UNIVERSITI SAINS MALAYSIA

Peperiksaan Kursus Semasa Cuti Panjang
Sidang Akademik 2006/2007

Jun 2007

MAT 101 – Calculus
[Kalkulus]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of FIVE pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi LIMA muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions : Answer **all four** [4] questions.

[Arahan *:* *Jawab* ***semua empat*** *[4] soalan.]*

1. (a) Find the following limits:

(i) $\lim_{x\to 0} \frac{1}{x}\left(\frac{1}{x+4}-\frac{1}{4}\right)$

(ii) $\lim_{x\to 1} \frac{\sqrt{x}-\sqrt{2x-1}}{x-1}$

(iii) $\lim_{x\to 2^-} \frac{|x-2|}{4x^2-9x+2}$

(iv) $\lim_{x\to 0} (x^2-2x)\sin\frac{1}{x}$

(b) Consider the equation $\sqrt{5x^2+12x}-3x=1$.

(i) Show that this equation has a solution in $\mathbb{R}$.
(ii) Show that its solution is not unique.

(c) A triangle is inscribed in a semicircle of radius 3 so that one side is along the diameter. Find the maximum area of the triangle and the dimension of the triangle.

[100 marks]

2. (a) Given that $f(x)=\begin{cases}\sin x, & x\le 0\\ x, & 0<x\le 2\\ \frac{ax-4}{x-2}, & x>2\end{cases}$.

(i) Find the constant a so that f is continuous at $x=2$.

(ii) Find $f'(x)$ wherever it exists and identify x at which $f'(x)$ does not exist.

(iii) If $g(x)=\tan^{-1}x$, $\forall x\in\mathbb{R}$, find $(g\circ f)'(0)$.

(b) (i) State the mean value theorem.

(ii) Use mean value theorem to approximate $e^{0.1}$ and estimate the error. (You may take e to be 2.7183)

(c) Find the largest set $A\subset\mathbb{R}$ such that $h: A\to\mathbb{R}$ is a function where

$$h(x)=\frac{\sqrt{x^2+2x}}{\sin x}.$$

[100 marks]

1. (a) *Cari had yang berikit:*

(i) $\lim_{x\to 0} \frac{1}{x}\left(\frac{1}{x+4}-\frac{1}{4}\right)$

(ii) $\lim_{x\to 1} \frac{\sqrt{x}-\sqrt{2x-1}}{x-1}$

(iii) $\lim_{x\to 2^-} \frac{|x-2|}{4x^2-9x+2}$

(iv) $\lim_{x\to 0} (x^2-2x)\sin\frac{1}{x}$

(b) *Pertimbangkan persamaan* $\sqrt{5x^2+12x}-3x=1$.

(i) *Tunjukkan bahawa persamaan ini mempunyai suatu penyelesaian pada* $\mathbb{R}$.

(ii) *Tunjukkan bahawa penyelesaiannya tidak unik.*

(c) *Suatu segitiga terterap di dalam suatu semibulatan yang berjejari 3 supaya suatu sisinya terletak pada diameter. Cari luas maksimum segitiga dan dimensi segitiga.*

[100 markah]

2. (a) *Diberi* $f(x)=\begin{cases}\sin x, & x\le 0\\ x, & 0<x\le 2\\ \frac{ax-4}{x-2}, & x>2\end{cases}$.

(i) *Cari pemalar* a *supaya* f *adalah selanjar pada* $x=2$.

(ii) *Cari* $f'(x)$ *di mana-mana ia wujud dan camkan* x *di mana* $f'(x)$ *tidak wujud.*

(iii) *Jika* $g(x)=\tan^{-1}x$, $\forall x\in\mathbb{R}$, *cari* $(g\circ f)'(0)$.

(b) (i) *Nyatakan teorem nilai min.*

(ii) *Gunakan teorem nilai min untuk mengganggarkan* $e^{0.1}$ *dan anggarkan ralat yang berkenaan .*
(Anda boleh anggap e *sama dengan 2.7183)*

(c) *Cari set* $A\subset\mathbb{R}$ *yang terbesar supaya* $h:A\to\mathbb{R}$ *ialah suatu fungsi di mana* $h(x)=\frac{\sqrt{x^2+2x}}{\sin x}$.

[100 markah]

3. (a) Prove by using the definition of limit that $\lim_{x\to 1} \frac{4}{x+2} = \frac{4}{3}$.

(b) (i) Show that $e^x \geq 1+x, \ \forall x \geq 0$.

(ii) Hence show that $\frac{4}{3} \leq \int_0^1 e^{x^2}\,dx \leq e$.

(c) Given $f(x) = \frac{4}{1+e^{-x}}, \ x \in \mathbb{R}$.

(i) Find all the asymptotes of the graph of f.

(ii) Determine where f is increasing, descreasing, convex or concave.

(iii) Find the local extrema of f and the inflection points of f if exist.

(iv) Sketch the graph of f using the above features.

[100 marks]

4. (a) Find the following integrals:

(i) $\int \frac{1}{\sqrt{x}\,(1+\sqrt{x})^2}\,dx$ (ii) $\int \frac{7x^2-x+2}{x(x^2+1)}\,dx$

(iii) $\int \frac{dx}{(4+x^2)^2}$ (iv) $\int \frac{\ell n\, x}{x^2}\,dx$

(b) Suppose that the function g is continuous on $(0,\infty)$ and

$$\int_1^x g(t)e^t\,dt = \ell n\, x - x e^x + \int_1^x \sqrt{1+t^3}\,dt.$$

Find $g(x), \ \forall x > 0$.

(c) (i) Given that $f(x) = \frac{1}{\sqrt{1+t^3}}$ on $[0,2]$ and the partition

$P = \{0, \frac{1}{2}, 1, 2\}$.

Find the lower sum $L(P;f)$ and the upper sum $U(P;f)$ of f with respect to the partition P.

What is the relation between $L(P;f)$, $U(P;f)$ and $\int_0^2 \frac{1}{\sqrt{1+t^3}}\,dt$?

(ii) Function $h:[0,1] \to \mathbb{R}$ is defined as

$$h(x) = \begin{cases} 3, & x \text{ rational} \\ 0, & x \text{ irrational} \end{cases}.$$

Does $\int_0^1 h(x)\,dx$ exist ? Support your answer with reason.

[100 marks]

3. (a) *Dengan menggunakan takrif had buktikan bahawa* $\lim_{x\to 1} \frac{4}{x+2} = \frac{4}{3}$.

(b) (i) *Tunjukkan bahawa* $e^x \geq 1+x, \ \forall x \geq 0$.

(ii) *Dengan ini tunjukkan bahawa* $\frac{4}{3} \leq \int_0^1 e^{x^2}\, dx \leq e$.

(c) *Diberi* $f(x) = \frac{4}{1+e^{-x}}, \ x \in \mathbb{R}$.

(i) *Cari semua asimptot bagi graf f.*

(ii) *Tentukan di mana f adalah menokok, menyusut, cembung atau cekung.*

(iii) *Cari ekstrema setempat bagi f dan titik lengkok balas bagi f jika wujud.*

(iv) *Lakarkan graf bagi f dengan menggunakan ciri atas.*

[100 markah]

4. (a) *Cari kamiran yang berikut:*

(i) $\int \frac{1}{\sqrt{x}\,(1+\sqrt{x})^2}\, dx$ (ii) $\int \frac{7x^2 - x + 2}{x(x^2+1)}\, dx$

(iii) $\int \frac{dx}{(4+x^2)^2}$ (iv) $\int \frac{\ell n\, x}{x^2}\, dx$

(b) *Andaikan fungsi g selanjar pada* $(0, \infty)$ *dan*

$$\int_1^x g(t)e^t\, dt = \ell n\, x - x e^x + \int_1^x \sqrt{1+t^3}\, dt.$$

Cari $g(x), \ \forall x > 0$.

(c) (i) *Diberi* $f(x) = \frac{1}{\sqrt{1+t^3}}$ *pada* $[0, 2]$ *dan partisi* $P = \{0, \frac{1}{2}, 1, 2\}$.

Cari hasil tambah bawah $L(P; f)$ *dan hasil tambah atas* $U(P; f)$ *bagi f terhadap partisi P.*

Apakah hubungan di antara $L(P; f)$, $U(P; f)$ *dan* $\int_0^2 \frac{1}{\sqrt{1+t^3}}\, dt$ *?*

(ii) *Fungsi* $h : [0, 1] \to \mathbb{R}$ *ditakrifkan sebagai*

$$h(x) = \begin{cases} 3, & x \text{ rasional} \\ 0, & x \text{ tak rasional} \end{cases}.$$

Wujudkah $\int_0^1 h(x)\, dx$ *? Berikan sokongan bagi jawapan anda.*

[100 markah]

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