i) $(V^{\perp})^{\perp}=V$,

(ii) $U\subseteq V\Rightarrow V^{\perp}\subseteq U^{\perp}$.