LOCAL CONTRAST ENHANCEMENT UTILIZING BIDIRECTIONAL SWITCHING EQUALIZATION OF SEPARATED AND CLIPPED SUB-HISTOGRAMS

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UNIVERSITI SAINS MALAYSIA 2014

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by

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Thesis submitted in fulfilment of the requirements for the degree of Master of Science

SEPTEMBER 2014

ACKNOWLEDGEMENTS

Many people have played important roles during my postgraduate study. It is time to express my gratitude to those whose support and encouragement were essential components for the successful completion of this thesis.

First and foremost, I would like to thank my supervisor Dr. Haidi Ibrahim for his guidance, support and encouragement throughout my research at Universiti Sains Malaysia (USM). I consider myself very fortunate to have work under Dr Haidi's supervision. During the duration of this postgraduate study, it was his great ideas, vision, motivation and invaluable inputs that have brought me to this phase. Not only that, Dr Haidi is always ready and willing to sacrifice his invaluable time especially during weekends to teach and guide me in image processing and C++ programming. Besides, I am extremely grateful to Dr Haidi for undertaking the behemoth tasks of proof-reading this thesis and several of my previous manuscripts.

I would also like to thank my parents Hoo Pak Chin and Saw Lay Hwa for their immense support given to me to pursue my goals. Without their unparalleled love and encouragement, this thesis could not have materialized. Further, I am greatly indebted to my loving wife, Khoo Wei Mei, for providing me with infinite courage, care and support.

Further, I would like to thank the viva voce committees for their time in evaluating and giving constructive comments in this research. They are Dr. John Chiverton (the external examiner from University of Portsmouth, UK), Dr. Dzati Athiar Ramli (the internal examiner), Associate Professor Dr. Shahrel Azmin Sundi @ Suandi (the Dean's representative during the viva voce), Professor Dr. Mohd Zaid Abdullah (the Dean) and Associate Professor Dr. Mashitah Mat Don (the chairman during the viva voce, from the school of Chemical Engineering).

Next, I would like to extend my thanks to USM. This research was supported in part by the USM's Research University: Individual (RUI) with account number 1001/PELECT/814169.

Finally, my acknowledgement would not be completed without expressing my personal belief in and gratitude towards GOD. I feel very blessed to have the opportunity to do my postgraduate study in USM.

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LISTS OF ABBREVIATIONS

AHE	Adaptive Histogram Equalization
AMBE	Absolute Mean Brightness Error
BBHE	Brightness Preserving Bi-Histogram Equalization
BHEPL	Bi-Histogram Equalization with a Plateau Limit
BLS	Black Level Stretch
BMHE	Bin Modified Histogram Equalization
BO	Bin Overflow
BOHE	Block Overlapped Histogram Equalization
BU	Bin Underflow
BUBOHE	Histogram Equalization with Bin Underflow and Bin Overflow
CDF	Cumulative Density Function
CLAHE	Contrast Limited Adaptive Histogram Equalization
CR	Contextual Region
СТ	Computed Tomography
DLP	Digital Light Processing
DSIHE	Dual Sub-Image Histogram Equalization
Е	Entropy
GHE	Global Histogram Equalization
HE	Histogram Equalization
HVP	Human Visual Perception

IAHE	Interpolated Adaptive Histogram Equalization
IQM	Image Quality Measure
LCD	Liquid Crystal Display
LCE-BSESCS	Local Contrast Enhancement Utilizing Bidirectional Switching Equalization of Separated and Clipped Sub-Histograms
LED	Light Emitting Diode
LHE	Local Histogram Equalization
MBE	Mean Brightness Error
MBPHE	Mean Brightness Preserving Histogram Equalization
MLBOHE	Multiple Layers Block Overlapped Histogram Equalization
MMBEBHE	Minimum Mean Brightness Error Bi-Histogram Equalization
MRI	Magnetic Resonance Imaging
MSSIM	Mean Structure Similarity Index Map
NOBOHE	Non-Overlapped Block Histogram Equalization
PDF	Probability Density Function
PSNR	Peak Signal-to-Noise Ratio
RMSHE	Recursive Mean-Separate Histogram Equalization
RSIHE	Recursive Sub-Image Histogram Equalization
SDCIQM	Song-Der Chen's Image Quality Measure
SI	Speckle Index
SNS	Speckle Noise Strength
SSIM	Structure Similarity Index Map
WLS	White Level Stretch

PENYERLAHAN BEZA JELAS SETEMPAT MENGGUNAKAN PENYERAGAMAN PENSUISAN DWIARAH SUB-HISTOGRAM TERPISAH DAN TERPOTONG

ABSTRAK

Kaedah penyerlahan beza jelas imej digit berdasarkan teknik penyeragaman histogram adalah berguna dalam penggunaan produk elektronik pengguna disebabkan pelaksanaan yang mudah. Walau bagaimanapun, kebanyakan kaedah penyerlahan yang dicadangkan adalah menggunakan teknik proses sejagat dan tidak menekan kepada kandungan setempat. Oleh itu, satu kaedah penyerlahan beza jelas imej yang baru, iaitu Penyerlahan Beza Jelas Setempat Menggunakan Penyeragaman Pensuisan Dwiarah Sub-Histogram Terpisah dan Terpotong (Local Contrast Enhancement Utilizing Bidirectional Switching Equalization of Separated and Clipped Sub-Histograms (LCE-BSESCS)) telah dicadangkan. Kaedah yang dicadangkan ini adalah hasil lanjutan daripada Penyeragaman Dwi-Histogram Pemeliharaan Kecerahan (Brightness Preserving Bi-Histogram Equalization (BBHE)) dan Penyeragaman Dwi-Histogram dengan Had Paras (Bi-Histogram Equalization with a Plateau Limit (BHEPL)). LCE-BSESCS mempunyai persamaan dengan BBHE dari segi penggunaan metodologi min-pemisahan yang sama. Tidak seperti BBHE yang menggunakan purata nilai keamatan sejagat sebagai titik pemisahan, titik pemisahan setempat untuk LCE-BSESCS adalah berdasarkan purata keamatan daripada sampel di rantau kontekstual. Di samping itu, serupa dengan BHEPL, LCE-BSESCS memotong histogram dengan nilai ambang yang diperolehi daripada nilai purata sub-histogram. Tetapi, tidak seperti BHEPL, pendekatan secara pensuisan telah digunakan. Pendekatan secara pensuisan ini dapat mengurangkan masa pemprosesan kerana rangkap pindah hanya dikira daripada satu sub-histogram sahaja berbanding dengan dua sub-histogram. Pelaksanaan LCE-BSESCS bermula dengan mendefinisikan rantau kontekstual, dan kemudian diikuti dengan mewujudkan histogram setempat, memisahkan histogram kepada dua sub-histogram, memotong sub-histogram yang berkaitan, mewujudkan rangkap pemetaan keamatan dwiarah dan akhir sekali pemetaan piksel pusat. Daripada analisis di kalangan semua kaedah yang dinilai, LCE-BSESCS mempunyai nilai yang terendah bagi Ralat Min Kecerahan (iaitu 3.4570) dan Kekuatan Bunyi Bintik (iaitu 6.4639).

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ABSTRACT

Digital image contrast enhancement methods that are based on histogram equalization (HE) technique are useful for the use in consumer electronic products due to their simple implementation. However, almost all the suggested enhancement methods are using global processing technique, which does not emphasize local contents. Therefore, a novel local histogram equalization method, which is Local Contrast Enhancement Utilizing Bidirectional Switching Equalization of Separated and Clipped Sub-Histograms (LCE-BSESCS) has been proposed. This proposed method is an extension to Brightness Preserving Bi-Histogram Equalization (BBHE) and Bi-Histogram Equalization with a Plateau Limit (BHEPL). LCE-BSESCS is similar to BBHE in terms of using the same mean-separation methodology. Unlike BBHE that uses global average intensity value as the splitting point, the local splitting point in LCE-BSESCS is the average intensity value from the samples within the contextual region. In addition, similar to BHEPL, LCE-BSESCS clip the histogram with a threshold value obtained using the average value from subhistogram. However, unlike BHEPL, a switching approach has been used. This switching approach reduces the processing time as the transfer function is calculated from one sub-histogram only instead of two sub-histograms. The implementation of LCE-BSESCS method starts by first defining the contextual region, and then followed by creating local histogram, separating the histogram into two subhistograms, clipping the corresponding sub-histogram, creating the bidirectional intensity switching mapping function and finally mapping the center pixel. From the analysis for all the methods evaluated, LCE-BSESCS has the lowest value for Mean Brightness Error (i.e. 3.4570) and Speckle Noise Strength (i.e. 6.4639).

CHAPTER 1

INTRODUCTION

1.1 Research Background

Image processing is a technique to improve the quality of pictorial information in raw images taken from cameras or sensors. Image processing systems are widely used in various applications due to easy access to powerful computers, large size memory devices and graphical software. Typically, there are two types of image processing methods which are analog and digital image processing (Reddy et al., 2012). Analog image processing is defined as image alteration by varying the electrical signal magnitude continuously. On the other hand, digital image processing relates to processing image with a digital computer.

Image processing is an interesting field of study because there are many applications that make use of image processing or analysis techniques. Generally, almost every discipline uses cameras or sensors to collect image data from the surrounding around us. Examples of disciplines using image processing are astronomy, autonomous navigation, microscopy, oceanography, radar, radiology, robot guidance, surveillance and many other fields (Bovik, 2009). Usually, the image data is multidimensional and thus need to be converted into a suitable format for human viewing.

Images can be digitalized for human viewing. There are several types of digital images which include binary image, grayscale image and color image. A binary image has only two level of brightness, for example a black and white image. A grayscale image contains brightness information whereby the brightness graduation can be observed. For color image, the brightness for each pixel is characterized with three values. Several common color models are RGB (red, green, blue), CMY (cyan, magenta, yellow) and CMYK (cyan, magenta, yellow, black).

There are two common approaches for digital image representation which are spatial domain approach and frequency domain approach. The term spatial domain indicates that the pixels in an image are manipulated directly. On the other hand, frequency domain approach is based on modifying the frequency components of an image. The frequency components of the image can be obtained via some transformation, such as Fourier transform. Yet enhancement techniques based on various combinations of methods from these two categories are common (Gonzalez and Woods, 2002). In this thesis, only the spatial domain approach is considered.

The field of image processing is very wide. Image processing can be expanded further to image restoration, image segmentation and image enhancement techniques. Image enhancement is one of the most interesting and important research areas in digital image processing field. The principal objective of this area is to improve the quality of visual information in an image for human observer as well as the transformation of an image into a more suitable format for computer-aided process (Koschan and Abidi, 2008). Image enhancement is rather subjective because it depends strongly on the specific information the user is hoping to get from the image (Solomon and Breckon, 2011).

Due to the wide scope of image processing, this thesis is focused on image enhancement with contrast manipulation only. Hence, the terms of image enhancement and contrast enhancement will be used interchangeably in this thesis. Contrast enhancement plays an active role in digital image processing applications, such as in digital photography, medical image analysis, remote sensing, liquid crystal display processing, and scientific visualization (Arici et al., 2009). Some of the reasons of an image or video showing poor contrast are poor illumination conditions, low quality imaging sensors, user operation errors and media deterioration (Wu, 2011). These effects cause under utilization of the offered dynamic range. Hence, such images and videos may have a washed out and unnatural look and may not reveal all the details in the captured scene (Arici et al., 2009). For improved human interpretation of image semantics and higher perceptual quality, contrast enhancement is often performed to eliminate the aforementioned problems. It has been an active research topic since early days of digital image processing, consumer electronics and computer vision (Wu, 2011).

1.2 Problem Statement

Histogram Equalization (HE) is one of the popular digital image contrast enhancement methods (Cheng and Xu, 2000). This method is simple and easy to be implemented. HE enhances the given input image by using a monotonic transform function, where this transform function is derived from the intensity histogram of the input image. Alternatively, this transform function can be derived from the probability density function, which is basically the normalized version of the histogram (Yang et al., 2003). In order to produce an overall contrast enhancement, HE stretches the dynamic range of the image's histogram.

Global Histogram Equalization (GHE) is a HE method used to stretch and expand the dynamic range of an image by using only one transfer function calculated from the whole pixels in the image (Gonzalez and Woods, 2002). As a result, GHE expands gray levels with large pixel populations to occupy a larger range of gray levels while compress gray levels with fewer pixels to smaller ranges. Despite of GHE simple and powerful implementation, it cannot effectively enhance the contrast of local regions of an image (Solomon and Breckon, 2011). Hence, some parts of the image suffer from contrast stretching ratio limitation, and resulting in significant contrast loses in the background and other small regions (Kim et al., 2001).

Local Histogram Equalization (LHE) based enhancement methods have been developed to overcome the limitations mentioned above. LHE is a process of applying histogram modification to each pixel sequentially based on the histogram of pixels within a moving filter neighborhood (Pratt, 2007). The gray level mapping for enhancement is applied only to the center pixel of the filter which effectively increasing local contrast. However, there are some problems associated with LHE that need to be solved. First, computing the transform at every pixel is computationally intensive and time consuming. For example, for a 2,448 \times 3,264 pixels image, the histogram equalization must be performed maximally 7,990,272 times. Consequently, there is a need to reduce the processing time. Second, LHE methods are not able to preserve the mean intensity brightness of an input image, hence leading to saturation effect (Russ, 2011). Therefore, saturation effect reduction process should be included in the implementation of LHE contrast enhancement methods. Third, LHE sometimes produce unnatural enhancement and thus leading to

overly enhance speckle noise in homogenous area or background (Kong and Ibrahim, 2011). As a result, it is essential to reduce noise amplification in an enhanced image. Besides, owning to the fundamentally different nature of one image from the next, it is difficult and there is no general guide to choose the suitable filter size for LHE contrast enhancement (Solomon and Breckon, 2011). Therefore, in this research, a novel method, which is called as the Local Contrast Enhancement Utilizing Bidirectional Switching Equalization of Separated and Clipped Sub-Histograms (LCE-BSESCS) has been proposed to address the aforementioned problems.

1.3 Objectives of Study

Given the drawbacks of LHE methods as mentioned above, the objectives of this research are:

- To reduce the processing time. Switching approach has been used so that instead of calculating transfer functions from both sub-histograms, LCE-BSESCS calculates one transfer function from one sub-histogram only. Moreover, two modifications based on local histogram creation and sliding direction of the LCE-BSESCS filter further reduce the processing time.
- ii) To reduce saturation effect. LCE-BSESCS uses bidirectional equalization, where the intensity mappings move towards the local mean.
- iii) To reduce noise amplification. LCE-BSESCS utilizes clipped sub-histograms in order to avoid abrupt intensity changes.

1.4 Scope of Study

The scopes of this research are as below:

- 1. The research only deals with images taken from optical camera. Other types of images such as medical or satellite images will not be considered as part of this research.
- 2. This research is limited to HE based method only although there are still many more approaches for contrast enhancement.

3. This research is limited to 8-bit-per-pixel grayscale images and 24-bit-perpixel color images.

1.5 Structure of the Thesis

The structure of this thesis is as follow. Chapter 1 presents an introduction to contrast enhancement and histogram. Chapter 2 gives a brief review of the types of image, physics of color, image processing, types of image enhancement and image quality measures. Moreover, Chapter 2 also highlight about histograms and the previous works on the extensions to HE method which are GHE, mean brightness preserving HE, bin modified HE, and local HE. Chapter 3 introduces the new proposed method, which is LCE-BSESCS. Chapter 4 shows the experimental results and discussions of the proposed method. Besides, the evaluation of different filter sizes towards the performance of LCE-BSESCS is discussed. Chapter 5 concludes with a summary about the entire work and possible future research directions.

CHAPTER 2

LITERATURE REVIEW

This chapter gives a review on types of images, physics of color, image processing, types of image enhancement, histograms, various types of Histogram Equalization (HE) methods and image quality measures. This chapter is divided as follows. Section 2.1 gives a brief introduction about digital image as this research is carried out and tested based on this type of input. Section 2.2 reviews about the physic of color. This section also explains several types of color model that are normally used in digital image processing. Section 2.3 describes about image processing. Section 2.4 explains about the types of image enhancement. Section 2.5 narrates about histograms. Section 2.6 describes the extensions of HE which are Global HE, Mean Brightness Preserving HE, Bin Modified HE and Local HE. Section 2.7 conveys about the image quality measures used in Block Overlapped HE experiment (as in *APPENDIX A*) to benchmark the method with other types of HE extensions. Finally, section 2.8 remarks about the various types of HE extensions in a table.

2.1 Digital Image

A digital image is an image X(i, j), where i, j and the amplitude values of X are all finite, discrete quantities. A digital image that has M rows and N columns can be denoted as an M×N array of elements. Each element of the array is called picture element, image element, pel or pixel. There are no requirements on M and N, as long as they are positive integers (Gonzalez and Woods, 2002). An example of M×N digital image is shown in the following matrix format:

$$X(i,j) = \begin{bmatrix} X(0,0) & X(0,1) & \cdots & X(0,N-1) \\ X(1,0) & X(1,1) & \cdots & X(1,N-1) \\ \vdots & \vdots & & \vdots \\ X(M-1,0) & X(M-1,1) & \cdots & X(M-1,N-1) \end{bmatrix}$$
(2.1)

with $0 \le X(i, j) \le L - 1$, where usually L is expressed as:

$$L = 2^k \tag{2.2}$$

Here, k is a positive integer. Thus the discrete levels are equally spaced and they are integers in the interval [0, L-1]. For example, in an 8-bit-per-pixel digital image, the pixels' values are in the range of [0,255] because L is equal to 256 (i.e., 2^8).

The processing of an image by means of a computer is generally referred as digital image processing. The use of computers to process images provides more flexibility and adaptability in terms of no hardware modifications are required to reprogram digital computers to solve different tasks. Besides, the digital image can be effectively stored using image compression algorithms. Moreover, the transmission of digital data from one computer to another computer in other place is easier (Jayaraman et al., 2009).

2.1.1 Binary Image

A binary image takes only two levels of brightness. A threshold value, T, that corresponds to a brightness level is selected. Then, at any point (i, j) for which $X(i, j) \ge T$ is set to white and represented in the image memory as '1'. Otherwise, the remaining points (i, j) for which X(i, j) < T are set to black and represented as '0'. Thus, brightness graduation cannot be differentiated in a binary image. The process time for a binary image is quick because it only requires a very small amount of memory to store an image (Akeel and Watanabe, 1999). Binary image can be used to extract easily the geometric properties of an object, like the orientation of the object or the location of the centroid of the object (Jayaraman et al., 2009).

2.1.2 Grayscale Image

A grayscale image consists of a single channel to carry brightness or intensity information. Each pixel in the image memory corresponds to an amount or quantity of light. The brightness graduation can be differentiated in a grayscale image (Jayaraman et al., 2009). A grayscale image that uses 8 bit-per-pixel has intensity value from the range of [0,...,255], where '0' represents the minimum brightness (i.e., black) and '255' represents the maximum brightness (i.e., white). However, 8

bit-per-pixel is not sufficient for many print applications and professional photography, as well as in astronomy and medicine. Usually for these types of domain, image depths of 12, 14 and even 16 bits are used (Burger and Burge, 2009). Although the processing time for grayscale image is slower than binary image, it has significant advantage in reliability and is fast enough for many applications (Akeel and Watanabe, 1999).

2.1.3 Color Image

The perception of color is very important to human beings. Color information enables human to distinguished objects, materials, food, places, and even the time of the day (Shapiro and Stockman, 2001). Each pixel in a color image has three values to measure the intensity and chrominance of light (Shi and Sun, 2000). Intensity is an attribute of visible light, while chrominance can be described as the combination of both hue and saturation (Gonzalez and Woods, 2002). The hue of a color is characterized by the dominant wavelength in a mixture of light waves. Saturation is the measure of white light mixed with a hue or in other words, purity of a color. Each pixel in an image is a vector of color components. Color image can be modeled as three-band monochrome image data, where each band of data corresponds to a different color (Jayaraman et al., 2009). For a color image with an 8-bit depth, each component of color requires 8 bits. Hence, each pixel requires (3 channels) \times (8 bits / (channel) = 24 bits to encode all three components, and the range of each individual color component is [0,...,255]. For professional applications, color images with 30, 36, and 42 bits per pixel are used (Burger and Burge, 2009). Different color models are used in a variety of applications. For color video cameras and monitors, RGB (red, green, blue) model is used. In color printing, CMY (cyan, magenta, yellow) and CMYK (cyan, magenta, yellow, black) models are used. Similarly with the way human interpret and describe color, the HSI (hue, saturation, intensity) model is introduced (Gonzalez and Woods, 2002).

2.2 Physics of Color

In the 17th century, Sir Isaac Newton reported that white light from the sun consists of a continuous spectrum of colors ranging from violet at one end to red at the other. Basically, the light reflected from an object determines the colors perceive by humans and some other animals. Only a narrow section of the electromagnetic spectrum is visible to human, with wavelength λ from approximately 380nm to 740nm (Sonka et al., 2008). Colors can be represented as combinations of the so called primary colors red (R), green (G), and blue (B). In the year of 1931, for the purpose of standardization, the CIE (Commission Internationale de l'Eclairage-the International Commission on Illumination) had designated the following specific wavelength values to the three primary colors: blue = 435.8nm, green = 541.6nm, and red = 700nm (Gonzalez and Woods, 2002). However, this standardization does not mean that all colors can be synthesized as combinations of these three primary colors.

Secondary colors of light – magneta (red plus blue), cyan (green plus blue), and yellow (red plus green) are produced by adding the primary colors. A proper combination of the three primaries, or a secondary with its opposite primary color in the right intensities, will give rise to white light. This is illustrated in Figure 2.1(a), which shows the three primary colors and their combinations to produce the secondary colors (Gonzalez and Woods, 2002).

The primary colors of light and the primary colors of pigments or colorants are different in meaning. In the latter, a primary color is defined as one that absorbs or subtracts a primary color of light and reflects or transmits the other two. Hence, magenta, cyan and yellow are the primary colors of pigments, and red, green and blue are the secondary colors. Mixing the three pigment primaries in equal amount, or a secondary with its opposite primary, produces black (Gonzalez and Woods, 2002). These colors are shown in Figure 2.1(b).

Color models provide a standard way to specify a particular color by defining a coordinate system and a subspace within that system where each color is represented by a single point. Any color can be specified using a model. Each color model is orientated either towards hardware (such as for color monitors and printers) or towards specific software application (such as the creation of color graphics for animation) (Gonzalez and Woods, 2002).



Figure 2.1: Primary and secondary colors of light and pigments

2.2.1 RGB Color Model

The RGB color model encodes colors as combinations of the three primary colors: red (R), green (G) and blue (B). This model is used widely on both analog devices such as television sets and digital devices such as computers, digital cameras and scanners for representation, transmission and storage of color images. Besides, this model is an additive color system because all colors start with black and are created by adding the primary colors. Different colors can be created by modifying the intensity of each of these primary colors independently. Each primary color with its distinct intensity is used to control the shade and brightness of the resulting color. White and gray colors are created by mixing the three primary colors at the same intensity (Burger and Burge, 2009).

A three-dimensional unit cube as shown by Figure 2.2 can be used to visualize the RGB color model. The RGB colors form the coordinate axis; cyan, magenta, and yellow are at three other corners; black is at the origin, and white is at the corner farthest from the origin. In this model, gray scale intensities lie along the line connecting black and white colors. Other colors in this model can be defined by extending vectors from the origin to the point inside or on the cube. For convenience, RGB values are often normalized to the interval [0,1] so that the resulting color space forms a unit cube (Gonzalez and Woods, 2002).



Figure 2.2: Schematic representation of the color cube. Reproduced from: http://zone.ni.com/reference/en-XX/help/372916P-01/nivisionconcepts/color_spaces

The trichromatic RGB encoding in graphics systems usually uses three bytes or 24-bit to produce roughly 16 million colors (i.e. $(2^8)^3 = 16,777,216$). Each 3-byte or 24-bit RGB pixel includes one byte for each of red, green, and blue. This 24-bit RGB image is denoted as full color image shown in Figure 2.3 (Gonzalez and Woods, 2002).



Figure 2.3: RGB 24-bit color cube. Reproduced from: http://www.cs.ru.nl/~ths/rt2/col/h2/colorcube.jpg

2.2.2 CMY Color Model

CMY is an abbreviation of cyan (C), magenta (M) and yellow (Y) which are the secondary colors of light or, alternatively, the primary colors of pigments. This color model uses subtractive color mixing that is found in most devices like printers and copiers (Jayaraman et al., 2009). It describes the appropriate reflections when a printed image is illuminated with white light. For example, yellow absorbs blue light from reflected white light, which itself consists of the same amount of red, green and blue light. Normally, the RGB to CMY conversion in printing devices is performed by using the following simple operation:

$$\begin{bmatrix} C\\ M\\ Y \end{bmatrix} = \begin{bmatrix} 1\\ 1\\ 1 \end{bmatrix} - \begin{bmatrix} R\\ G\\ B \end{bmatrix}$$
(2.3)

where all color values are assume to be normalized in the range [0,1]. Equation above shows that red is absorbed when a pure cyan surface is illuminated with white light (that is, C = 1 - R in the equation). Similarly, pure magenta absorbs green, and pure yellow absorbs blue (Shapiro and Stockman, 2001).

When equal amount of cyan, magenta, and yellow are combined, it will produce a muddy looking black. In order to produce a true black, which is used abundantly in printed documents, a fourth color, black, is added. Thus, this gives rise to the CMYK color model also known as "four-color printing". For clarity, CMYK refers to the three colors in CMY color model plus black (Gonzalez and Woods, 2002).

2.2.3 HSI Color Model

In order to describe colors that are suitable for human interpretation, HSI color model is introduced. HSI stands for hue, saturation and intensity. Hue can be described as the dominant color perceived by an observer. It is an attribute associated with the dominant wavelength. Saturation measures the degree in which a pure color is diluted by white light. Intensity reflects the brightness of the color. HSI decouples the intensity component from the color, while hue and saturation correspond to human perception, hence it is very useful to develop image processing algorithm with this representation. HSI is a popular color model because it is based on human color perception (Jayaraman et al., 2009).

The double cone model of HSI is shown in Figure 2.4. The Hue (H) corresponds to the angle 0, varying from 0° to 360° . Saturation (S) represented to the radius, varying from 0 to 1. Intensity varies along the *z* axis in the range of 0 being black to 1 being white (Singh et al., 2003).



Figure 2.4: Double cone HSI color model. Reproduced from Singh et al. (2003)

2.2.3.1 RGB to HSI Color Conversion

In a RGB color image format, the H component of each RGB pixel is obtained by using (Gonzalez and Woods, 2002):

$$H = \begin{cases} \theta & \text{if } B \le G \\ 360 - \theta & \text{if } B > G \end{cases}$$
(2.4)

where

$$\theta = \cos^{-1} \left\{ \frac{\frac{1}{2} [(R-G) + (R-B)]}{[(R-G)^2 + (R-B)(G-B)]^{\frac{1}{2}}} \right\}$$
(2.5)

The saturation component is given by:

$$S = 1 - \frac{3}{(R+G+B)} [\min(R,G,B)]$$
(2.6)

Finally, the intensity component is given by:

$$I = \frac{1}{3} (R + G + B)$$
 (2.7)

The RGB values are assumed to be normalized in the range [0,1] and that θ is measured with respect to the red axis of the HSI space. By dividing all values from equation (2.4) with 360°, hue values that are normalized in the range of [0,1] are

obtained. The other two HSI components are in this range if the given RGB values are in the interval [0,1] (Gonzalez and Woods, 2002).

2.2.3.2 HSI to RGB Color Conversion

For HSI values in the interval [0,1], the corresponding RGB values in the same range can be found. The equations used in this conversion depend on the values of H. Referring to Figure 2.4, there are three sectors of interests corresponding to the 120° intervals in the separation of primaries. Initially, the H component is multiplied by 360° so that the hue is returned to its original range of [0°,360°] (Gonzalez and Woods, 2002).

RG sector $(0^{\circ} \le H \le 120^{\circ})$: If the given value of *H* is in this range, the RGB components are:

$$B = I(1-S) \tag{2.8}$$

$$R = I \left(1 + \frac{S \cos H}{\cos(60^\circ - H)} \right)$$
(2.9)

$$G = 3I - (R+B) \tag{2.10}$$

GB sector ($120^{\circ} \le H \le 240^{\circ}$): When H is in this sector, first subtract 120° from it:

$$H = H - 120^{\circ} \tag{2.11}$$

Then, the RGB components are:

$$R = I(1-S) \tag{2.12}$$

$$G = I \left(1 + \frac{S \cos H}{\cos(60^\circ - H)} \right)$$
(2.13)

and

$$B = 3I - \left(R + G\right) \tag{2.14}$$

BR sector $(240^{\circ} \le H \le 360^{\circ})$: Finally if H is in this sector, subtract 240° from it:

$$H = H - 240^{\circ} \tag{2.15}$$

Then, the RGB components are:

$$G = I(1-S) \tag{2.16}$$

$$B = I \left(1 + \frac{S \cos H}{\cos(60^\circ - H)} \right)$$
(2.17)

and

$$R = 3I - (G + B) \tag{2.18}$$

2.3 Image Processing

The scope of digital image processing is wide. Therefore, in this section, only four digital image processing branches will be discussed. They are image restoration, image segmentation, image compression and image enhancement.

Image restoration is used to reverse the distortions undergone by an image to recover the original image. The distortions are due to motion blur, noise and camera misfocus (Jayaraman et al., 2009). The techniques of image restoration are very mathematical in nature, which can be formulated either in spatial or frequency domain (Gonzalez and Woods, 2002).

Image segmentation can be used to extract certain characteristics of an image in order to obtain the region of interest for further analysis and interpretation (Petrou and Petrou, 2010). Segmentation deals with dividing an image by splitting it up into connected areas. There are three different approaches for image segmentation which are edge approach, boundary approach and region approach (Jayaraman et al., 2009).

Nowadays, digital images comprise of an enormous amount of data due to the advances in technology. As a result, image compression is needed to reduce the amount of data required to store a digital image (Gonzalez and Woods, 2002). In other words, image compression is related to the mapping from higher dimensional space to a lower dimensional space. Image compression can be classified into lossless compression and lossy compression (Jayaraman et al., 2009).

The goal of image enhancement is to improve the perception and interpretability of the information present in images for human viewers. The scope of image enhancement includes contrast enhancement, pseudo coloring, interpolation, sharpening and smoothing (Jain, 1989). Contrast enhancement is used to process an input image such that the visual content of the output image is more pleasing or more useful for machine vision applications by changing the intensity values of the input image (Celik, 2012). Therefore, the dynamic range of the image is expanded, besides enlarging the intensity difference among objects and background. Pseudo color image processing involves assigning colors to gray level values based on a specified criterion (Jain, 1989). This is because human eye perceive color more easily compare to variations in intensity. Image interpolation is essentially a process of magnifying image to ensure that the resultant image has better visual effect for special need (Zhou et al., 2012). Sharpening refers to any enhancement techniques that highlight edges and fine details in an image (Bovik, 2009). Usually highpass filters are associated with sharpening that significantly amplify high frequencies without attenuating lower frequencies. Smoothing process is related to noise reduction and blurring of an image (Jain, 1989). Lowpass filters are used to reduce high frequency components, and hence smoothing an image.

2.4 Types of Image Enhancement

There are several types of image enhancement methods. Nonetheless, only several types of image enhancement methods are discussed such as image negative, log transformation, power law transformation, linear spatial filter, non-linear spatial filter and histogram processing.

The negative transformation inverses the intensity of an image in the range [0, L-1]. The negative transformation function is given by (Jayaraman et al., 2009):

$$Y(i, j) = L - 1 - X(i, j)$$
(2.19)

where X(i, j) is the input image and Y(i, j) is the output image. A photographic negative image is obtained by reversing the intensity levels of an image. Negative images are useful in producing negative prints of medical images.

The log transformation is used to spread out a narrow range of low intensity levels in the input image into a wider range of output levels. The log transformation is defined as (Gonzalez and Woods, 2002):

$$Y(i, j) = c \log(1 + X(i, j))$$
(2.20)

where c is a constant. For example, this transformation function spreads the values of dark pixels while compressing the higher values. The opposite is true for the inverse log transformation whereby the transformation function maps the narrow range of high intensity levels in the input image into a wider range of output levels.

The intensity of light generated by cathode ray tube monitors has an intensity-to-voltage response that is not linear. Hence, this non-linearity must be compensated by a power law function to achieve correct reproduction of intensity. The power law transformation is obtained by (Gonzalez and Woods, 2002):

$$Y(i, j) = c(X(i, j))^{\gamma}$$
 (2.21)

where c and γ is a constant. The power law transformation function is the same as the log transformation albeit it is much more versatile and a family of possible transformation can be obtained by varying the γ value. For $\gamma < 1$, the low intensity levels in the input image are spread out and the opposite is true for $\gamma > 1$.

Spatial filtering involves neighbourhood operation. This process is implemented by simply sliding the filter from one pixel to another pixel of an image. The response of the filter at each point (i, j) is calculated using a predefined relationship. For linear spatial filter, each pixel value in the output image is the average of the pixels in the neighbourhood of the corresponding pixel in the input image (Gonzalez and Woods, 2002). This filter sometimes is called the mean filter. On the other hand, non-linear spatial filter replaces each pixel with the ranking result based on the pixels enclose by the filter (Jayaraman et al., 2009). An example of this kind of filter is the median filter which reduces impulse noise in an image.

Besides the image enhancement methods mentioned above, the visual quality of an image can be improved by manipulating the histogram of an input image. The following sections give the idea about histogram and the histogram equalization techniques used to enhance the quality of an image.

2.5 Histograms

Histograms play an important role in contrast enhancement as well as other image processing applications (Gonzalez and Woods, 2002). The histogram is used to represent image statistics in an easily interpreted visual format (Burger and Burge, 2009). In order to define a histogram, first, assume that $\mathbf{X} = \{X(i, j)\}$ is a digital image with L discrete gray levels in the intensity range of $\{X_0, X_1, ..., X_{L-1}\}$. X(i, j) represents the intensity of the image at spatial location (i, j) with the condition that $X(i, j) \in \{X_0, X_1, ..., X_{L-1}\}$. The histogram of a digital image is a discrete function as all the intensities are in discrete values. Thus, the histogram h is defined as:

$$h(X_k) = n_k, \text{ for } k = 0, 1, ..., L-1$$
 (2.22)

where X_k is the k-th gray level and n_k shows the number of pixels in the image having gray level X_k . In general, histogram depicts the frequency of the intensity values that occur in an image (Burger and Burge, 2009). The histogram provides more insight about image contrast and brightness, but it is unable to convey any information regarding spatial relationships between pixels (Jayaraman et al., 2009). Usually, the histogram of an image **X** is presented as a graph plots of $h(X_k)$ versus X_k . Shown in Figure 2.5 are some examples of images and their respective histograms.

These images show four basic gray level characteristics which are dark, bright, low contrast and high contrast. Figure 2.5(a) corresponds to a dark image dominated by low intensity pixels. Thus, the components of histogram are biased toward the left side of the gray scale as shown in Figure 2.5(b). On the other hand, the *snow* image as shown in Figure 2.5(c) is a bright image dominated by high intensity pixels. Hence, as shown in Figure 2.5(d), the components of histogram tend to be concentrated on the right side of the gray scale. A dull and washed-out image is shown in Figure 2.5(e). This is a low contrast image which has a histogram of narrow gray scale range as depicted in Figure 2.5(f). Finally, a high contrast image is show in Figure 2.5(g). The components of histogram cover almost all the entire range of

possible gray levels. Besides, the distribution for most of the pixels is not too far from uniform. Therefore, a high contrast has a large variety of gray tones.



(a) Image of *Hong Kong night view*



(b) Histogram of (a)



(c) Image of snow



(d) Histogram of (c)



(e) Image of *turtle*



(f) Histogram of (e)







00 150 Intensity values

Figure 2.5: Example of images and their corresponding histograms

2.6 Extensions of HE

There are various extensions of HE method. Generally, these variations of HE can be classified into four groups as shown in Figure 2.6. These extensions are Global Histogram Equalization, Mean Brightness Preserving Histogram Equalization, Bin Modified Histogram Equalization and Local Histogram Equalization.



Figure 2.6: Block diagram of HE's extension

Thus, this section provides a literature review on some of the extensions of HE. First, Global Histogram Equalization is introduced in Section 2.6.1. Then, the Mean Brightness Preserving Histogram Equalization methods will be presented in Section 2.6.2. After that, Section 2.6.3 will discuss about the Bin Modified Histogram Equalization. Section 2.6.4 is about Local Histogram Equalization. Finally, all the methods discussed will be summarized in the last section.

2.6.1 Global Histogram Equalization

Global Histogram Equalization (GHE) is a HE method that uses only one transform function calculated from the whole pixels in the image (Gonzalez and Woods, 2002). The algorithm for GHE is described briefly as below. For a given image \mathbf{X} , the Probability Density Function (PDF) for intensity X_k , $p(X_k)$, is given as:

$$p(X_k) = \frac{n_k}{N},$$
 for $k = 0, 1, ..., L-1$ (2.23)

where N is the total number of samples in the image. The PDF is actually a normalized version of the histogram.

The sum of all components of the normalized histogram or PDF results a Cumulative Density Function (CDF) of an image. Based on the PDF in equation (2.23), the CDF for intensity X_k , $c(X_k)$, is given as:

$$c(X_k) = \sum_{j=0}^k p(X_j), \quad \text{for } k = 0, 1, ..., L-1$$
 (2.24)

By definition, $c(X_{L-1})=1$. Similar to PDF, CDF of an image also can be represented as a plot of $c(X_k)$ versus X_k . In GHE, CDF is used as the intensity transform function for intensity value mapping.

GHE is a scheme that maps the input image into the entire dynamic range, $[X_0, X_{L-1}]$, by using CDF as its transformation function. Now, let $x = X_k$. The transform function, f(x), is defined based on CDF as (Kim, 1997):

$$f(x) = X_0 + (X_{L-1} - X_0) \times c(x)$$
(2.25)

From here, the output image produced by GHE, $\mathbf{Y} = \{Y(i, j)\}$, can be expressed as:

$$\mathbf{Y} = f(x) = \left\{ f(X(i,j)) \mid \forall X(i,j) \in \mathbf{X} \right\}$$
(2.26)

GHE usually causes brightness saturation effect and washed out appearance. This happens because GHE extremely pushes the intensities towards the right (i.e., bright) or the left (i.e., dark) side of the histogram. GHE is applied to images in Figures 2.5(a), (c), (e) and (g). Notice that for a dark image, it is clearly visible that the output of GHE image shown in Figure 2.7(a) suffers from brightness saturation. Hence, the histogram is pushed to the right side as depicted in Figure 2.7(b). On the other hand, GHE causes dark saturation effect on bright image shown in Figure 2.7(c). The corresponding histogram is shifted towards the left side as in Figure 2.7(d). This saturation effect, not only degrades the appearance of the image, but also leads to information loss (Wadud et al., 2007). The result of applying GHE to low contrast image is shown in Figure 2.7(g) shows a high contrast image processed by

GHE. By comparing its histogram in Figure 2.7(h) with the original histogram in Figure 2.5(h), it is concluded that GHE fail completely when the original image is already occupying the full dynamic range. Generally, important objects have a higher and wider histogram region, so the contrast of these objects is stretched. On the other hand, the contrast of lower and narrower histogram regions, such as background is lost (Kim et al., 2001).

The enhancement process is done globally in GHE method without considering the local contents of the image. Therefore, GHE is effective in enhancing the low contrast image when the input image contains only one big single object, or when there is no appearance contrast change between the object and the background in the image (Cheng and Shi, 2004). In addition, GHE causes over enhancement in the part of a histogram which has high probabilities of intensity levels, while loss of contrast for low probabilities of levels (Kim et al., 1998; Cheng and Shi, 2004; Wang and Ward, 2007; Wadud et al., 2007; Kim and Chung, 2008). Thus, GHE might enhances parts of the image which are unimportant for the viewer such as the background area of the image (Zimmerman et al., 1988; Csapodi and Roska, 1996). As a result, GHE method is not applicable to many image modalities, such as infrared image, because this method usually enhances the image's background instead of the object that occupies only a small portion of the image (Wang et al., 2006). With the limitations of GHE as discussed above, many researches actively develop various extensions to HE method. These extensions include Mean Brightness Preserving HE, Bin Modified HE and Local HE.

2.6.2 Mean Brightness Preserving Histogram Equalization (MBPHE)

Mean Brightness Preserving Histogram Equalization (MBPHE) is an extension to HE. Generally, these methods separate the histogram of the input image into several sub histograms, and the equalization is carried out independently in each of the subhistograms (Kong and Ibrahim, 2008). The basic idea of MBPHE is to separate the histogram into several segment boundaries based on certain type of threshold value. Examples of MBPHE are Brightness Preserving Bi-Histogram Equalization, Dual Sub-Image Histogram Equalization, Minimum Mean Brightness Error Bi-Histogram Equalization, Recursive Mean-Separate Histogram Equalization, and Recursive SubImage Histogram Equalization. The following subsections describe each of the MBPHE methods in details.





(a) Enhanced image of *Hong Kong night view*





(c) Enhanced image of *snow*



(d) Histogram of (c)



(e) Enhanced image of *turtle*



(f) Histogram of (e)



(g) Enhanced image of *fruit stall* (h) Histogram of (g) Figure 2.7: Example of images produced by GHE and their corresponding histograms