ENHANCED MICRO GENETIC ALGORITHM-BASED MODELS FOR MULTI-OBJECTIVE OPTIMIZATION

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ENHANCED MICRO GENETIC ALGORITHM-BASED MODELS FOR MULTI-OBJECTIVE OPTIMIZATION

by

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LIST OF ABBREVIATIONS

AoC	Apportionment of Credit
CD	Crowding distance
EA	Evolutionary Algorithm
J48	J48 decision tree
Logistic	Logistic regression model
MAS	Multi-Agent System
MDR	Mediated Decision Rule
mMDR	modified Mediated Decision Rule
MOEA	Multi-Objective Evolutionary Algorithm
MOGA	Multi-Objective Genetic Algorithm
MOP	Multi-objective Optimization Problem
mGA	micro Genetic Algorithm
mGA2	micro Genetic Algorithm II
MmGA	Modified micro Genetic Algorithm
NPGA	Niched-Pareto Genetic Algorithm
NSGA	Non-dominated Sorting Genetic Algorithm
NSGA-II	Non-dominated Sorting Genetic Algorithm II
PAES	Pareto Archived Evolution Strategy
RBF	Radial Basis Function neural network
RL	Reinforcement Learning
SHD	Statlog Heart Disease
SOP	Single-objective Optimization Problem
SPEA	Strength Pareto Evolutionary Algorithm
SPEA2	Strength Pareto Evolutionary Algorithm 2
SVM	Support Vector Machine
TNC	Trust-Negotiation-Communication
VEGA	Vector Evaluated Genetic Algorithm
WBC	Wisconsin Breast Cancer
WEKA	Waikato Environment for Knowledge Analysis

LIST OF SYMBOLS

$accuracy$	An accuracy rate of classification performance measure
$archiveSize$	The size of archive for the adaptive grid process
bid	A bid of AoC scheme
$BiSection$	The depth for the adaptive grid archive
$cbid$	A bid coefficient of AoC scheme
$ebid$	An effective bid of AoC scheme
$payment$	A reward payment procedure at clearing house of the AoC scheme
$\preceq_{CD}$	An CD-based crowded comparison operator
$\prec$ or $\preceq$	The weak and strict of Pareto dominance concept
I_{gd}	An MOP indicator of Generational Distance
I_{igd}	An MOP indicator of Inverted Generational Distance
I_s	An MOP indicator of Spread
mv	An RL reward/penalty signal
$\mathbf{m}$	A population memory size
$\mathbf{pf}$	A Pareto front
$\mathbf{pf}_{true}$	A true Pareto front
$\mathbf{rm}$	A replaceable memory component
$\mathbf{irm}$	An irreplaceable memory component
$ratio$	A replacement ratio for **rm**
$\mathbf{ratio}$	A set of replacement *ratio*
$spec$	A specificity rate of classification performance measure
$sens$	A sensitivity rate of classification performance measure
$strength$	A strength score of MmGA under the AoC scheme
$numfea$	A number of features of the classification problem

LIST OF SYMBOLS FOR LEMMAS

c_0 A maximum outlier generation cycle

c_1 A maximum nominal evolution cycle

c_2 A size of existing cumulative non-dominated front before generation cycle t

c_3 A total number of ensemble members (MmGAs)

c_4 A size of **irm**

c_5 An element size from **rm** and **irm** components for the extended population formation in MmGA

c_6 An element size from **rm** and **m** components for the extended population formation in MmGA

c_7 An element size from random generated solutions for the extended population formation in MmGA

c_8 A number of elements of MmGA in the AoC scheme

c_9 A number of elements of MmGA in the RL scheme

c_{10} An element size after feedback at outlier generation cycle fb

c_{11} An element size after feedback at outlier generation cycle $fb-1$

c_{12} An element size after feedback at outlier generation cycle $fb-2$

c_{13} A number of MmGA ensembles

c_{14} A number of MOP indicators

M The number of objectives

N The number of population sizes

MODEL TERTINGKAT BERASASKAN ALGORITMA GENETIK MIKRO UNTUK PENGOPTIMUMAN BERBILANG OBJEKTIF

ABSTRAK

Masalah pengoptimuman berbilang objektif (*Multi-objective Optimization Problem-MOP*) melibatkan berbilang objektif yang perlu dipenuhi serentak. Sekumpulan penyelesaian optimuman alternatif diperlukan untuk memenuhi kesemua objektif yang menunju ke arah barisan Pareto. Di samping itu, kualiti penyelesaian optimuman yang baik perlu diseimbangkan antara penumpuan dan kepelbagaian ke arah barisan Pareto sebenar (*true Pareto front*). Penyelidikan ini berkenaan cara algoritma evolusi digunakan untuk menangani masalah pengoptimuman berbilang objektif dengan kewujudan sifat penumpuan and kepelbagaian yang baik kepada penyelesaian berkenaan dengan barisan Pareto sebenar. Algoritma genetik mikro (*micro Genetic Algorithm-mGA*) dijadikan sebagai blok asas untuk mereka bentuk serta membangun ketiga-tiga model yang dicadangkan. Algoritma genetik mikro terubah suai (*Modified micro Genetic Algorithm-MmGA*) merupakan model pertama yang dicadangkan dengan matlamat mencari penyelesaian optimuman Pareto optimum secara cekap dan meningkatkan skor penumpuan (*convergence score*) penyelesaian ke arah barisan Pareto (*Pareto front*) dalam penyelesaian masalah pengoptimuman berbilang objektif. Seterusnya, ensembel MmGA (*MmGA ensemble*) dicadangkan untuk meningkatkan keteguhan model MmGA individu dan, pada masa yang sama mencepatkan skor penumpuan ke arah barisan Pareto. Seterusnya, demi mengambil kira kedua-dua ukuran penumpuan serta kepelbagaian penyelesaian (*diversity measure*) optimum Pareto, sistem berbilang agen berasaskan model penaakulan Amanah-Rundingan-Komunikasi (*Trust, Negotiation, Communication-TNC*) dieksploitasikan. Sehubungan itu, berbilang pe-

tunjuk prestasi digunakan dalam model MmGA berasaskan TNC untuk pengukuran skor penumpuan dan kepelbagaian. Kekompleksan masa komputasi untuk ketiga-tiga model diperiksa dengan analisis tatatanda O (*O-notation analysis*). Kekompleksan masa komputasi untuk ketiga-tiga model adalah berasimptot lebih tinggi berbanding dengan yang untuk mGA, namun serupa dengan yang untuk Algoritma Genetik Isihan Tak Terdominan II (*Non-dominated Sorting Genetic Algorithm II-NSGA-II*), yang terbenam dalam semua model berasaskan MmGA demi untuk meningkatkan prestasi mereka. Semua model yang dicadangkan mencatatkan keputusan yang baik berbanding dengan model pra-pengganti dalam skor penumpuan and skor kepelbagaian penyelesaian optimum. Model-model yang dicadangkan juga melaporkan peningkatan prestasi pengelas, dan pengurangan bilangan ciri yang digunakan dalam masalah pengelasan berbilang objektif. Potensi praktikal model berasaskan MmGA yang dicadangkan dalam menangani empat masalah dunia sebenar turut ditunjukkan. Semua petunjuk prestasi model berasaskan MmGA diukur menggunakan kaedah bootstrap. Hasil daripada kajian ini berjaya menentukan kebergunaan model berasaskan MmGA yang dicadangkan bagi menangani masalah pengoptimuman berbilang objektif.

ENHANCED MICRO GENETIC ALGORITHM-BASED MODELS FOR MULTI-OBJECTIVE OPTIMIZATION

ABSTRACT

Multi-objective Optimization Problems (MOPs) entail multiple conflicting objectives to be satisfied simultaneously. As such, a set of alternative solutions that is able to satisfy all objectives with respect to the Pareto optimality principle is desired. Besides that, the quality of good MOP solutions needs to strike a balance between convergence and diversity against the true Pareto front (i.e. distribution of the ideal Pareto optimal solutions). This research is concerned with how evolutionary algorithms can be employed to undertake MOPs with good convergence and diversity properties of the solutions with respect to the true Pareto front. Specifically, three evolutionary models based on the micro Genetic Algorithm (mGA) have been developed in a sequential manner for undertaking MOPs. Firstly, a Modified mGA (MmGA) model is introduced. MmGA aims to search for the Pareto optimal solutions efficiently and improve the convergence score of the solutions towards the Pareto front in tackling MOPs. Secondly, an ensemble of MmGA models is proposed to improve the robustness of individual MmGA models and, at the same time, to accelerate the convergence score of the solutions towards the Pareto front. Thirdly, to take both convergence and diversity scores of the Pareto optimal solutions into consideration, a multi-agent system that utilizes the Trust-Negotiation-Communication (TNC) reasoning scheme is exploited. Multiple performance indicators are incorporated into the TNC-based MmGA model, in order to achieve good convergence and diversity scores. The computational time complexity of three proposed evolutionary models is examined using the O-notation analysis. It is found that, while the computational time complexity of the three proposed models is higher than that of mGA, all proposed models have the same computational

time complexity with that of the Non-dominated Sorting Genetic Algorithm II (NSGA-II), which is embedded into all proposed models to improve their performances. Based on a number of MOP benchmark problems, all proposed models are able to yield favorable results as compared with those from their predecessors in term of convergence and diversity scores of the solutions. A number of experiments with multi-objective classification problems also indicate improvement of the classifier performances and, at the same time, reduction of the number of features used in classification, as compared with the results from standard classifiers as well as from other methods published in the literature. The potential of the proposed MmGA-based models in undertaking practical problems has also been demonstrated using four real-world MOPs. All performance indicators of the proposed MmGA-based models are quantified using the bootstrap statistical method. The outcomes positively ascertain the usefulness of the proposed evolutionary models in undertaking MOPs.

CHAPTER 1

INTRODUCTION

In this chapter, the research background is first presented. Then, the research questions, research objectives, and research scope are clarified. The organization of this thesis is presented at the end of this chapter.

1.1 Research Background

The research background includes an introduction to Multi-objective Optimization Problems (MOPs) and the Pareto optimality principle that is used as the yardstick to measure the effectiveness of various Evolutionary Algorithms (EAs) developed in this research. Specifically, Multi-Objective Evolutionary Algorithms (MOEAs) that constitute the core of this research are described.

1.1.1 Multi-Objective Optimization

Real-world problems often entail multiple and yet conflicting objectives. As a result, many optimization methods have been developed and employed to help identify and resolve the relationships among different, possibly contradictory, objectives since decades ago (Purshouse and Fleming, 2003). In general, optimization refers to finding the best possible solution of a problem with respect to a given set of constraints (Coello, 2006). The solution produced by an optimization model usually contains trade-offs, i.e. improvement in one objective could lead to degradation of another objective. As an example, saving cost by minimizing the number of workers in a production line could result in a longer production time.

A literature survey reveals that MOPs exist in a variety of domains. Examples include minimizing energy used and broadcasting time while maximizing the coverage achieved in a mobile ad hoc network (Ruiz et al., 2013); maximizing strength and elastic modulus while minimizing the cost of designing titanium alloys in prosthetic applications (Datta et al., 2013); maximizing profits in production scheduling under different environmental and economic concerns (Capón-García et al., 2013); minimizing operation cost, emission, and transmission losses of scheduling in a dispatch problem (Li, Das, Pahwa and Deb, 2013). These problems are known as MOPs because they require multiple objectives to be satisfied at the same time. As such, a set of alternative solutions in tackling all objectives is required. The solution set is known as the Pareto optimal solutions (Fonseca and Fleming, 1995). This means that the solutions are non-dominated, non-inferior, admissible, or efficient solutions (Fonseca and Fleming, 1995). This situation gives rise to the issue of what the optimal solution for an MOP is.

As an example, consider a manufacturing operation where the board of management would like to have a high profit and a low operation cost. Logically, the production cost has a direct relationship with the profit generated from manufacturing operations, where more capital investments lead to more revenue under an ideal, risk-free environment. In such a case, the cost-profit relationship can be viewed as a positive slope of a line chart, as shown in Figure 1.1. The cost-decreasing activities move downwards from the top until the Pareto front (as explained in the next section) is reached. Similarly, the profit-increasing activities move to the right towards the Pareto front. As such, a decision is said to be Pareto inefficient, i.e. all coordinates reside on the Pareto front, if one activity (such as increasing profit) can be conducted without harming any other activities (such as reducing cost). On the contrary, a solution is said to be Pareto efficient or Pareto optimal if improvement in an activity always leads to degradation of one or more activities.

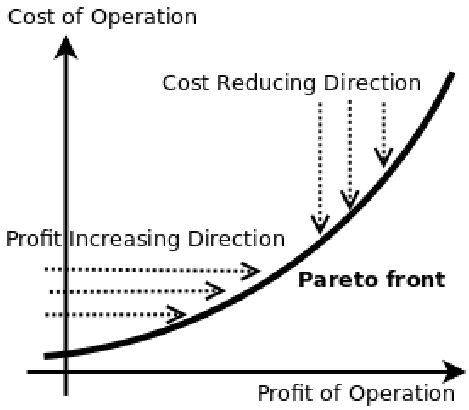

Figure 1.1: An example of the typical cost-profit operation problem and the Pareto front.

1.1.2 The Pareto Optimality Principle

Originated from Vilfredo Pareto (an Italian economist), the Pareto principle claimed that 20% of the population held 80% of wealth, i.e. the 80-20 rule (Pareto and Schwier, 1971). In other words, the Pareto principle describes that 80% of effects come from 20% of causes (Pareto and Schwier, 1971). This principle has been successfully used in explaining the dynamics underlying many different problems. As an example, in a study for ranking data from twelve types of sports, the results showed that only a few top-ranked players and teams are accumulating the majority of the prizes through the sport ranking model (Deng et al., 2012). In minimization problems, the concept of "Pareto optimal" is defined when there are no other feasible solutions which can decrease an objective without causing a simultaneous increase in at least one other objective (Coello, 2006). The opposite scenario applies to maximization problems. As such, the solutions for an MOP entail trade-offs, and when the Pareto principle is observed, they are known as the Pareto optimal set of solutions.

In essence, the Pareto optimal set refers to a set of Pareto efficient points known as the "Pareto front" (**pf**).The use of **pf** to evaluate efficacy of optimization trade-offs has been investigated in the literature. As an example, **pf** was employed to examine conflicting objectives in a plethora of biological observations ranging from morphological features like bird beak and bat wing shapes (Shoval et al., 2012).

A variety of Pareto-based evolutionary methods for solving MOPs are available. In the pharmacological area, drug designs with multiple targets using evolutionary methods for determining the molecular targets of drugs were reported in Lounkine et al. (2012). An adaptive Pareto-based evolutionary method was introduced to optimize the design of ligands against polypharmacological profiles (Besnard et al., 2012). In a Pareto-based evolutionary method pertaining to an acetylcholinesterase inhibitor drug, the associated multi-objective prioritization activity was performed by calculating the parameters involved as a multi-dimensional coordinate. The output was ranked by the magnitude between multi-dimensional coordinates of the predicted values and the ideal objective points, which are known as the true Pareto front ($\mathbf{pf}_{true}$) (Besnard et al., 2012).

1.1.3 Evolutionary Algorithms

Evolution by natural selection is a compelling area in modern science, and it has been shown to be able to solve problems and to model natural phenomena since the early 1990s (Forrest, 1993). There are many independent efforts to incorporate ideas from natural evolution into computation, and to realize a logical theory of adaptive systems (Holland, 1962). To date, many useful EAs are available to deal with different challenges. Novel reproduction operators (Ishibuchi et al., 2010), fitness function management strategies (Zhang and Li, 2007; Zhang et al., 2010), and weight assignments (Soylu and Koksalan, 2010) have been introduced to improve the convergence score to the Pareto optimal solutions, as well as the spread of solutions along **pf**. The key objective of evolutionary computation is to establish an effective and efficient computing paradigm by exploiting natural selection phenomena and the learning capability of problem solving. This, in turn, has led to the development of EAs for tackling optimization problems (Coello and Lamont, 2004).

An MOP is also known as a global optimum problem (Coello, 2006) whereby it needs to find the best possible solution available, or at least a good approximation to the best solution. In this research, the focus is on tackling MOPs using multi-objective EA-based models.

1.1.4 Multi-Objective Evolutionary Algorithms

MOEAs have been investigated (Van Veldhuizen and Lamont, 2000; Zhou et al., 2011) and used in undertaking MOPs since more than a decade ago. In general, multi-objective optimization methods can be broadly classified into three types according to two different stages of the MOP solution (Coello et al., 2007), i.e. stage (i) the objective function of optimization involved, and stage (ii) the process of deciding the trade-offs in solutions from the decision maker perspective, or known as the multi-criteria decision making process.

The three types of methods (Van Veldhuizen and Lamont, 2000; Coello et al., 2007) that are used for searching and making multi-criterion decisions are:

- the priori preference method (decide $\rightarrow$ search),
- the posteriori preference method (search $\rightarrow$ decide), and
- the progressive preference method (decide $\leftrightarrow$ search).

The "decide $\rightarrow$ search" method makes a decisions before searching, while the "search $\rightarrow$ decide" method performs searching before making a decision. The "decide $\leftrightarrow$ search" method combines search and decision making in its operation. A recent survey (Giagkiozis et al., 2014) claimed that the "search $\rightarrow$ decide" method is frequently employed owing to a higher degree of separation between the algorithm and decision-making process. This optimization method allows the testing process of population-based MOEAs to be conducted independently from the target MOP, without involving the decision maker in its operation. In other words, the decision maker is presented with a set of Pareto optimal solutions, i.e. non-dominated solutions, and the decision is chosen therein.

1.2 Research Questions

As reported in Giagkiozis et al. (2014), insufficient information and/or advice is often provided for practitioners to choose a suitable EA in practical applications. As a result, a comparison guideline for the number of publications per year was conducted (Giagkiozis et al., 2014). The data were extracted from the ISI Web of Knowledge from 1991 to 2012. The outcomes showed that the Genetic Algorithm (GA) recorded the highest number of publications as compared with other EAs. Meanwhile, the number of publications on the use of GA for MOPs also appeared to be the highest as compared with other problems such as continuous problems, discrete problems, and combinatorial problems.

On the other hand, population-based MOEA has been recognized as a useful optimisation methodology for tackling MOPs due to its versatility (Giagkiozis et al., 2014). The use of a large population size has been a commonly used strategy in improving the performance of an EA. However, Chen et al. (2012) reported that a large population leads to a larger probability of finding optimum solutions at the local basin by an EA.

In this research, the main research focus is on a popular GA-based model with the "search $\rightarrow$ decide" method. Specifically, this research uses a GA-based model that operates with a small population using the "search $\rightarrow$ decide" method. The micro Genetic Algorithm (mGA) (Coello and Pulido, 2005), which offers a good convergence score towards $\mathbf{pf}_{true}$ by using only a small population size of 3-6 chromosomes, forms the core model in this research.

Three enhanced mGA-based models are proposed, i.e. a Modified micro Genetic Algorithm (MmGA), an MmGA ensemble, and an agent-based MmGA model, for undertaking MOPs. The key research questions undertaken in this research are:

- how to improve the convergence properties of the MmGA solutions towards the Pareto front while preserving the salient features of the original mGA model?
- how to achieve better convergence properties of solutions from single MmGA model towards the Pareto front by using an ensemble of MmGA models?
- how to incorporate multiple MOP performance indicators into a hierarchical agent-based MmGA model and enhance both convergence and diversity properties of the solutions towards the Pareto front?

It has been reported in (Coello and Pulido, 2005) that since mGA possesses a very small population size, its population reinitialization procedure is crucial in avoiding the pre-matured convergence problem. In this research, since the MmGA-based models are developed by using mGA as the building block, the salient properties of mGA are preserved. In particularly, the population re-initialization procedure is maintained in the proposed MmGA-based models, therefore inheriting the capability of avoiding the pre-matured convergence problem. In addition, issue related to maintaining the computational time complexity of the resulting algorithms of MmGA-based models are analyzed in detail.

1.3 Research Objectives

This research is devoted to the design, development, and application of three enhanced mGA-based models for undertaking MOPs. The Pareto front, which contains all optimal (non-dominated) solutions, is used as the yardstick for evaluating the performances. The bootstrap method is employed to quantify the results statistically (Efron, 1982; Hall, 1992). In addition, the computational time complexity of the resulting models is analyzed using the O-notation method (Kleinberg and Tardos, 2006; Cormen et al., 2009). The specific research objectives are as follows:

- to improve the original mGA with three proposed models:
  - an MmGA model that is able to provide a set of solutions for tackling MOPs with good convergence properties towards the true Pareto front,
  - an ensemble of MmGA models that is able to improve the solutions from a pool of individual MmGA models with better convergence properties towards the true Pareto front, and
  - an agent-based MmGA model that is able to provide good solutions which consider both convergence and diversity properties towards the true Pareto front.
- to systematically analyse the computational time complexity of the three proposed mGA-based models,
- to comprehensively evaluate the effectiveness and applicability of individual, ensemble, and agent-based MmGA models using a series of benchmark and real-world MOPs.

1.4 Research Scope

In this research, an GA-based of MOEA framework with three multi-objective optimization models are introduced. These models, i.e. MmGA, an MmGA ensemble, and an agent-based MmGA model, are escalated and explained sequentially in chapters to undertake MOPs. A series of benchmark and real MOPs are employed to evaluate the effectiveness of the proposed models. The effectiveness of the proposed models is assessed within the Pareto optimality concept. The measures of effectiveness are constrained within the convergence and diversity properties of the solutions with respect to the Pareto front. To quantify the performance statistically, the experimental results of each model are evaluated using the bootstrap method, i.e. a statistical evaluation method that does not rely on the underlying data statistics, and is useful for small data samples. The performance of the models is compared with those from other related methods published in literature. In addition, the models are analyzed with the worst-case computational time complexity analysis using the O-notation method. The detail methods of each model in solving the MOP and its assessment in robustness constitute the main research scope of this thesis.

1.5 Research Contribution

This research contributes three enhanced mGA-based models to tackle MOPs, namely MmGA, MmGA ensemble, and agent-based MmGA model. Systematic benchmark evaluations on efficacy the three models are conducted. Applicability of the agent-based MmGA model to four real-world case studies is demonstrated. In addition, the computation time complexity of the three models are analyze using O-notation method.

Seven steps are conducted to realize the contributions of this research, as follows.

1. enhancing the mGA model: formulating MmGA, and deriving its computational time

complexity with the O-notation method,

2. evaluating MmGA using benchmark MOP functions,
3. improving MmGA with a better convergence score: formulating an MmGA ensemble, and deriving its computational time complexity using the O-notation method,
4. evaluating the MmGA ensemble with benchmark MOP functions and multi-objective classification problems,
5. improving the MmGA ensemble with better convergence and diversity scores: formulating the agent-based MmGA model using the Trust-Negotiation-Communication (TNC) structure, and deriving its computational time complexity using the O-notation method,
6. evaluating the TNC-based MmGA model with benchmark MOP functions and multi-objective classification problems, and
7. evaluating efficacy of the TNC-based MmGA model with real-world multi-objective classification and optimization problems.

1.6 Thesis Organization

The organization of this thesis is as follows: in Chapter 2, a literature review is presented. The review covers MOPs, MOEAs, mGA, and the Pareto principle. Besides that, the performance indicators of MOEAs, the O-notation analysis for computational time complexity measurement, and the bootstrap sampling of statistical method are explained.

The details of the proposed three MmGA-based models are elaborated in Chapters 3, 4, and 5, respectively. In Chapter 3, the mGA model and the O-notation analysis of the resulting model are described. A detailed description of the first proposed model, i.e. MmGA and its O-notation analysis is presented. In addition, an evaluation of the proposed MmGA model with a benchmark MOP is conducted. The results are compared with those published in the literature.

In Chapters 4 and 5, detailed descriptions pertaining to the second and third proposed models are presented, i.e. MmGA ensemble and agent-based MmGA model, respectively. The O-notation analyses for the resulting models are provided. Evaluations on the proposed models are conducted with benchmark MOPs and multi-objective classification problems. Again, the results are compared with those published in the literature.

In Chapters 6, real-world case studies are presented to assess the effectiveness of the MmGA-based models. The case studies include a multi-objective job shop problem, a multi-objective circuit design problem, a multi-objective human motion detection and classification problem, and a multi-objective myocardial infarction classification problem. The results are compared with the enumeration method for the job shop problem, and Simulated Annealing and GA from the commercial Agilent ADS software for the circuit design problem. On the other hand, popular standard classifiers in the literature, i.e. J48 decision tree, Logistic Regression model, Radial Basis Function neural network, and Support Vector Machine, are used for comparison purposes in both the human motion detection and myocardial infarction classification problems.

Conclusions and recommendations for further work are presented in Chapter 7.

CHAPTER 2

LITERATURE REVIEW

A literature review pertaining to principles, models, and methods that are closely related to this research is presented in this chapter. Specifically, Multi-Objective Evolutionary Algorithms (MOEAs), micro Genetic Algorithm (mGA) and its variants, as well as the Pareto principle are reviewed and explained. Then, various MOEA performance indicators, the *O*-notation analysis, as well as the bootstrap method used to evaluate the proposed models are described in detail.

2.1 Multi-Objective Evolutionary Algorithms

In general, search and optimization schemes can be grouped into three schemes, i.e. enumerative, deterministic, and stochastic (Coello et al., 2007). The enumerative scheme explores a defined finite search space with all possible solutions. The deterministic scheme limits the search space for the required solution in a given time constraint. The stochastic scheme requires a fitness function to carry out search and evaluate the possible solutions, as well as a mechanism to map the target problem into the model parameters. EAs belong to the stochastic scheme (Coello et al., 2007). They are able to search for a set of possible solutions that forms **pf**.

There are four conventional EA-based models (Iba and Aranha, 2012a):

- Genetic Algorithm (GA): The GA is the earliest form of the evolutionary paradigm. Its operators are used to change and improve upon a population of solutions to a problem (Holland, 1992),

- Evolutionary Strategy (ES): Similar to the GA, ES focuses more on the mutation activity (Bäck, 1996),
- Genetic Programming (GP): GP is also similar to the GA. It includes an extension of genetic elements using expressions in terms of trees and graphs (Koza et al., 2005), and
- Evolutionary Programming (EP): EP uses interaction of "species", rather than "individuals", as used in the other three paradigms (Fogel, 1994).

Summing the objective function scores is a common method to turn an MOP into a Single-objective Optimization Problem (SOP). In SOP, a scalar function is used to represent the new optimization criterion (Smajic et al., 2009). Each scalar factor relies on a number of parameters, where trade-offs among the original objectives can be specified, i.e. the (i) weighted sum, (ii) Tchebycheff, and (iii) ε-constraint (Zitzler, 2012). An MOP differs from a SOP as it contains multiple objectives that require simultaneous optimization, and has acceptable performance ranges for all objective functions. Figure 2.1 shows the pseudo-code of an EA for undertaking SOP, which can be extended for tackling MOPs. The explanation is as follows:

$t = 0$
initialize $\mathbf{p}(0) = \left\{ \{a_1(0), \ldots, a_\mu(0)\} \in I^{\mu(0)} \right\}$ /**A parent population at the initial state.**/
repeat
    the recombination process with the Crossover Operator: $\mathbf{p}'(t) = r^{(t)}_{\theta_r^{(t)}}\left(\mathbf{p}(t)\right)$
    the mutation process with the Mutation Operator: $\mathbf{p}''(t) = m^{(t)}_{\theta_m^{(t)}}\left(\mathbf{p}'(t)\right)$
    the selection process with the Selection Operator:
    if χ **then**
        $\mathbf{p}(t+1) = s^{(t)}_{(\theta_s^{(t)}, \Phi)}\left(\mathbf{p}''(t)\right)$ /**A parent-offspring population, after the recombination and mutation processes, upon the criterion of selection process is filled.**/
    else
        $\mathbf{p}(t+1) = s^{(t)}_{(\theta_s^{(t)}, \Phi)}\left(\mathbf{p}''(t) \cup \mathbf{p}(t)\right)$ /**A parent-offspring population, after the recombination and mutation processes, upon the criterion of selection process is not filled.**/
    end if
    $t = t + 1$
until $\iota\left(\{\mathbf{p}(0), \ldots, \mathbf{p}(t)\}\right) \neq true$ is met /**The termination criterion of evolution process.**/
return $\mathbf{pf} \equiv \mathbf{p}$

Figure 2.1: The pseudo-code of an Evolutionary Algorithm, adopted from Coello et al. (2007)

Let I be a non-empty set with an individual space, $\mu(i)$ be a sequence in $\mathbb{Z}^{+}$ for $i \in \mathbb{N}$. Both I and μ denote a parent population at the initial stage, as $i = 0$ (line 2). Note that $\mu^{'}(i)$ and $\mu^{''}(i)$ for $i \in \mathbb{N}$ are the offspring population in a sequence in $\mathbb{Z}^{+}$ (lines 4, 5, 8, and 10). On the other hand, $\chi \in \{true, false\}$ is the criterion of the selection operator, $\theta_r^{(i)} \in \mathbb{X}_r^{(i)}$, $\theta_m^{(i)} \in \mathbb{X}_m^{(i)}$ and $\theta_s^{(i)} \in \mathbb{X}_s^{(i)}$ are the crossover, mutation, and selection operators, respectively. Besides that, r is a sequence in $r^{(i)}$ of the crossover operator, $r^{(i)} : \mathbb{X}_r^{(i)} \to \tau\left(\Omega_r^{(i)}, \tau(I^{\mu^{(i)}}, I^{\mu^{'(i)}})\right)$ (line 4), m is a sequence in $m^{(i)}$ of the mutation operator, $m^{(i)} : \mathbb{X}_m^{(i)} \to \tau\left(\Omega_m^{(i)}, \tau(I^{\mu^{'(i)}}, I^{\mu^{'(i)}})\right)$ (line 5), and s is a sequence in $s^{(i)}$ of the selection operator, $s^{(i)} : \mathbb{X}_s^{(i)} \times \tau(I, \mathbb{R}) \to \tau\left(\Omega_s^{(i)}, \tau(I^{\mu^{'(i)}+\chi\mu^{(i)}}, I^{\mu^{(i+1)}})\right)$ (lines 8 and 10). With the termination criterion, $\iota : \bigcup_{i=1}^{\infty}(I^{\mu})^{(i)} \to \{true, false\}$ (line 13), the fitness function of the EA can be denoted as Equation 2.1. Similarly, in MOEAs, the single-objective fitness function is substituted with a multi-objective fitness function, i.e., Equation 2.2 (Coello et al., 2007). Note that $n(n \geq 2)$ represents multiple objective functions, and I is the initial parent population of MOEAs.

$$\Phi : I \to \mathbb{R} \tag{2.1}$$

$$\Phi^{'} : I \to \mathbb{R}^{n} \tag{2.2}$$

The first implementation of MOEA was in 1985, i.e. the Vector Evaluated Genetic Algorithm (VEGA), to solve problems in machine learning (Schaffer, 1985). According to Giagkiozis et al. (2014), an EA comprises five elements in undertaking MOPs: (i) the main algorithm, (ii) an extension to deal with constrained optimization problems, (iii) an element to maintain promising solutions, (iv) an elitism determination criterion, and (v) a stopping criterion to stop the algorithm execution using one or more pre-defined conditions.

The main algorithm consists of three operators: (i) one that combines information within the population (crossover operator), (ii) one that perturbs some individuals to enhance search space exploration (mutation operator), and (iii) one that selects promising individuals to be part of the new generation (selection operator). A variety of EA-based models with different characteristics have been used to solve MOPs in various domains, as shown in Table 2.1.

Table 2.1: EA-based models in undertaking MOPs

Model	Characteristic	Usage
Artificial Immune Systems (Dasgupta, 2006)	It is a model that mimics the mechanisms of the biological immune system, and simulates dynamic behaviors in the presence of antigens and pathogens.	Novel MOEA design and formation (Shang et al., 2012), and distributed system reconfiguration (Ahuja et al., 2007).
Ant Colony Optimization (Dorigo and Di Caro, 1999)	It is a swarm intelligence-based model inspired from the foraging behavior of ants and the pheromone on the ground followed by other members of the colony.	Novel MOEA design and formation (Lopez-Ibanez and Stutzle, 2012; Ke et al., 2013), electromagnetic design (Ho and Yang, 2008), robot wall-following control (Hsu and Juang, 2013), and distributed system reconfiguration (Ahuja et al., 2007).
Bees Algorithm (Pham et al., 2006)	It is based on the behavior of honey bees in a colony. Artificial bees are grouped into three types: employed bees, onlookers, and scouts. An onlooker bee waits in the dance area to make a decision to choose a food source. An employed bee goes to the food source it visited previously. The scout bees carry out random searches.	Novel MOEA design and formation (Zou et al., 2011), sizing and distribution system reconfiguration (Nasiraghdam and Jadid, 2012), DNA sequence design (Chaves-González et al., 2013), copper strip production (Zhang et al., 2012), and power flow optimization (Khorsandi et al., 2013).
Cuckoo Search Algorithm (Yang and Deb, 2009)	It is based on the breeding strategy of cuckoos in combination with the Lévy flight behaviours of birds and fruit flies.	Hysteresis parameter estimation in Jiles-Atherton vector (Coelho, Guerra, Batistela and Leite, 2013), symmetric linear array element optimization (Rani et al., 2012), and job shop scheduling (Hanoun et al., 2012).
Differential Evolution (Storn, 1996; Pampara et al., 2006)	It is simple in concept, and has a small number of tuning parameters. It is used to handle real and binary representations in objective functions for global search.	Novel MOEA design and formation (Ali et al., 2012; Wang and Cai, 2012), fuel economy and emissions optimization for hybrid electric vehicles (Wu et al., 2011), and transformer design (Coelho, Mariani, Ferreira da Luz and Leite, 2013).
Particle Swarm Optimization (Eberhart and Kennedy, 1995; Kennedy and Eberhart, 1997; Kennedy, 2010)	It is inspired by the flocking behavior of birds as well as swarm theory. The decision vectors, i.e. particles, are updated based on the velocity through a set of rules using a pre-defined fitness. An archive, which is similar to the selector operator in the GA, is used to keep the best achieved objective function values for each particle.	Novel MOEA design and formation (Wang and Yang, 2010; Daneshyari and Yen, 2011), hysteresis parameter estimation in Jiles-Atherton vector (Coelho et al., 2012), and path following footstep optimization for humanoid robots (Lee and Kim, 2013).

2.1.1 Generation of MOEAs

Research in MOEAs can be broadly divided into two generations, as shown in Figure 2.2. The first generation of MOEAs focuses on simplistic methodologies in validation, while the second generation concentrates on efficiency at the algorithmic and data structure levels (Coello, 2006). Further explanations are given in the subsequent sub-sections.

First Generation
- Vector Evaluated Genetic Algorithm (VEGA)
- Multi-Objective Genetic Algorithm (MOGA)
- Niched-Pareto Genetic Algorithm (NPGA)
- Non-dominated Sorting Genetic Algorithm (NSGA)

Second Generation
- Strength Pareto Evolutionary Algorithm (SPEA)
- Strength Pareto Evolutionary Algorithm 2 (SPEA2)
- Pareto Archived Evolution Strategy (PAES)
- Non-dominated Sorting Genetic Algorithm II (NSGA-II)

Figure 2.2: Generations of Multi-Objective Evolutionary Algorithms.

2.1.1.1 The First Generation

VEGA (Schaffer, 1985) extends the GA principles for tackling MOPs by considering all objectives simultaneously. The limitation of VEGA is its inability to retain solutions with good candidates under the modified selection scheme. MOGA (Fonseca and Fleming, 1993) utilizes the Pareto dominance ($\preceq$) concept to combine multiple objectives, and incorporates the "decide $\leftrightarrow$ search" method (Van Veldhuizen and Lamont, 2000) to involve decision makers interactively. Individuals in the population are ranked based on their dominance relationship, and the selection procedure is guided by the ranked scores (Fourman, 1985).

Niched-Pareto Genetic Algorithm (NPGA) (Horn et al., 1994) utilizes a tournament selection scheme based on the $\preceq$ concept. The $\preceq$ concept is used to determine the winner of non-dominated individuals based on two randomly chosen individuals from the entire population. This method leads to the success of other MOEAs, such as mGA (Coello and Pulido, 2005), Strength Pareto Evolutionary Algorithm (SPEA) (Zitzler and Thiele, 1999), and Strength Pareto Evolutionary Algorithm 2 (SPEA2) (Zitzler et al., 2001). Based on both competitors in NPGA, the winning decision is determined using fitness sharing. This method brings a uniform distribution of individuals in the objective function space. NPGA extends research in the rank-niche evolution strategy (Chen and Hsu, 2006) and utilizes the sharing concept to maintain a uniformly distributed solution. On the other hand, Non-dominated Sorting Genetic Algorithm (NSGA) (Srinivas and Deb, 1994) is based on several layers of individual classifications. The population is ranked on the basis of non-domination before the selection activity. Diversity of solutions is maintained by sharing the dummy fitness scores among different layers of non-dominated individuals. NSGA is not computational efficient, because repeating the Pareto ranking process causes the growth of computational time complexity to $O(MN^3)$, where M is the number of objectives and N is the population size (Deb et al., 2002).

2.1.1.2 The Second Generation

In Pareto Archived Evolution Strategy (PAES) (Knowles and Corne, 2000), the crowding procedure is used in the objective space to maintain diversity of the Pareto optimal set. An adaptive grid is used to locate the solution based on its objectives. The concept of an adaptive grid archive is also used in other MOEAs, such as mGA (Coello and Pulido, 2005) and mGA2 (Toscano Pulido and Coello, 2003). SPEA (Zitzler and Thiele, 1999) uses the archive approach to keep the non-dominated solutions with the strength value labeling during each generation. The fitness of each individual, determined by the closeness to the true Pareto front and an even distribution of solutions, is computed according to the strength scores in the archive. The $\preceq$ concept is used to ensure the quality of the Pareto optimal set distribution. Efficiency of SPEA is jeopardized when the archive size grows too large, which slows down the search process. A pruning technique can be applied to ensure that the archive size remains below a certain threshold without affecting the selection pressure (Coello, 2006). SPEA2 (Zitzler et al., 2001) is an extension of SPEA (Zitzler and Thiele, 1999) in three aspects: (i) a more fine-grained fitness assignment for individual domination, (ii) a guided search of the nearest neighbor density estimation, and (iii) an enhanced archive pruning procedure to preserve the boundary of the Pareto optimal set.

NSGA-II (Deb et al., 2002) uses the Crowding Distance (*CD*) to estimate the density of solutions surrounding a particular solution in the population by computing the average distance of two points. It uses the crowded comparison operator ($\preceq_{CD}$) to determine the chosen solution. A non-dominated solution is preferred over a dominated solution, as well as the one that resides in the less crowded region is preferred among non-dominated solutions. Note that $i \preceq_{CD} j$ when $i_{rank} < j_{rank}$ or $(i_{rank} = j_{rank}$ and $i_{CD} < i_{CD})$, both i and j are the individuals in the population, whereby each of them has the attributes of a non-dominated rank (i_{rank}) and crowding distance (i_{CD}). NSGA-II also uses the elitism strategy to combine the best parents with the best offspring. The *CD* measure is used in other MOEAs, such as mGA, mGA2 (Coello and Pulido, 2001) , the niching-based differential-evolution algorithm (Qu et al., 2012), and the preference-based solution selection algorithm (Kim et al., 2012). A summary of the reviewed MOEAs is shown in Table 2.2.

Table 2.2: A Summary of MOEAs spanning two generations

Year	Model	Characteristic	Usage
1985	VEGA	It is a simple GA with a modified selection mechanism.	Personnel assignment problem (Toroslu and Arslanoglu, 2007), process planning and scheduling (Zhang and Fujimura, 2012), and satellite scheduling optimization (Mao et al., 2012).
1993	MOGA	It utilizes the $\preceq$ concept to combine multi-objectives, and incorporates the "decide $\leftrightarrow$ search" method to involve decision makers interactively.	Histone deacetylase inhibitor design (Fang et al., 2012), springback control of sheet metal forming (Wei, Yuying, Zhongwen and Lihong, 2009), sensor array design (Xu and Lu, 2011), and spectrum sensing design for cognitive radio (Balieiro et al., 2013).
1994	NPGA	It is a tournament-based niched Pareto GA.	Environmental-economic dispatch problem (Abido, 2003), heat pipe design (Zhang et al., 2009), and rule discovery problem (Lu et al., 2011).
2000	PAES	It is an evolution strategy with a single parent generating a single offspring using an elitism-based archive approach.	Wireless sensor networks (Manjarres et al., 2013), semantic features extraction in image retrieval (Zhang and Izquierdo, 2007), and voltage regulation in power distribution network (Montoya et al., 2010).
1999, 2001	SPEA, SPEA2	SPEA uses fitness assignment based on the principles of co-evolution and the niching technique founded on the $\preceq$ concept; SPEA2 is an improved version of the SPEA.	Heavy equipment design (Wei, Yang, Wang and Wang, 2009), water distribution systems (Kurek and Ostfeld, 2013), distributed power generation (Sheng et al., 2012), and dispatching of electrical network (Aribia et al., 2013).
1994, 2002	NSGA, NSGA-II	NSGA implements the non-dominated sorting and maintenance procedure to overcome the bias distribution of population as in VEGA; NSGA-II is an improved version of NSGA.	Automobile conformal antenna design (Kim and Walton, 2006), project scheduling (Ghoddousi et al., 2013), automatic test task scheduling (Lu et al., 2013), computer-communication networks (Lin and Yeh, 2012), and broadband reflector antenna satellite design (Bora et al., 2012).

2.1.2 Categories of MOEAs

According to Zhou et al. (2011), MOEAs can be grouped into six categories, i.e. decomposition-based, preference-based, indicator-based, hybrid-based, memetic-based, and co-evolution-based. The details are as follows:

In the first category, MOPs are decomposed into a set of single-objective sub-problems using a scalar function, and each sub-problem is weighted in an aggregated manner (Zhang and Li, 2007). Neighborhood relations among these sub-problems are defined based on the distances among the aggregated weight vectors. Each sub-problem is optimized using information from its neighboring sub-problems.

The second MOEAs category is designed based on preference (Fonseca and Fleming, 1993). The rank of a population member is determined by both the $\preceq$ concept and preference information, which are conducted using the "decide $\rightarrow$ search", "search $\rightarrow$ decide", or "decide $\leftrightarrow$ search" methods, from the decision makers.

The third category is indicator-based MOEAs (Zitzler et al., 2003; Zitzler and Künzli, 2004). The MOEA performance indicators are used within the evolution process to measure the quality of the resulting Pareto optimal set approximation. The theoretical foundations and practical implications of indicator-based MOEAs are reviewed in Auger et al. (2012). This category of MOEAs implicitly allows incorporation of preferences into the search process. The goal changes from optimizing a set of objective functions simultaneously to finding a set of solutions that maximizes the underlying performance indicators. This implies that the performance indicators co-exists with the $\preceq$ concept in the search process.

The fourth category is hybrid-based MOEAs (Zhou et al., 2011). Hybridization exploits the advantages of different MOEAs to deal with complicated MOPs. There are three types of hybridization strategies: (i) hybridizing different search method, (ii) hybridizing search and updating methods, and (iii) hybridizing different methods in different search phases. The first type combines global and local search methods, known as the memetic approach (Castro et al., 2013). It also uses the idea of combining the search operators of different algorithms (Liu, Jiang and Geng, 2013). The second type integrates different components from different algorithms, i.e. the PSO operator is inserted into the main loop of an EA (Elhossini et al., 2010). The third type is partitioned into three phases (Yang et al., 2009), i.e. to emphasize the dominated solutions, to balance dominated and non-dominated solutions, and to focus on non-dominated solutions, respectively. As an example, NSGA-II and a local incremental search algorithm are used to achieve these goals in Yang et al. (2009). It includes a search process into different evolutionary phases, and uses different search strategies in different phases.

The fifth category is memetic-based MOEAs (Zhou et al., 2011). It is a special case of hybrid MOEAs that incorporates local search methods. As an example, the resilient back-propagation algorithm with a local search technique is combined with NSGA-II for designing a neural network model (Fernandez Caballero et al., 2010).

The sixth category is MOEAs based on co-evolution (Zhou et al., 2011). Co-evolution can be regarded as evolving multiple sub-populations simultaneously to tackle a complicated MOP. Two co-evolving populations, i.e. EA population and archive population, are used in the MOEA evolution process (Jiao et al., 2013).

These six categories of MOEAs are used to undertake different MOPs, as shown in Table 2.3.

Table 2.3: Categories of MOEAs

MOEA Category	Usage
Decomposition-based	Novel MOEA design and formation (Li and Zhang, 2009; Ke et al., 2013), and arc routing problem (Mei et al., 2011a).
Preference-based	Novel MOEA design and formation (Thiele et al., 2009; Liu, Wang, Liu, Fang and Jiao, 2013; Wagner and Trautmann, 2010).
Indicator-based	Novel MOEA design and formation (Wagner and Trautmann, 2010; Bader and Zitzler, 2011), and nurse scheduling (Basseur et al., 2012).
Hybrid-based	Novel MOEA design and formation (Elhossini et al., 2010; Yang et al., 2009), vehicle routing problem (Cattaruzza et al., 2014), arc routing problem (Liu, Jiang and Geng, 2013), and traveling salesperson problem (Castro et al., 2013).
Memetic-based	Novel MOEA design and formation (Soliman et al., 2009; Fernandez Caballero et al., 2010), arc routing problem (Mei et al., 2011b), environmental power unit commitment design (Li, Pedroni and Zio, 2013), permutation flow shop scheduling (Chiang et al., 2011), job shop scheduling (Frutos and Tohmé, 2013).
Co-evolution-based	Novel MOEA design and formation (Soliman et al., 2009; Wang et al., 2013), ship design (Cui and Turan, 2010), knapsack problem (Jiao et al., 2013).

2.2 The micro Genetic Algorithm (mGA)

The mGA model, i.e. a GA with a small population size, was derived from the theoretical investigation in Goldberg (1989b); Goldberg et al. (1989). In essence, a small population size (i.e. three to six chromosomes) can be used to achieve convergence in solving non-linear optimization problems, regardless of the chromosome length (Goldberg, 1989b; Coello and Pulido, 2005; Mendoza et al., 2009; Chen, 2011). In addition to a small population size, the mechanism to maintain diversity in mGA and traditional GA is different (Abu-Lebdeh and Benekohal, 1999). Unlike the conventional mutation operation, mGA uses a restart strategy to maintain genetic diversity in the population. As the current population converges, a new population is generated. The new population has the same population size. It consists of the best individual from the previous population and other new randomly generated individuals. The evolutionary procedure continues until the global optimum is found, or the number of maximum evolution is reached. As reported in Abu-Lebdeh and Benekohal (1999), mGA operates on a small population size and achieves fast convergence in a few generations as compared with traditional GA. A review on mGA and its variants in tackling SOPs and MOPs is conducted, as presented in the next sub-section.

2.2.1 mGA for SOPs

The original mGA model (Krishnakumar, 1990) used a population size of five, a crossover rate of one, and a mutation rate of zero. The mGA model is capable of avoiding premature convergence and is better at reaching the optimal region than the traditional GA model. It adopts an elitist strategy that transfers the best string found in the current population to the next. The selection operator is used to allow competition among adjacent chromosomes in the population. The individual with the highest fitness score is declared the winner. It was reported that mGA performed better than GA on two stationary functions and a real-world wind-shear controller study (Krishnakumar, 1990).

Inspired by mGA, a number of investigations on tackling SOPs were conducted. In Johnson and Abushagur (1995), mGA was used to optimize diffractive-optic components of the dielectric grating structure design. This was accomplished using an initial set of five randomly initialized chromosomes. The Mean-Squared Error (MSE) was used to evaluate the fitness of chromosomes, and rank them from the strongest to the weakest. The best chromosome was saved, and the (four) remaining ones competed among themselves in the evolution cycle. The worst chromosome was removed from the population. It was reported that mGA was able to reduce the number of variables and simplify the complex grating structures of the design process (Johnson and Abushagur, 1995).

In Ali and Ramaswamy (2009), two EAs, i.e. mGA and PSO, were used to optimize the parameters of a Fuzzy Logic Controller (FLC) applied to a multi-degree of freedom (MDOF) system. The optimized FLC was used to monitor voltage input to a non-linear damper system attached to a three-storey building. The weighted sum method was adopted to combine three objective functions. The result showed that PSO required a longer processing time and a higher number of function evaluations as compared with mGA. The weighted sum method was also used in Smajic et al. (2009) to combine two objective functions, i.e. power loss and shunt volume, into a single objective function. Similar to mGA used in Smajic et al. (2008); Hafner et al. (2007), a hybrid binary-real coded model was applied in the design of the magnetic shunt topology.

In Itoh et al. (2012), mGA was bundled with the real-coded genes and the finite-difference time-domain (FDTD) method. It was used to evaluate the objective function, i.e. the slot length, in designing the waveguide slot antenna with dielectric lenses. The objective function was computed using FDTD, and antenna parameters were coded in real numbers for chromosome formation. On the other hand, a similar mGA but with bit-coded genes was used in Watanabe et al. (2010). The model was coupled with an immune algorithm to optimize the parameters and topology of an inductor shape design. The aim of the model was to reduce the inductor size and, at the same time, to satisfy the specifications of inductance under weak and strong biases.

In Chu et al. (2013), mGA was used to optimize the heat transfer coefficients and fraction of heat flux of disk specimen in a washer-on-disk wear test. A temperature-based error metric to rank the population, and to eliminate the worst chromosome (with the highest error) in mGA evolution was computed. This error metric represented the average difference in the measured and computed values as well as the stopping condition of the mGA operation. The proposed model avoided previous assumptions (Lin et al., 1996) in insulating unworn surface as well as in handling the temperature distribution in the disk specimen. A summary of mGA-based models is shown in Table 2.4.

Table 2.4: A Summary of mGA-based models for undertaking SOPs

Year	Model	Characteristics and Usage
1990	mGA (Krishnakumar, 1990)	An original mGA implementation for undertaking benchmark SOPs.
1995	mGA (Johnson and Abushagur, 1995)	A model used to optimize the diffractive-optic components of dielectric grating structure design.
2009	mGA with FLC (Ali and Ramaswamy, 2009)	A model used to optimize the parameters of an FLC of a MDOF system.
2009	mGA with a hybrid binary-real coded scheme (Smajic et al., 2009)	A model used to optimize power loss and shunt volume in the design of magnetic shunt topology.
2010	mGA with IA (Watanabe et al., 2010)	A model used to optimize antenna parameters of an inductor shape design.
2012	mGA with FDTD (Itoh et al., 2012)	A model used to optimize the slot length of a waveguide antenna design.
2013	mGA (Chu et al., 2013)	A model used to optimize the heat transfer coefficients to the ambient atmosphere and fraction of heat flux of the washer-on-disk test.