IMPROVEMENT OF DISCRIMINATION POWER AND WEIGHT DISPERSION IN MULTI-CRITERIA DATA ENVELOPMENT ANALYSIS

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by

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LIST OF ABBREVIATIONS

Abbreviation	Description
AP	Super-efficiency technique proposed by Andersen & Petersen (1993)
AR	Assurance region
BCC	The classical DEA model proposed by Banker, Charnes, & Cooper (1984) under VRS technique
BiO-FWeR	Bi-objective fuzzy weight restriction
BiO-MCDEA	Bi-objective multiple criteria DEA
BiO-WeR	Bi-objective weight restriction
CCR	The classical DEA model proposed by Charnes, Cooper, & Rhodes (1978) under CRS technique
CO_2	Carbon dioxide
CRS	Constant returns to scale
CSW	Common set of weights
DEA	Data envelopment analysis
DM	Decision maker
DMU	Decision making unit
DRS	Decreasing returns to scale
EU	European Union
GPDEA-CCR	Goal programming DEA model under CRS technique
GPDEA-CCR	Goal programming DEA model under VRS technique
IRS	Increasing returns to scale
LP	Linear programming
MCDEA	Multiple criteria data envelopment analysis
MCLP	Multiple criteria linear programming
MOLP	Multi-objective linear programming
VRS	Variable returns to scale

LIST OF PUBLICATIONS

1. Ghasemi, M. R., Ignatius, J., & Davoodi, S. M. (2014). Ranking of Fuzzy Efficiency Measures via Satisfaction Degree. In A. Emrouznejad& M. Tavana (Eds.), *Performance Measurement with Fuzzy Data Envelopment Analysis* (Vol. 309, pp. 157-165): Springer Berlin Heidelberg.

2. Ghasemi, M. R., Ignatius, J., & Emrouznejad, A. (2014). A bi-objective weighted model for improving the discrimination power in MCDEA. *European Journal of Operational Research, 233*, 640-650.

PENAMBAHBAIKAN KUASA DISKRIMINASI DAN SERAKAN PEMBERAT DALAM ANALISIS PENYAMPULAN DATA BERBILANG KRITERIA

ABSTRAK

Kekurangan keupayaan mendiskriminasi dan kelemahan pengagihan pemberat kekal sebagai isu utama dalam Analisis Penyampulan Data (DEA). Semenjak model DEA berbilang kriteria (MCDEA) pertama yang dibentuk pada akhir tahun 1990an, hanya pendekatan pengaturcaraangol; yakni, GPDEA-CCR dan GPDEA-BCC telah diperkenalkan bagi menyelesaikan masalah berkenaan dalam konteks berbilang kriteria. Kajian ini mendapati bahawa model GPDEA adalah tidak sah dan seterusnya menunjukkan bahawa model DEA berbilang criteria dwi-objektif (BiO-MCDEA) yang dicadangkan adalah lebih baik daripada model GPDEA dalam aspek kuasa mendiskriminasi dan pengagihan pemberat, di samping memerlukan kod komputasi yang sedikit. Sebagai tambahan, kajian ini mencadangkan suatu model susulan yang dikenali sebagai BiO-WeR yang membenarkan penggunaan sekatan pemberat tambahan (WeR) supaya nilai pemberat input-output dapat diagihkan dengan lebih saksama berbanding dengan pengagihan yang diperolehi pada permulaan penggunaan model BiO-MCDEA. Akhir sekali, konsep teori set kabur digunakan untuk mengambil kira ketidakpastian pemberat output yang berkaitan. Kajian ini kemudiannya melaksanakan model BiO-WeR dengan sekatan kabur terhadap pemberat output sebagai usaha untuk mengurangkan bilangan *DMU* yang cekap dan meningkatkan kuasa mendiskriminasi. Suatu aplikasi dalam kajian kebergantungan tenaga di antara 25 negara Kesatuan Eropah digunakan untuk menjelaskan keberkesanan dan menunjukkan cara perlaksanaan kaedah yang dicadangkan.

IMPROVEMENT OF DISCRIMINATION POWER AND WEIGHT DISPERSION IN MULTI-CRITERIA DATA ENVELOPMENT ANALYSIS

ABSTRACT

Lack of discrimination power and poor weight dispersion remain major issues in Data Envelopment Analysis (DEA). Since the initial multiple criteria DEA (MCDEA) model developed in the late 1990s, only goal programming approaches; that is, the GPDEA-CCR and GPDEA-BCC were introduced for solving the said problems in a multi-objective framework. This study finds GPDEA models to be invalid and demonstrates that the proposed bi-objective multiple criteria DEA (BiO-MCDEA) outperforms the GPDEA models in the aspects of discrimination power and weight dispersion, as well as requiring less computational codes. In addition, this study proposed an extension model named as BiO-WeR that provides additional weight restrictions (WeR) in order to distribute the values of input-output weights more evenly than those obtained by the initial BiO-MCDEA model. Lastly, the concept of fuzzy set theory is used to account for the uncertainty in the corresponding output weights. This study then implements the BiO-WeR model with fuzzy restrictions on the output weights as a means to further reduce the number of efficient *DMU*s and improve the discrimination power. An application of energy dependency among 25 European Union member countries is further used to describe the efficacy and demonstrate the implementation of the proposed approaches.

CHAPTER 1

INTRODUCTION

Data envelopment analysis (DEA) was first proposed by Charnes, Cooper, and Rhodes (1978) and remained the leading technique for measuring and evaluating the relative efficiencies of a set of homogenous decision making units (*DMU*s) based on their respective multiple inputs and outputs. The inputs can consist of labour, materials, energy, machines, and other resources, while the by-product of outputs may consist of finished products, services, customer satisfaction, and other forms of outcomes.

DEA has been the fastest growing discipline in the past three decades covering easily over a thousand papers in the *Operations Research* and *Management Science* discipline. There are a large number of DEA applications in environmental performance, especially at the national level. Many researchers began to provide a variation of one of the following carbon models and measures: CO_2 emission intensity, CO_2 emissions per capita, carbonization index, and energy intensity.

A common value of relative efficiency when there are multiple inputs and outputs can be expressed as

$$\frac{The\ weighted\ sum\ of\ ouputs}{The\ weighted\ sum\ of\ inputs}.$$

By using this notion, the efficiency measurement is generalized for a set of homogenous *DMU*s from a single-output and single-input to multiple-outputs and multiple-inputs. The *DMU* under evaluation (the target *DMU*) is designated as DMU_o where o ranges over $1, 2, \ldots, n$.

1.1 DEA model

Consider the relative efficiency of *n DMU*s which use *m* inputs ($x_{ij}, i = 1, \ldots, m, j = 1, \ldots, n$) to produce *s* outputs ($y_{rj}, r = 1, \ldots, s, j = 1, \ldots, n$). By assuming that the inputs-outputs data are nonnegative and at least one input and one output are positive, we solve the following fractional programming problem for each *DMU* to achieve measures of the input weights ($v_i,\ i = 1, \ldots, m$) and the output weights ($u_r,\ r = 1, \ldots, s$) as variables.

$$max\ \theta_o = \frac{u_1 y_{1o} + u_2 y_{2o} + \cdots + u_s y_{so}}{v_1 x_{1o} + v_2 x_{2o} + \cdots + v_m x_{mo}}$$

subject to:

$$\frac{u_1 y_{1j} + u_2 y_{2j} + \cdots + u_s y_{sj}}{v_1 x_{1j} + v_2 x_{2j} + \cdots + v_m x_{mj}} \leq 1, \quad j = 1, \ldots, n,$$

$$u_1, u_2, \ldots, u_s \geq 0,$$

$$v_1, v_2, \ldots, v_m \geq 0, \qquad (1.1)$$

where

y_{rj} =amount of output *r* assigned to DMU_j

u_r =weight assigned to output *r*

x_{ij} =amount of input *i* assigned to DMU_j

v_i =weight assigned to intput *i*.

In DEA model (1.1), we use the optimal value of the objective function to evaluate the efficiency value of DMU_o, which is equal to

$$\theta_o^* = \frac{u_1^* y_{1o} + u_2^* y_{2o} + \cdots + u_s^* y_{so}}{v_1^* x_{1o} + v_2^* x_{2o} + \cdots + v_m^* x_{mo}}.$$

According to the transformation approach proposed by (Charnes & Cooper, 1962), a "linear fractional programing problem" can be modified into an equivalent linear programming problem, thus Model (1.1) can be replaced by the following linear programming problem,

$$max\, \theta_o = u_1 y_{1o} + u_2 y_{2o} + \cdots + u_s y_{so}$$

subject to:

$$v_1 x_{1o} + v_2 x_{2o} + \cdots + v_m x_{mo} = 1,$$

$$u_1 y_{1j} + u_2 y_{2j} + \cdots + u_s y_{sj} \leq v_1 x_{1j} + v_2 x_{2j} + \cdots + v_m x_{mj}, \quad j = 1, \dots, n,$$

$$u_1, u_2, \dots, u_s \geq 0,$$

$$v_1, v_2, \dots, v_m \geq 0. \qquad (1.2)$$

We note that the scores of efficiency are independent of the units, in which the inputs-outputs are measured, thus establishing these units to be the same for every *DMU*.

1.2 Problem Statement

DEA has been one of fastest growing discipline in performance evaluation methods since the past three decades. Although DEA offers many advantages relative to other statistical methods, there are some drawbacks such as lack of discrimination power and the unrealistic weight distribution, which are still considered to be major issues that limit the interpretation and confidence on the generalizability of DEA results.

The problems above are more pronounced in environmental performance evaluation. Although DEA provides a readily available framework, it is not so straight forward as outputs in environmental efficiency models make up both desirable and undesirable outputs. For instance, higher GDP index tend to come with higher CO_2 emissions. This means that desirable outputs have to be sacrificed so that inputs can be reallocated for minimization of undesirable outputs (Hernandez-Sancho, Picazo-Tadeo, &Reig-Martinez, 2000).

Despite existing approaches such as assurance region (AR) procedure (Khalili, Camanho, Portela, & Alirezaee, 2010), cone ratio envelopment (Cao & Kong, 2010), super-efficiency model (Andersen & Petersen, 1993), and cross-efficiency evaluation technique (Wang & Chin, 2011) claiming to solve the drawbacks, they still possess the same problems. AR and cone ratio techniques are highly dependent on the measurement of the inputs-outputs units, which may lead to infeasible solutions. On the other hand, both the methods incorporate extra constraints to the DEA model; therefore, making it harder to solve the problem. The super-efficiency DEA model may obtain infeasible solutions for efficient *DMU*s; particularly, under variable returns to scale (VRS) model. With respect to cross-efficiency evaluation techniques, the non-uniqueness of DEA weights could provide a large number of multiple optimal solutions for DEA models.

It can be concluded that the existing methods still possess the following problems:

- Lack of discrimination among efficient *DMU*s, hence yielding many *DMU*s to be efficient,
- The unrealistic and poor weight distribution which may reveal that some input or output weights to possess zero values, hence implying that some of the variables were not used in the evaluation judgment in achieving the final ranking, and

- The need to sacrifice desirable outputs in the presence of undesirable outputs when keeping input levels at a minimal range in the context of sustainability and environmental performance.

1.3 Research Objectives

The research objectives of this study are as follows:

- To improve the existing methods such as multi-criteria DEA (MCDEA) model (Li & Reeves, 1999) and goal programming DEA (GPDEA) models (Bal, Örkcü, & Çelebioglu, 2010) in terms of discrimination power and weight dispersion. This is achieved by proposing approaches that distribute the values of input-output weights more evenly thus reducing the number of efficient *DMU*s,
- To propose the use of weight restriction by providing the upper bounds and fuzzy restrictions on the weights in DEA models, and
- To provide a solution for the above in the context of sustainability and environmental performance.

1.4 Theoretical Contribution

This thesis addresses the gaps in the MCDEA framework that was first proposed by Li and Reeves (1999). First, this research could provide an optimal solution to the problem, whereas the original authors (Li and Reeves, 1999) considered a series of solutions in interactive programming manner. In cases where a series of solutions are needed, the proposed bi-objective multiple criteria DEA (BiO-MCDEA) method could still handle weight adjustments to better discriminate the efficiency scores among *DMU*s. This has wider implications to the theoretical aspect of mathematical programming, where there are too many multiple optimal solutions that are present when one structures a multiple objective program. Compared to the goal programming versions known as GPDEA models (Bal et al., 2010), the proposed method has a greater advantage in terms of weight dispersion and discriminant power. Thus, it provides other researchers seeking to address weight dispersion and discriminant power problems to revisit the MCDEA framework.

Next, it can be shown that the proposed BiO-MCDEA performs better than the GPDEA model in terms of requiring lesser computational effort. The proposed method can be further extended by imposing restrictions on the input-output weights, and named as BiO-MCDEA model with weights restriction (BiO-WeR). The proposed BiO-WeR model is able to better discriminate the input-output weights among *DMU*s than the BiO-MCDEA model.

To account for uncertainty, the BiO-WeR model can be integrated with the fuzzy concept. This study models the constraints of fuzzy restrictions corresponding to the output weights. By adding the constraints to the BiO-MCDEA model, a DEA model with fuzzy restrictions on the output weights is obtained and named as BiO-FWeR model. By using α-cut set for the triangular fuzzy number associated to each output weight in BiO-FWeR model and solving the problem across different α-levels, different values of efficiency for each *DMU* can be obtained. The number of efficient *DMU*s can be decreased or increased by varying the value of α. Thus this is a good opportunity for the decision maker (*DM*) to decide on which value of α is the best for the scenario under his or her interpretation. In comparison to the BiO-MCDEA and BiO-WeR models, the proposed BiO-FWeR model is more informative and it can also provide a more balanced dispersion of input-output weights.

1.5 The Thesis Outline

The rest of the thesis is organized in the following way. Chapter 2 gives a literature review on DEA and carbon emission efficiency evaluation. The DEA literature includes a description of the basic DEA models and the drawbacks of DEA such as lack of the discrimination power and poor weight distribution. Several documented approaches such as AR, cone ratio envelopment, super-efficiency model, and cross-efficiency evaluation technique are further outlined in the literature to deal with the difficulties. It will also be noted that the recent approaches may suffer from some drawbacks in certain cases. Furthermore, a brief description of the MCDEA model (Li&Reeves, 1999) and the more recent GPDEA model (Bal et al., 2010) as a procedure for MCDEA is given.

We then highlight the drawbacks of using GPDEA to represent MCDEA analysis in Chapter 3. We therefore introduced BiO-MCDEA model to improve the discrimination power of MCDEA.

Chapter 4 presents BiO-WeR model as a way to improve the weights dispersion in BiO-MCDEA model. This is because we found that the proposed BiO-MCDEA model in Chapter 3 produces poor input or output weights in some cases.

In Chapter 5, the concepts of fuzzy numbers are used to define a triangular fuzzy number associated with the output weights in BiO-MCDEA model. In this Chapter, we first recall some basic definitions on fuzzy sets theory and introduce the main concepts needed for the remainder part of the chapter. Then the BiO-FWeR model is introduced to improve the weight dispersion and discrimination power of BiO-MCDEA model in Chapter 3. In comparison with BiO-WeR model, the proposed BiO-FWeR model is more informative in terms of giving opportunity to the decision maker to decide on the best scenario under his or her interpretation.

Some numerical examples and an application of energy dependency among 25 European Union (EU) member countries are given to describe the efficacy and demonstrate the implementation of each approach in Chapters 3 to 5. Concluding remarks and a discussion of the future research directions are given in Chapter 6.

CHAPTER 2

LITERATURE REVIEW ON DEA

2.1 Introduction

This chapter provides a literature review on DEA and carbon emission efficiency evaluation. The literature review on DEA includes the basic DEA models, the drawbacks of DEA such as the lack of discrimination among efficient decision making units (*DMU*s) and unrealistic input-output weights, and several techniques which were addressed in the literature as strategies to increase the discrimination power of *DMU*s and solve problems arising from unrealistic weight distribution. Special attention is given to the MCDEA model (Li & Reeves, 1999) and GPDEA models (Bal et al., 2010) because they are the closest to the proposed method in this thesis.

2.2 Basic DEA models

Data envelopment analysis (DEA) was first proposed by Charnes et al. (1978) and remained the leading technique for measuring the relative efficiency of *DMU*s based on their respective multiple inputs and outputs. DEA has been the fastest growing discipline in the past three decades covering easily over a thousand papers in the *Operations Research* and *Management Science* discipline (Emrouznejad, Parker, & Tavares, 2008; Hatami-Marbini, Emrouznejad, &Tavana, 2011). The efficiency of a *DMU* is defined as a weighted sum of its outputs divided by the weighted sum of its inputs on a bounded ratio scale.

Consider we are interested to evaluate the relative efficiency of n DMUs which use m inputs to produce $s$output. The m-input-s-output data can be expressed as $x_{ij} (i = 1, \dots, m, j = 1, \dots, n)$ and$y_{rj} (r = 1, \dots, s, j = 1, \dots, n)$. The envelopment form and dual (multiplier) form of input-oriented CCR model can be formulated as the following linear programming (LP) problems:

<u>The envelopment form of CCR model:</u>

$min\ \theta_o$

subject to:

$$\sum_{j=1}^{n} \lambda_j x_{ij} \leq \theta_o x_{io}, \qquad i = 1, \dots, m,$$

$$\sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{ro}, \qquad r = 1, \dots, s,$$

$$\lambda_j \geq 0, j = 1, \dots, n. \qquad (2.1)$$

<u>The dual (multiplier) form of CCR model:</u>

$$max\ \theta_o = \sum_{r=1}^{s} u_r y_{ro}$$

subject to:

$$\sum_{i=1}^{m} v_i x_{io} = 1,$$

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0, \quad j = 1, \dots, n,$$

$$u_r \geq 0, \qquad r = 1, \dots, s,$$

$$v_i \geq 0, \qquad i = 1, \dots, m, \qquad (2.2)$$

where j is the *DMU* index, $j=1,\dots, n$; r is the output index, $r=1,\dots, s$; i is the input index, $i= 1,\dots, m$; y_{rj} is the value of the r^{th} output for the j^{th} *DMU*, x_{ij} is the value of the i^{th} input for the j^{th} DMU, u_r is the weight given to the r^{th} output; v_i is the weight given to the i^{th} input, and θ_o is the relative efficiency of DMU_o, the DMU under evaluation.

Definition 2.1. DMU_o is efficient relative to the other units if the optimal value of the objective function (θ_o^*) is equal to one, otherwise if $\theta_o^* < 1$, DMU_o is inefficient.

Definition 2.2. Returns to scale (RTS) refers to increasing or decreasing efficiency based on size. The scale returns can be a variable, either increasing or decreasing, or constant. If a proportional increase in all the inputs results in a more or less than proportional increase in the single output, RTS will be increasing returns to scale (IRS) or decreasing returns to scale (DRS). Combining the two IRS and DRS ranges would necessitate variable returns to scale (VRS). Constant returns to scale (CRS) means that a proportional increase in the inputs consumed leads to a proportional increase in the outputs produced.

The CCR model is widely known as the CRS model. However, BCC model was proposed by Banker, Charnes, & Cooper (1984) to extend the CCR model by accommodating for VRS. CRS tends to lower the relative efficiency scores while VRS tends to raise relative efficiency scores. The envelopment form and dual (multiplier) form of input-oriented BCC model can be formulated as the following LP problems:

The envelopment form of BCC model:

$min\ \theta_o$
subject to:

$$\sum_{j=1}^{n} \lambda_j x_{ij} \leq \theta_o x_{io}, \qquad i = 1, \dots, m,$$

$$\sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{ro}, \qquad r = 1, \dots, s,$$

$$\sum_{j=1}^{n} \lambda_j = 1,$$

$$\lambda_j \geq 0, j = 1, \dots, n. \qquad (2.3)$$

<u>The dual (multiplier) form of BCC model:</u>

$$max\ \theta_o = \sum_{r=1}^{s} u_r y_{ro} - c_o$$

subject to:

$$\sum_{i=1}^{m} v_i x_{io} = 1,$$

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} - c_o \leq 0, \quad j = 1, \dots, n,$$

$$u_r \geq 0, \qquad r = 1, \dots, s,$$

$$v_i \geq 0, \qquad i = 1, \dots, m,$$

$$c_o \text{ free in sign}, \qquad (2.4)$$

where c_o indicates returns to scale. θ_o, $u_r (r = 1, \dots, s)$ and $v_i (i = 1, \dots, m)$ are defined as in the CCR model.

2.3 Discrimination power and weight dispersion problems in DEA

One of the drawbacks of DEA is the lack of discrimination among efficient *DMU*s, hence yielding many *DMU*s to be efficient. The problem is highlighted when the number of *DMU*s evaluated is significantly lesser than the number of inputs and outputs used in the evaluation. The post-hoc weights derived from a DEA analysis may reveal that some inputs or outputs have zero values. This is counter-intuitive especially in a decision making exercise, where one expects to use all the inputs and output values that are rated for the *DMUs*. Hence, it further implies that some of the variables were not used in the evaluation judgment in achieving the final ranking. On the contrary, the unrealistic weight distribution for DEA also occurs when some *DMUs* are rated as efficient due to extremely large weights in a single output and/or extremely small weights in a single input.

Thompson, Singleton Jr., Thrall, & Smith (1986) are among the first authors to propose the use of weight restriction to increase the discrimination power of *DMUs*. The issue was immediately picked up by many authors, including Dyson and Thanassoulis (1988), Charnes, Cooper, Huang, & Sun (1990), Thanassoulis and Allen (1998), and Saati (2008). Hence, several methods such as assurance region (AR) procedure and cone ratio envelopment were addressed in the literature as strategies to solve problems arising from unrealistic weight distribution. Other DEA models were also introduced in the literature to overcome the discriminant power problems, such as the super-efficiency model and cross-efficiency evaluation technique.

Drawing from a multiple objective decision making framework, the multiple criteria (or multi-objective) DEA model was suggested as a means to overcome discriminant power and weight dispersion problems.

2.3.1 Improving discrimination power in DEA

Super-efficiency model was introduced as one of the techniques in the literature to overcome the discriminant power problems. Super-efficiency technique first proposed by Andersen & Petersen (1993) is well known as the AP model which enables an extreme efficient DMU_o to obtain an efficiency value greater than one by removing the o^{th} constraint in the DEA models. The AP model based on multiplier form of CCR model can be formulated as follows:

<u>The Super-efficiency method of multiplier form of CCR model:</u>

$$max\ \theta_o = \sum_{r=1}^{s} u_r y_{ro}$$

subject to:

$$\sum_{i=1}^{m} v_i x_{io} = 1,$$

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0, \quad j = 1, \dots, n, j \neq o,$$

$$u_r \geq 0, \quad r = 1, \dots, s,$$

$$v_i \geq 0, \quad i = 1, \dots, m. \tag{2.5}$$

More details on the super-efficiency technique can be found in the following research (see Chen, 2005; Chen, Du, &Huo, 2013; Lee, Chu, & Zhu, 2011).The super-efficiency DEA model may obtain infeasible solutions for efficient DMUs; particularly, under VRS model. However, attempts had been made to solve the infeasibility problem in super efficiency methods. Chen (2005) proposed an approach in which both input-oriented and output-oriented super-efficiency models are used to fully characterize the super-efficiency model, thus claiming that the approach kept infeasibility to a rare occasion. However, Soleimani-damaneh, Jahanshahloo, &Foroughi (2006) presented some counter examples to negate Chen's (2005) claims without any proposed alternative. Drawing from two main sources (i.e. Chen, 2005; Cook, Liang, Zha, & Zhu, 2008), Lee et al. (2011) later provided a solution by a two-stage process catering to adjustments in input saving and output surpluses. Chen and

Liang (2011) subsequently formulated a one-model solution to the two-stage process. Lee and Zhu (2012) found that the solution can still be infeasible should some of the input variables have zero values.

Cross-efficiency evaluation technique was also introduced as another technique in the literature to overcome the discriminant power problems. Cross-efficiency approach was first proposed by Sexton, Silkman, & Hogan (1986) and it is often computed in two phases. The first phase is to obtain the value of input weights and output weights using the CCR model (2.2). Suppose that u^*_{ro} and v^*_{io} are the optimal values of the r^{th} output and the i^{th} input respectively for DMU_o (DMU under evaluation). The next phase is to achieve the cross-efficiency of DMU_t using the optimal weight values which were determined for DMU_o in model (2.2) as

$$E_{ot} = \sum_{r=1}^{s} u^*_{ro} y_{rj} / \sum_{i=1}^{m} v^*_{io} x_{ij} , \qquad o, t = 1, \dots, n. \qquad (2.6)$$

The values from (2.6) can be listed in a matrix, known as cross-evaluation matrix (see Table 2.1). To rank the *DMUs* using the cross-efficiency technique, the average of the cross-efficiency score is calculated as $\bar{E}_{ot} = \frac{1}{n}\sum_{t=1}^{n} E_{ot}$, which is assigned to the cross-efficiency value for DMU_o ($o=1,\dots,n$). For more details, the interested reader is referred to Anderson, Hollingsworth, &Inman (2002), Doyle & Green (1995), Green, Doyle, & Cook (1996), and Wang & Chin (2010, 2011).

Table 2.1
Cross-evaluation matrix

	DMU_1	DMU_2	.	.	.	DMU_n
DMU_1	E_{11}	E_{12}	.	.	.	E_{1n}
DMU_2	E_{21}	E_{22}	.	.	.	E_{2n}
.	.	.				.
.	.	.				.
.	.	.				.
DMU_n	E_{n1}	E_{n2}	.	.	.	E_{nn}

With regards to cross-efficiency evaluation technique, the non-uniqueness of the DEA weights could provide a high degree of multiple optimal solutions for DEA models. Although recent improvements of cross-efficiency evaluation techniques were proposed (e.g. Angiz & Sajedi, 2012), the solution is computationally expensive with the need to solve a series of linear programming problems. The suggestion of imposing secondary goals to improve variability of cross efficiency scores still leaves the non-uniqueness problem looming (see Cook & Zhu, 2013).

2.3.2 Strategies for solving problems arising from unrealistic weight distribution

AR and cone ratio techniques were addressed in the literature as strategies to solve problems arising from unrealistic weight distribution; they are highly dependent on the measurement of the inputs-outputs units, which may lead to infeasible solutions. In other words, both the methods incorporate extra constraints to the model; thus, making it harder to solve the problem. We briefly outline both methods in this section. Special attention is further given to the common set of weights (CSW) approach proposed by Saati (2008).

2.3.2.1 Cone ratio technique

Unrealistic weight distribution is a major problem in DEA and it occurs when some *DMU*s are rated as efficient due to extreme or zero value of input and/or output weights. The cone ratio model was developed by Charnes et al. (1990) and Charnes, Cooper, Wei, & Huang (1989) which arose from the observation of the space of the input-output weights as a strategy to solve the problem of unrealistic weight dispersion.

Suppose that $V = A^T\alpha$, where $A^T = [a_1, \dots, a_n] \epsilon \mathbb{R}^{m\times n}$ and $\alpha = \begin{bmatrix}\alpha_1\\ \vdots \\ \alpha_n\end{bmatrix} \epsilon\ \mathbb{R}^{n\times 1}$ be the polyhedral cone for input weight v, where $v = \begin{bmatrix}v_1\\ \vdots \\ v_m\end{bmatrix} \epsilon\ \mathbb{R}^{m\times 1}$. In the same manner, the polyhedral cone for output weight u, where $u = \begin{bmatrix}u_1\\ \vdots \\ u_s\end{bmatrix} \epsilon\ \mathbb{R}^{s\times 1}$ can be defined as: $U = B^T\beta$, where $B^T = [b_1, \dots, b_n] \epsilon\ \mathbb{R}^{s\times n}$ and $\beta = \begin{bmatrix}\beta_1\\ \vdots \\ \beta_n\end{bmatrix} \epsilon\ \mathbb{R}^{n\times 1}$. By adding the cone ratio restriction to CCR model (2.2), the CCR model can be transformed as

$max\ \theta_o = u^T y_o$

subject to:

$v^T x_o = 1,$

$u^T Y - v^T X \leq 0,$

$u \epsilon U, \quad v \epsilon V,$ (2.7)

where, $X = (x_{ij})_{m\times n}$, $Y = (y_{rj})_{s\times n}$, ${x_o}^T = [x_{1o}, \dots, x_{mo}]$, and ${y_o}^T = [y_{1o}, \dots, y_{so}]$.

According to the above definition of U and V, the LP model (2.7) can be written as following (in terms of α and β variables):

The Cone Ratio of multiplier form of CCR model:

$max\ \theta_o = \beta^T(By_o)$

subject to:

$\alpha^T(Ax_o) = 1,$

$\beta^T(BY) - \alpha^T(AX) \leq 0,$

$$\alpha \geq 0,\ \ \beta \geq 0. \tag{2.8}$$

In this case, the cone ratio model can be treated as a CCR model that evaluates the same *DMUs* with transformed data. However, the results must be transformed back into the original form in order to interpret the results and it would be considered a disadvantage. More details on the issue can be found in Cao&Kong (2010) and Charnes et al. (1990).

2.3.2.2 Assurance region (AR) method

Poor weight dispersion is a major issue in DEA. The problem is highlighted when some *DMU*s are rated as efficient due to extremely large weights in a single output and/or extremely small weights in a single input. To overcome the issue, weight restriction techniques such as AR was first developed by Thompson et al. (1986), which consider an upper and lower bound for its weights. The AR restrictions are defined as follows:

$a_r u_t \leq u_r \leq b_r u_t,\quad r < t,\qquad r, t = 1, \ldots, s,$

$$\alpha_i v_k \leq v_i \leq \beta_i v_k,\quad i < k,\qquad i, k = 1, \ldots, m. \tag{2.9}$$

where a_r and b_r are the lower and upper bounds on the ratios of output weights and α_i and β_i are the lower and upper bounds on the ratios of input weights, which are provided by the decision maker (*DM*).

Normally, one of the inputs (say x_1) can be selected as an input numeraire and one of the outputs (say y_1) can be selected as an output numeraire. Therefore, the above constraints (2.9) can be modified into the following form of AR constraints:

$$a_r u_1 \leq u_r \leq b_r u_1, \quad r = 1, \dots, s,$$
$$\alpha_i v_1 \leq v_i \leq \beta_i v_1, \quad i = 1, \dots, m. \tag{2.10}$$

By adding the above constraint to the CCR model (2.2), the model will be transformed into a CCR model which has bound relating to weights. This type of AR is known as assurance region I (ARI). It can be pointed out that ARI is a special case of cone ratio.

Another class of AR, which is called assurance region II (ARII) considers the relationship between input and output weights, i.e., bounds are set on the ratios of output weights to input weights. In ARI method, there will always be at least one efficient *DMU* whereas; in ARII case there is no certainty it will result in at least one efficient *DMU*. For more details we refer the reader to Khalili, Canmanho, Portela, & Alirezaee (2010), Mecit & Alp (2013), Thompson, Langemeier, Lee, & Thrall (1990).

2.3.2.3 Common set of weights (CSW) approach

In the extreme cases, when no flexibility is allowed, a CSW is applied in the literature for the evaluation of all *DMU*s. However, there are some drawbacks to the method – for instance, applying a CSW for the assessment of all *DMU*s limits the flexibility of DEA in assigning individual sets of weights to each of the participating *DMU*s.

Saati (2008) developed a technique, in which he suggests to find a CSW across *DMUs*. In this method, upper levels of the weights are first determined based on the optimal solution of some LP problems. Then by solving a linear programming problem, a CSW is determined. To determine the upper bounds of the output weights, the following LP problem can be considered.

$$\begin{aligned}
& max\ u_r \\
& \text{subject to:} \\
& \sum_{i=1}^{m} v_i x_{ij} \leq 1, \quad j = 1, \dots, n, \\
& \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0, \quad j = 1, \dots, n, \\
& u_r \geq 0, \qquad r = 1, \dots, s, \\
& v_i \geq 0, \qquad i = 1, \dots, m,
\end{aligned} \qquad (2.11)$$

where k ranges over $1, 2, \dots, s$and the constraint $\sum_{i=1}^{m} v_i x_{io} \leq 1$, is a constraint which normalizes the factor weights and the maximum value of each factor weight is obtained in such a way that the efficiency of each *DMU* does not exceed 1(Saati, 2008).

In the same manner the upper bounds of the input weights are determined by solving the following LP problem.

$$\begin{aligned}
& max\ v_l \\
& \text{subject to:} \\
& \sum_{i=1}^{m} v_i x_{ij} \leq 1, \qquad j = 1, \dots, n,
\end{aligned}$$

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0, \quad j = 1, \dots, n,$$
$$u_r \geq 0, \qquad r = 1, \dots, s,$$
$$v_i \geq 0, \qquad i = 1, \dots, m, \qquad (2.12)$$

where l ranges over $1, 2, \dots, m$.

Using the above models (2.11) and (2.12), the upper bounds of input-output weights are characterized by solving $s+m$ LP problems. It can be noted that these problems are feasible and their optimal values are bounded and positive (Saati, 2008). Furthermore, it was claimed by Satti (2008) that in the above models (2.11) and (2.12), $\sum_{r=1}^{s} u_r y_{rj} \leq 1$ and $\sum_{r=1}^{m} v_i x_{ij} \leq 1$ ($j=1,\dots,n$). Therefore, the values of upper bounds of output and input weights can be achieved as follows:

$$u_r^* = 1/\max_{1\leq j\leq n}\{y_{rj}\}, \quad (r = 1, \dots, s),$$
$$v_i^* = 1/max_{1\leq j\leq n}\{x_{ij}\}, \quad (i = 1, \dots, m). \qquad (2.13)$$

By assuming bounds on factor weights, the CCR model (2.2) can be expressed as follows:

The bounded CCR model:

$$max\ \theta_o = \sum_{r=1}^{s} u_r y_{ro}$$

subject to:

$$\sum_{i=1}^{m} v_i x_{io} = 1,$$
$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0, \quad j = 1, \dots, n,$$
$$U_r^l \leq u_r \leq U_r^u, r = 1, \dots, s,$$

$$V_i^l \leq v_i \leq V_r^u, r = 1, \dots, m, \tag{2.14}$$

where U_r^l, U_r^u, V_i^l, and V_r^u are the lower and upper bounds of output and input weights respectively.

By considering the bounded DEA model (2.14), a CSW can be provided by representing the deviation from either bound as a fraction of the range between the upper and lower bounds. By assuming the same deviation from bounds across all *DMUs*, the following problem can be considered.

$$\begin{aligned} & max\ \Delta \\ & \text{subject to:} \\ & \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0, \ \ j = 1, \dots, n, \\ & U_r^l + \Delta(U_r^u - U_r^l) \leq u_r \leq U_r^u - \Delta(U_r^u - U_r^l), r = 1, \dots, s, \\ & V_i^l + \Delta(V_i^u - V_i^l) \leq v_i \leq U_r^u - \Delta(V_i^u - V_i^l), r = 1, \dots, m. \end{aligned} \tag{2.15}$$

By setting the lower bounds of factor weights equal to zero ($U_r^l = V_i^l = 0$) and applying expressions (2.13), model (2.15) can be transformed into

$$\begin{aligned} & max\ \Delta \\ & \text{subject to:} \\ & \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0, \ \ j = 1, \dots, n, \\ & \Delta U_r \leq u_r \leq (1 - \Delta) U_r, r = 1, \dots, s, \\ & \Delta V_i \leq v_i \leq (1 - \Delta) V_i, r = 1, \dots, m, \end{aligned} \tag{2.16}$$

where $U_r (r = 1, \dots, s)$and $V_i (i = 1, \dots, m)$ are computed by expressions (2.13). The problem (2.16) is feasible and its optimal value is bounded and positive (Saati, 2008).

The efficiency value of DMU_o can be obtained as follows:

$$Eff_o = \frac{\sum_{r=1}^{s} u_r^* y_{ro}}{\sum_{i=1}^{m} v_i^* x_{io}},$$

where$u_r^* (r = 1, \dots, s)$and $v_i^* (i = 1, \dots, m)$ are the optimal values of LP problem (2.16) for DMU_o.

2.3.3 Multi-criteria approach to DEA

Drawing from a multiple objective decision making framework, the multiple criteria (or multi-objective) DEA model (Chen, Larbani, & Chang, 2009; Foroughi, 2011; Li & Reeves, 1999) can be used to improve discrimination power and also solving weight dispersion problems.

Li and Reeves (1999) first proposed the multiple criteria DEA (MCDEA) model as a means to improve the discrimination power of the classical DEA model. They developed their proposed model based on the basic DEA model (2.2) and they first represented the DEA model (2.2) equivalently in the following deviation variable form:

$$min\ d_o\ (or\ max\ \theta_o = \sum_{r=1}^{s} u_r y_{ro})$$

subject to:

$$\sum_{i=1}^{m} v_i x_{io} = 1,$$

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + d_j = 0, \quad j = 1, \dots, n,$$

$$u_r \geq 0, \qquad r = 1, \dots, s,$$

$$v_i \geq 0, \qquad i = 1, \dots, m,$$

$$d_j \geq 0, j = 1, \dots, n, \tag{2.17}$$

where d_o is a deviation variable for DMU_o and d_j is a deviation variable for DMU_j. The quantity d_o in the objective function is bounded on an interval [0, 1) and is regarded as a measure of inefficiency. DMU_o is efficient if d_o= 0 or, equivalently, θ_o= 1, thus θ_o= 1- d_o where θ_o is the efficiency measure in a classical DEA.

In their solution procedure, Li and Reeves (1999) suggested an interaction approach for solving three objectives. The first objective or criterion considers the classical definition of relative efficiency, thus capturing the classical DEA solution within the set of MCDEA solutions. The other two objectives, Minimax and Minsum objectives provide a more restrictive or lax efficiency solutions, respectively. This implies that a wider solution is possible with MCDEA, so as to gain more reasonable input and output weights.

In MCDEA, the three objectives are analyzed separately; one at a time, with no preference order set for those objectives. The solutions derived from each run are considered non-dominated in the multi-objective linear programming (MOLP) sense. Li and Reeves (1999) note that generally the Minimax criterion is more restrictive than the Minsum criterion, while the first criterion (i.e. Classical DEA objective) is considered to be the least restrictive of the three. Since the Minimax and Minsum criteria tend to provide less number of efficient *DMU*s as compared to the first criterion, it is said to provide better discrimination power than a classical DEA model. As such, the Minimax and Minsum criteria are helpful when the number of *DMU*s is not sufficiently larger than the number of inputs and outputs used for evaluation.