UNIVERSITI SAINS MALAYSIA

Peperiksaan Kursus Semasa Cuti Panjang
Sidang Akademik 2005/2006

Jun 2006

MAT 101E – Calculus
[Kalkulus]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of SIX pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi ENAM muka surat yang bercetak sebelum anda memulakan peperiksaan ini].

Instructions: Answer **all four** [4] questions.

*[**Arahan:** Jawab **semua empat** [4] soalan.]*

1. (a) Prove by using definition that $\lim_{x\to 3}\frac{x^2+1}{x+2}=2$

(b) Find the following limits :

(i) $\lim_{x\to 3}\frac{\sqrt{3x-5}-2}{x-3}$

(ii) $\lim_{x\to 0}\frac{\sin x\ \tan 3x}{x^2}$

(iii) $\lim_{x\to 5^-}\left(\frac{x-5}{\left|x^2-25\right|}+\left[\frac{2x}{5}\right]\right)$

where $[x]$ is the greatest integer function.

(c) If $f(x)=\frac{\ell n(x-2)}{\sqrt{(x+1)(x-4)}}$, find the largest subset of $\mathbb{R}$ such that f is defined.

[100 marks]

1. *(a)* *Buktikan* $\lim_{x\to 3}\frac{x^2+1}{x+2}=2$ *dengan menggunakan takrif had.*

(b) *Cari had yang berikut :*

(i) $\lim_{x\to 3}\frac{\sqrt{3x-5}-2}{x-3}$

(ii) $\lim_{x\to 0}\frac{\sin x\ \tan 3x}{x^2}$

(iii) $\lim_{x\to 5^-}\left(\frac{x-5}{\left|x^2-25\right|}+\left[\frac{2x}{5}\right]\right)$

di mana $[x]$ *merupakan fungsi integer terbesar.*

(c) *Jika* $f(x)=\frac{\ell n(x-2)}{\sqrt{(x+1)(x-4)}}$, *cari subset* $\mathbb{R}$ *yang terbesar supaya* f *tertakrif.*

[100 Markah]

2. (a) Given that $f(x)=\begin{cases} \dfrac{x^2-1}{x-x^2}, & x<-1 \\ 3x+3, & -1\le x<2. \\ a^2x^2-5a, & x\ge 2 \end{cases}$

(i) Show that f is continuous at $x=-1$.

(ii) Find a so that f is continuous on $\mathbb{R}$.

(b) By using Mean value Theorem or otherwise, show that

$$1+x>\sqrt{1+2x},\ \forall\, x>0.$$

(c) Suppose that $f(x)=e^x\sin x,\ x\in[0,2\pi]$.

(i) Find the maximum and minimum of f.

(ii) Determine the intervals where f increasing or decreasing.

[100 marks]

2. *(a) Diberi* $f(x)=\begin{cases} \dfrac{x^2-1}{x-x^2}, & x<-1 \\ 3x+3, & -1\le x<2. \\ a^2x^2-5a, & x\ge 2 \end{cases}$

(i) Tunjukkan bahawa f *adalah selanjar pada* $x=-1$.

(ii) Cari a *supaya* f *adalah selanjar pada* $\mathbb{R}$.

(b) Dengan menggunakan Teorem Nilai Min atau cara yang lain, tunjukkan bahawa

$$1+x>\sqrt{1+2x},\ \forall\, x>0.$$

(c) Andaikan $f(x)=e^x\sin x,\ x\in[0,2\pi]$.

(i) Cari maksimum dan minimum bagi f.

(ii) Tentukan selang yang mana f *menokok atau menyusut.*

[100 Markah]

3. (a) Given that $f(x)=\begin{cases} x^2\sin\frac{1}{x}, & x\neq 1 \\ 0, & x=0 \end{cases}$.

(i) Find $f'(0)$.

(ii) Show that f' is not continuous at 0.

(b) Consider the equation $6+\cos\pi x-\frac{5x}{1+x^2}=0$.

(i) Show that this equation has a solution in $\mathbb{R}$.

(ii) Show that the solution of this equation is not unique.

(c) Suppose that the function f is continuous on $[a,b]$ and differentiable on (a,b), and $f(a)=f(b)=0$. By considering the function $F(x)=e^{kx}f(x)$, show that there exists some $c\in(a,b)$ such that

$$f'(c)+kf(c)=0,$$

where k is a constant.

[100 marks]

3. (a) Diberi $f(x)=\begin{cases} x^2\sin\frac{1}{x}, & x\neq 1 \\ 0, & x=0 \end{cases}$.

(i) Cari $f'(0)$.

(ii) Tunjukkan bahawa f' *tidak selanjar pada* 0.

(b) Pertimbangkan persamaan $6+\cos\pi x-\frac{5x}{1+x^2}=0$.

(i) Tunjukkan bahawa persamaan ini mempunyai suatu penyelesaian dalam $\mathbb{R}$.

(ii) Tunjukkan bahawa penyelesaian ini tidak unik.

(c) Andaikan bahawa fungsi f *selanjar pada* $[a,b]$ *dan terbezakan pada* (a,b), *dan* $f(a)=f(b)=0$. *Dengan mempertimbangkan fungsi* $F(x)=e^{kx}f(x)$, *tunjukkan bahawa wujud* $c\in(a,b)$ *supaya*

$$f'(c)+kf(c)=0,$$

di mana k *ialah suatu pemalar.*

[100 Markah]

4. (a) Given that $f(x) = 2x+1$.

(i) Let $P_n = \left\{ 0, \frac{1}{n}, \frac{2}{n}, \cdots, \frac{n-1}{n}, 1 \right\}$ be a partition of the interval $[0, 1]$, where n is a positive integer. Find the lower sum $L(P_n, f)$ and the upper sum $U(P_n, f)$.
(You may assume $\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$)

(ii) Show by definition that f is integrable on $[0, 1]$.

(iii) What is the value of $\int_0^1 f(x)\,dx$?

(b) Find the following integrals :

(i) $\int_0^3 x^2 \cos(x^2)\,dx$

(ii) $\int \frac{x^2}{\sqrt{a^2 - x^2}}\,dx$, a is a constant.

(iii) $\int \frac{5x^2 + 2x + 1}{x(x^2+1)}\,dx$

(iv) $\int \sqrt{\frac{1-x}{x}}\,dx$

[100 marks]

4. (a) Diberi $f(x) = 2x+1$.

(i) Andaikan $P_n = \left\{ 0, \frac{1}{n}, \frac{2}{n}, \cdots, \frac{n-1}{n}, 1 \right\}$ *merupakan suatu partisi pada* $[0, 1]$, *di mana* n *ialah integer positif. Cari hasil tambah bawah* $L(P_n, f)$ *dan hasil tambah atas* $U(P_n, f)$.
(Anda boleh anggap $\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$*)*

(ii) Dengan menggunakan takrif, buktikan bahawa f *adalah terkamirkan pada* $[0, 1]$.

(iii) Apakah nilai $\int_0^1 f(x)\,dx$ *?*

(b) *Cari kamiran yang berikut :*

(i) $\int_0^3 x^2 \cos(x^2)\, dx$

(ii) $\int \frac{x^2}{\sqrt{a^2 - x^2}}\, dx$, *a ialah suatu pemalar.*

(iii) $\int \frac{5x^2 + 2x + 1}{x(x^2 + 1)}\, dx$

(iv) $\int \sqrt{\frac{1-x}{x}}\, dx$

[100 Markah]

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