ATTITUDE DETERMINATION SYSTEM FOR NANO SATELLITE USING EXTENDED KALMAN FILTER ON ARM7TDMI PLATFORM

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UNIVERSITI SAINS MALAYSIA

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ATTITUDE DETERMINATION SYSTEM FOR NANO SATELLITE USING
EXTENDED KALMAN FILTER ON ARM7TDMI PLATFORM

by

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<tr>
<td>CF</td>
<td>Compact Flash</td>
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<td>ACS</td>
<td>Attitude Control System</td>
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<td>ADCS</td>
<td>Attitude Determination and Control System</td>
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<tr>
<td>ADC</td>
<td>Analog-to-Digital Converter</td>
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</tr>
<tr>
<td>ADS</td>
<td>Attitude Determination System</td>
<td></td>
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<tr>
<td>ANGKASA</td>
<td>Malaysian National Space Agency</td>
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<tr>
<td>ATSB</td>
<td>Astronautics Technology Sdn. Bhd.</td>
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<tr>
<td>CHARM</td>
<td>CubeSat Hydrometric Atmospheric Radiometer Mission</td>
<td></td>
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<tr>
<td>CHIME</td>
<td>CubeSat Heliospheric Imaging Experiment</td>
<td></td>
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<tr>
<td>COTS</td>
<td>Commercial of The Shelf</td>
<td></td>
</tr>
<tr>
<td>DSP</td>
<td>Digital Signal Processor</td>
<td></td>
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<tr>
<td>DCM</td>
<td>Direction Cosine Matrix</td>
<td></td>
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<tr>
<td>DMA</td>
<td>Direct Memory Access</td>
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<tr>
<td>ECI</td>
<td>Earth Centered Inertial</td>
<td></td>
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<tr>
<td>ECEF</td>
<td>Earth Centered Earth Fixed</td>
<td></td>
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<tr>
<td>EKFS</td>
<td>Extended Kalman filter</td>
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<td>EPS</td>
<td>Electric Power Supply</td>
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**ExEMFP**  Experimental Earth Magnetic Field Probe

**ExPSS**  Experimental Pyramidal Sun Sensor

**FIFO**  First In First Out

**FPGA**  Field-Programmable Gate Array

**GUI**  Graphical User Interface

**I²C**  Inter-Integrated Circuit

**I\O**  Input\Output

**IP**  Intellectual Property

**I²S**  Inter-Integrated Circuit Sound

**IDE**  Integrated Drive Electronics

**IGRF**  International Geomagnetic Reference Field

**JTAG**  Joint Test Action Group

**LCD**  Liquid Crystal Display

**LED**  light Emitting Diode

**MCU**  Microprocessor

**MSE**  Mean Squared Error

**NASA**  National Aeronautics and Space Administration

**NORAD**  North American Aerospace Defense Command

**OBC**  On-Board Computer
<table>
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<tr>
<td>PC</td>
<td>Personal Computer</td>
</tr>
<tr>
<td>PLL</td>
<td>Phase-locked loop</td>
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<tr>
<td>PWM</td>
<td>Pulse-width modulation</td>
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<tr>
<td>RISC</td>
<td>Reduced Instruction Set Computing</td>
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<tr>
<td>RTC</td>
<td>Real Time Clock</td>
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<tr>
<td>SBC</td>
<td>Spacecraft Body Coordinate</td>
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<tr>
<td>SEADS</td>
<td>Space Experimental Attitude Determination System</td>
</tr>
<tr>
<td>SEE</td>
<td>Single Event Effects</td>
</tr>
<tr>
<td>SEB</td>
<td>Single Event Burnout</td>
</tr>
<tr>
<td>SEGR</td>
<td>Single Event Gate Rupture</td>
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<tr>
<td>SEL</td>
<td>Single Event Latchup</td>
</tr>
<tr>
<td>SEU</td>
<td>Single Event Upset</td>
</tr>
<tr>
<td>SET</td>
<td>Single Event Transient</td>
</tr>
<tr>
<td>SIO</td>
<td>Serial Input\Output</td>
</tr>
<tr>
<td>SPSS</td>
<td>Solar Panel Sun Sensor</td>
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<tr>
<td>SRAM</td>
<td>static random-access memory</td>
</tr>
<tr>
<td>STK</td>
<td>Satellite Toolkit</td>
</tr>
<tr>
<td>TID</td>
<td>Total Ionization Dose</td>
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<td>TLE</td>
<td>Two Line Element</td>
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**UART**  Universal Asynchronous Receiver/Transmitter

**USB**  Universal Serial Bus

**USM**  Universiti Sains Malaysia

**UTC**  Coordinated Universal Time
# LIST OF SYMBOLS

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<td>$A$</td>
<td>attitude matrix</td>
</tr>
<tr>
<td>$a_s$</td>
<td>semimajor axis</td>
</tr>
<tr>
<td>$B$</td>
<td>magnetic field vector</td>
</tr>
<tr>
<td>$B_{n,m}$</td>
<td>contribution of spherical harmonics of $n$ and $m$</td>
</tr>
<tr>
<td>$c_f$</td>
<td>distance from the ellipse center to the ellipse focal point (Earth)</td>
</tr>
<tr>
<td>$\Delta(t)$</td>
<td>time difference between the epoch and the last perigee passage before epoch</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>obliquity of the ecliptic plane</td>
</tr>
<tr>
<td>$E_a$</td>
<td>Eccentric Anomaly</td>
</tr>
<tr>
<td>$e_s$</td>
<td>eccentricity</td>
</tr>
<tr>
<td>$n_{rev}$</td>
<td>mean motion</td>
</tr>
<tr>
<td>$\hat{e}$</td>
<td>rotation axis</td>
</tr>
<tr>
<td>$i_s$</td>
<td>inclination</td>
</tr>
<tr>
<td>$JD_{now}$</td>
<td>current reference Julian date</td>
</tr>
<tr>
<td>$K_k$</td>
<td>Kalman gain</td>
</tr>
<tr>
<td>$lat$</td>
<td>Argument of latitude</td>
</tr>
<tr>
<td>$M_{Epoch}$</td>
<td>Mean anomaly since epoch</td>
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<tr>
<td>$M_{sun}$</td>
<td>Sun mean anomaly</td>
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Mean Anomaly

True Anomaly

Argument of Perigee

Earth angular velocity

orbital period

priori covariance

posteriori covariance

rotation angle

Euler pitch angle

Euler yaw angle

Right Ascension of Ascending Node

measurement covariance

distance of the satellite from Earth

position of the satellite in unit vector in the ECI frame

rotation from ECI to ECEF frame

sun position in ECI plane

satellite position with respect to Earth

sun position with respect to Earth

time since epoch
$\theta$ Euler roll angle

$Q$ process covariance

$q_1$ quaternion parameter 1

$q_2$ quaternion parameter 2

$q_3$ quaternion parameter 3

$q_4$ quaternion parameter 4

$w_k$ white process noise

$x(t)$ system state vector

$x_k$ linear system model

$\hat{x}_k$ posteriori state

$v_k$ white measurement noise

$x$ discrete linear stochastic equation

$\hat{x}_k^-$ priori state

$V_{sun}$ sun position in ECI frame

$z_k$ measurement vector

$z_k$ linear measurement model
SISTEM PENENTUAN KEDUDUKAN UNTUK SATELIT NANO
MENGUNAKAN PENAPISAN KALMAN LANJUTAN PADA PLATFORM ARM7TDMI

ABSTRAK

Penapisan Kalman Lanjutan (PKL) telah digunakan dengan jayanya di dalam misi satelit. Akan tetapi, jika dibandingkan dengan kaedah penentuan kedudukan lain yang telah dihasilkan, ia adalah salah satu daripada algoritma yang memiliki komputasi yang paling berat dan dengan perkembangan terkini yang menggunakan satelit nano yang mempunyai kuasa elektrik, berat, ruang dan biasanya peruntukan yang terhad, adalah menjadi satu keutamaan untuk merekabentuk sebuah Sistem Penentuan Kedudukan (SPK) yang dapat mematuhi kekangan ini. Maka, penambakan dari segi komputasi dibuat kepada projek InnoSAT dengan menggantikan mikropengawal RCM3400 dengan mikropemproses berasaskan ARM7TDMI untuk menguji kebolehan ARM7TDMI. ARM7TDMI telah dipilih kerana ia merupakan antara mikropemproses yang dikenali dan digunakan dalam kebanyakan peralatan elektronik kerana kerendahan kuasanya dan kerana ia tahan lasak. Papan pembangunan EMBEST S3CEV40 yang digunakan dilengkapi dengan S3C44B0X yang merupakan mikropemproses ARM7TDMI. Ujian dijalankan dengan menghubungkan papan pembangunan kepada komputer peribadi yang telah dipasang dengan perisian antara muka ARM yang di program berdasarkan Visual Basic 6.0. Papan pembangunan telah dibenamkan dengan perisian SPK berdasarkan PKL. Komunikasi antara kedua-dua sistem adalah
menggunakan komunikasi 3-wayar bersiri. Perisian antara muka ARM akan bertin-
dak sebagai penderia satelit dengan menghantar data penderia melalui komunikasi
bersiri. Data penderia yang diterima of papan pembangunan ARM7TDMI digunak-
an untuk menghitung kedudukan satelit. Setelah menghitung, papan pembangunan
ARM7TDMI akan menghantar kembali data kepada komputer peribadi untuk penyim-
panan dan analisis lanjutan. Penyimpanan data ini dijalankan oleh perisian antara muka
ARM. Ini adalah untuk mensimulasikan SPK untuk menerima data penderia, menghitung
dan menghantar kembali data kedudukan kepada sistem Pengendalian Data Atas Papan
(PDAP) yang dilakukan untuk 2 orbit satelit. Hasil pemodelan orbit satelit dan elemen
yang diukur iaitu arah matahari dan medan magnet adalah sesuai untuk dijadikan input
dan PKL. PKL menunjukkan hasil yang memberangsangkan kerana sisihan piawai
kedudukan satelit untuk setiap paksi adalah kurang dari 3° mengatasi toleransi 5° yang
diperlukan. Penambahbaikan yang diperlukan adalah penambahan kepada komputasi
yang memerlukan 3 saat untuk setiap komputasi SPK.
ATTITUDE DETERMINATION SYSTEM FOR NANO SATELLITE USING EXTENDED KALMAN FILTER ON ARM7TDI PLATFORM

ABSTRACT

Extended Kalman Filter (EKF) has been successfully utilized in satellites missions. However, in comparison to other attitude determination methods developed, it is one of the most computationally burdening algorithm and with the new development of using nano satellites which have limited electrical power, mass, space and usually budget, it is essential to design an Attitude Determination System (ADS) which conforms to this limitation. Therefore, in this thesis, as a computational improvement on the InnoSAT project, the RCM3400 microcontroller of the ADS is replaced with the ARM7TDI based microprocessor to test its capability. The ARM7TDI has been chosen since it is one of the well-known microprocessor used in various electronic equipment because it is low power and robust. The EMBEST S3CEV40 Development Board used, houses the S3C44B0X microprocessor which is an ARM7TDI microprocessor. The testing is done by connecting the development board to a personal computer which has been installed with the ARM Interface Software based on Visual Basic 6.0. The development board is embedded with the ADS software based on the EKF. The communication for the two system uses 3-wire serial communication. The ARM Interface software will act as the sensor of the satellite by sending sensor data via serial communication. Sensor data received by the ARM7TDI development board is used to calculate the attitude of the satellite. Once calculated, the ARM7TDI development
board will send attitude data back to the PC for storage and further analysis. Data storage is done by ARM Interface software. This is to simulate the ADS system to receive sensor data, calculate and send back attitude data to On Board Data Handling (OBDH) system which is done for 2 satellite orbits. Modelling results of satellite orbit and sensed elements consisting of sun position and magnetic field are suitable to be used for EKF input. The EKF shows promising results as the satellite attitude standard deviation in each axis is less than $3^\circ$ exceeding the $5^\circ$ required tolerance. The only improvement required is the computation which need 3 seconds for each computation on ADS.
CHAPTER 1
INTRODUCTION

1.1 Background

InnoSAT satellite program is a satellite program lead by Astronautics Technology Sdn. Bhd. (ATSB) and Malaysian National Space Agency (ANGKASA). InnoSAT is a 300 × 100 × 100 mm$^3$ or a 3 unit CubeSat nanosatellite design. CubeSat was collaboratively designed by California Polytechnic State University and Stanfords University’s Space Systems Development Laboratory (Lee et al., 2009) to provide a standard design for picosatellite to reduce cost and development time, increase accessibility to space, and sustain frequent launches. Many institutions use this platform as their platform for various mission. Some like the Danish AAUSAT-II uses the CubeSat satellite platform for educational purposes and at the same time for research in gamma radiation in space (Andresen et al., 2005). California Institute of Technology and National Aeronautics and Space Administration (NASA) teamed up to design the CubeSat Hydrometric Atmospheric Radiometer Mission (CHARM) satellite to be used for Earth radiometry study which replaces the use of large and costly satellites (Lim et al., 2012). Dickinson et al. (2011) uses the CubeSat platform for their satellite to predict and diagnose space weather events at Earth. Many researches have been done using the CubeSat platform which is why the CubeSat is a suitable start off for InnoSAT as a new satellite research in Malaysia. The program provides local universities with the opportunity to be involved with a real satellite project with ATSB as the project leader as each university takes part by developing a specific subsystem for InnoSAT.
1.2 Problem Statement

Universiti Sains Malaysia (USM) was given the opportunity to develop the Attitude Determination System (ADS) of InnoSAT. ADS is one of the crucial subsystems of a satellite. The satellite would be flying blindly and eventually out of control without this subsystem. Thus USM took the opportunity to develop an ADS which has been designated Space Experimental Attitude Determination System (SEADS) and a Rabbit RCM3400 based microcontroller board which process sensor measurement to generate InnoSAT attitude. SEADS consists of three sensors which has been installed on InnoSAT. First sensor consists of 4 sets of three sided pyramidal sun sensors designated Experimental Pyramidal Sun Sensor (ExPSS), second is a magnetometer designated Experimental Earth Magnetic Field Probe (ExEMFP) and finally a redundant sun sensor designated Solar Panel Sun Sensor (SPSS) which utilizes InnoSAT solar panels to generate sun vector. All sensors provide sun position and magnetic field vector of InnoSAT in Spacecraft Body Coordinate (SBC) frame.

Readings from the sensors are to be processed by the Rabbit RCM3400 microcontroller board to be stored in an on board flash memory and to generate attitude of InnoSAT. The Rabbit RCM3400 controller board has been embedded with an attitude determination algorithm consisting of Kepler orbit model which generates InnoSAT position vector in Earth Centered Inertial (ECI) frame, Keplerian sun model and International Geomagnetic Reference Field (IGRF) model which generates the sun and magnetic field vectors respectively in the ECI frame and a combination of Extended Kalman filter (EKF) and Q-method which calculate InnoSAT attitude using both sun and magnetic field vector in ECI and SBC frames.
Though, this was the plan for RCM3400 controller board, a combination of issues surfaced during the implementation of the algorithm which are improper Julian date representation, RCM3400 computation speed limit and memory burden of ADS. The first issue is the Julian date. Julian date is a crucial initial parameter to the orbit model where the orbit model is a crucial starting point for further attitude determination processing. However, during implementation of attitude determination methods in Attitude Determination and Control System (ADCS) of InnoSAT, it is found that the RCM3400 cannot represent Julian date variable accurately in 32 bits (floating point) but should be represented in double 64 bit numbers, because the date precision exceed the 32 bit decimal limit causing major calculation errors. Another issue is the computation speed limitation and memory burden of ADS on RCM3400 micro-controller. The EKF with the inverse of element matrix larger than 3x3 and the IGRF calculation with an order of 10 caused a significant overloading of RCM3400 memory and reduction in processing time.

Therefore, in order to improve on the capabilities of InnoSAT ADS, an ARM7TDMI microprocessor will be tested to see whether the capabilities of the ARM7TDMI would be suitable for InnoSAT ADS.

1.3 Objective of study

From above, the objectives of this research are

(i) To estimate the attitude of InnoSAT from combination of two position sensors using EKF
(ii) To embed the EKF attitude estimation algorithm in the ARM7TDMI based microprocessor

(iii) To compare the performance and result of the experimentation of the real time embedded EKF with Satellite Toolkit (STK) software.

1.4 Scope and Limitations

The scope of this research is limited to the ADS of InnoSAT only. Though a real satellite would include a control system, which completes the ADCS of a full satellite, adding the control system to the research would be too big for this research. Testing would only be done to the microprocessor by using a development board on ground. Sensor readings are simulated readings from STK by using proposed InnoSAT Two Line Element (TLE) provided by ATSB.

1.5 Thesis Organization

This thesis is divided into 5 chapters which entails the implementation EKF based ADS on ARM7TDMI. Chapter 1 introduces InnoSAT which is the satellite platform and base for this research. Research goals and overview is also included in this chapter. Chapter 2 presents a literature review of ADS used by various groups around the world. The fundamentals of ADS and EKF are also explained in this section as well space requirement of a satellite system. Chapter 3 presents the method of experimentation for the implementation of the EKF on the ARM7TDMI microprocessor. Chapter 4 discusses the results of the implementation of the EKF based ADS on the ARM7TDMI microprocessor. Finally, Chapter 5 concludes the finding of this thesis and the recommendations for future work.
CHAPTER 2
OVERVIEW OF ATTITUDE DETERMINATION SYSTEM AND ARCHITECTURE

2.1 Introduction

Attitude of a spacecraft is its orientation in space. Attitude determination is the process of computing the orientation of the spacecraft relative to either an inertial reference or some object of interest, such as Earth. This typically involves several types of sensors on each spacecraft and sophisticated data processing procedures (Wertz, 1978).

2.2 Attitude Determination System Architectures

Satellite subsystems are unique to one another. This is also true for the ADS. The configuration and combination of sensors and the built in electronic system depends on the mission, satellite size and shape.

One of the example is the Danish AAUSAT-II. It is a $10 \times 10 \times 10\text{cm}^3$ CubeSat weighting no more than $1kg$. The satellite ADS hardware consists of three single axis rate gyro and a 3-axis magnetometer. The sampling of sensor measurement is done by an Atmel AT89C51 Microprocessor (MCU). The ADS which is the EKF calculations of AAUSAT-II is placed in the On-Board Computer (OBC) since the OBC is relatively powerful. The overall AAUSAT-II structure can be seen in Figure 2.1 (Andresen et al., 2005).
1.3. AAUSAT-II SUBSYSTEMS

Figure 1.1: Block diagram depicting the subsystems in the AAUSAT-II and a conceptual visualization of their respective interfaces to other subsystems.

1.3.2 Subsystem Descriptions

MCC (Mission Control Center) is responsible for handling and storing all transmission data from the satellite and sending flight plans etc. to the satellite. The MCC provides a user interface and a database to store housekeeping data from the satellite. The mission control center is furthermore able to control multiple ground stations, both the ground station located in Aalborg and a ground station placed at Svalbard in Norway which is currently in the final stages of development.

GND (Ground Station) is responsible for the communication between the MCC and the satellite. The task of the ground station is to track the satellite throughout each pass and adjust the radio frequency, so data between the satellite and the MCC can be sent and received correctly. Furthermore, the ground station is designed to be autonomously controlled by the MCC, both for communicating with the AAUSAT-II satellite and the ESA SSETI Express satellite 2.

COM (Communication System) is designed to function as a pipeline for the communication between the ground station and the CDH. COM modulates and sends data from CDH to the ground station. Data received from the ground station is demodulated and sent via the CAN bus to the CDH subsystem.

EPS (Electrical Power Supply) is responsible for generating power from the solar cells and storing it in the batteries in order to be able to deliver continuous power during eclipse and peak demands. The EPS subsystem also conditions and distributes the power to other satellite subsystems.

ADCS (Attitude Determination and Control System) is responsible for determining and controlling the attitude of the satellite. This primarily implies detumbling and stabilization of the satellite.

P/L (Payload) consists of a gamma ray burst detector. The gamma ray burst detector is a newly developed detector crystal supplied by Danish National Space Center.

OBC (On-Board Computer) is the main computer on the satellite. The CDH subsystem software is executed on the OBC. The OBC subsystem also provides processing facilities for other satellite subsystems.

2http://sseti.gte.tuwien.ac.at/WSW4/express1.htm

AAUSAT-II is the next generation of the Danish satellite AAUSAT. AAUSAT has the same dimension as AAUSAT-II and the ADCS architecture is as seen in Figure 2.2. A PIC16C774 microcontroller from Microchip was selected for the ADCS. The microcontroller had to sample data from three sample types, send sensor data to the OBC via Inter-Integrated Circuit (I²C) bus, interface actuators and execute control algorithms (Krogh et al., 2002). Software developed for the PIC16C774 microcontroller is mainly for data sampling from sensors and to control the satellite. The ADS algorithm is implemented on the OBC of the satellite.

Another work done by Brand and Bakes (2007) took a general look at nanosatellite OBC. The authors considered various processing cores for a 30cm × 30cm × 20cm nanosatellite and finally concluded that an AT91SAM7A2 ARM7 based processor from Atmel was the best processor for a nanosatellite OBC. The ARM-based processor was chosen since ARM processors are proven reliable because it has been used in a lot of handheld devices (Brand and Bakes, 2007). Eventhough the ARM processor is a reliable processor for various handheld devices, the space environment poses a totally
different threat to a satellite when compared with the Earth’s environment. Brand and Bakes’s main concern are errors caused by radiation exposure in the space since the chosen processor is not a space qualified component. However, in recent years, satellite developers has begun experimenting and using Commercial of The Shelf (COTS) components which include processors as well. Eventhough these COTS components are susceptible to radiation errors, they are much cheaper compared to space qualified components. Brand and Bakes (2007)’s also suggest that the vital part which is the onboard memory the satellite should consists only of static random-access memory (SRAM) and flash memory only with the inclusion of some form of detection and correction hardware. According to the authors as well, the most computationally complex operation a nanosatellite would have to deal with using its limited hardware is the IGRF modelling of ADCS. This confirms the high computation of the IGRF modelling.

![Figure 2.2: AAUSAT overall satellite structure (Brand and Bakes, 2007)](image)

nCUBE is a Norwegian student satellite (Ose, 2004). It is in the same league as both AAUSAT satellite in terms of size and weight. The ADS or the attitude estimator also uses an EKF which is written in C programming language. The nCUBE team
chose an ATmega128L since it is the low power version. Ose (2004) also realized that EKF requires highly accurate variables, therefore the variables of the EKF in AAUSAT program is implemented as floating variables which has a range of $\pm3.403 \times 10^{308}$ and an accuracy of $\pm1.175 \times 10^{-308}$ (IEEE Std 754-2008, 2008; 754-1985, 1985). This also indicates the ATmega128L microcontroller is able to use variables in double precision format.

The CubeSat Heliospheric Imaging Experiment (CHIME) satellite as mentioned by Dickinson et al. (2011) uses a Gumstix Verdex computer for its OBC which also govern the attitude determination and control of CHIME. The CHIME satellite uses two sun sensors to determine its attitude.

Another architecture done by Ali et al. (2014) into the AraMiS-C1 satellite, is the integration of the Electric Power Supply (EPS) and the ADCS onto a single Cubesat standard tile. The AraMiS-C1 satellite system uses COTS components. Sensors on board consists of sun detector and magnetometer. System management of the satellite is done by a commercial MSP430F5438A ultra power 16-bit Reduced Instruction Set Computing (RISC) architecture microcontroller which support up to 25Mhz system clock.

Yang et al. (2012) used a TMS320C54 series Digital Signal Processor (DSP) from Texas Instruments to perform the duty of controller by implementing sampling, computing and actuating algorithms. The system performs a dual-vector attitude determination based on solar and Earth-magnetic sensor reading. The ADCS manages to stabilize the satellite the attitude with Earth-pointing precision of $5^\circ / \text{sec}$ when the sun in
2.3 Satellite Orbit Description

A satellite orbiting the Earth is considered a two point mass which are influenced by their mutual gravitational attraction (Jensen et al., 2010). This system can be described by using Kepler’s three empirical laws of planetary motion (Wertz, 1978).

2.3.1 Keplerian Orbit

An orbit can be described using the method developed by Johannes Kepler which gives a description of the orbit size, shape and orientation, as well a spacecraft’s position (Sellers et al., 2003). The method requires only 6 orbital elements which are:

- Semimajor axis, \( a_s \)
- Eccentricity, \( e_s \)
- Inclination, \( i_s \)
- Right Ascension of Ascending Node (\( \Omega_s \))
- Argument of Perigee, \( \omega_s \)
- True Anomaly, \( \nu_s \)

A size and shape of an orbit is defined respectively by the semimajor axis, \( a_s \) and eccentricity, \( e_s \). From Sellers et al. (2003), the eccentricity is given by

\[
e_s = \frac{c_f}{a_s}
\]  \hspace{1cm} (2.1)
where $c_f$ is the distance from the ellipse center to the ellipse focal point (Earth) as in Figure 2.3.

![Figure 2.3: Geometric properties of an elliptical orbit with the Earth as focal point](image)

The shape of the orbit either a circle, ellipse, parabola or a hyperbola depends on the eccentricity, $e_s$ and the relationship can be seen in Table 2.1.

<table>
<thead>
<tr>
<th>Orbit Shape</th>
<th>Eccentricity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Circle</td>
<td>$e_s = 0$</td>
</tr>
<tr>
<td>Ellipse</td>
<td>$0 &lt; e_s &lt; 1$</td>
</tr>
<tr>
<td>Parabola</td>
<td>$e_s = 1$</td>
</tr>
<tr>
<td>Hyperbola</td>
<td>$e_s &gt; 0$</td>
</tr>
</tbody>
</table>

The orbit plane orientation is determined by two elements which are inclination and right ascension of ascending node as in Figure 2.4.

The inclination, $i_s$, is the angle between orbit plane and the Earth’s equatorial plane as seen in Figure 2.4. If $i_s = 0^\circ$, the orbit is Equatorial orbit and if $i_s = 90^\circ$ it becomes a Polar orbit. Orbits in between the two values are prograde orbits and when the $i_s > 90^\circ$
As seen from Figure 2.4, $\Omega_s$, is the angle measured from the Vernal Equinox to the ascending node which is the point at which the satellite crosses the equator plane going from south to north.

The argument of perigee, $\omega_s$ as in Figure 2.4, determines the orientation of orbit in the plane. It is the angle measured in the satellite’s direction of motion from the ascending node to the perigee. Perigee is the closest approach to the satellite to the Earth.

True anomaly, $\nu_s$ will be calculated from the mean anomaly, $M_a$. Mean anomaly, $M_a$ is an angle with no physical meaning but mathematically it represents the angle of satellites position in its orbital path given by,

$$M_a = 360 \cdot \left( \frac{\Delta(t_e)}{P_{orb}} \right)[deg] \quad (2.2)$$
where:

$\Delta(t_e)$ is the time difference between the epoch and the last perigee passage before epoch and $p_{orb}$ is the orbital period.

Figure 2.5: Relationship between True Anomaly and Eccentric Anomaly

Mean anomaly is linked to the true anomaly by an intermediate variable, the Eccentric anomaly, $E_a$ which can be seen in Figure 2.5. The mean and eccentric anomalies are related by Kepler’s equation as

$$M_a = E_a - e \sin E_a \quad (2.3)$$
Then Gauss’s equation relates the eccentric anomaly to the True Anomaly, \( v_s \) by

\[
\tan \left( \frac{v_s}{2} \right) = \left( \frac{1 + e_s}{1 - e_s} \right)^{1/2} \tan \left( \frac{E_s}{2} \right) \quad (2.4)
\]

From Equation 2.4, True Anomaly, \( v_s \) can be expressed directly as a function of Mean Anomaly, \( M_a \) by expanding in a power series in orbital Eccentricity, \( e_s \) to yield

\[
v_s = M_a + 2e_s \sin(M_a) + 5e_s^2 \frac{\sin(2M_a)}{4} \quad (2.5)
\]

2.3.2 Two-Line Element

TLE or short for two-line element is a simple data format of two line sets having 69 characters which describes the orbit of an Earth satellite. This general perturbation element sets is available for all space objects and is maintained by North American Aerospace Defense Command (NORAD). An example of a TLE format is as seen in Figure 2.6 (Krogh et al., 2002).

![TLE Format](Image)

**Figure 2.6**: TLE for ØRSTED (Krogh et al., 2002)

Other than the orbital parameters explained previously in Section 2.3.1, the TLE includes other following parameters:

**Time of Epoch**: This represents the time when the orbital parameters were ob-
tained. Epoch year in described in the first two numbers. The remaining numbers in
the integral part is the Julian day and the fraction represent the fractional portion of the
day.

**1st Derivative of Mean Motion:** This represents the change in the mean motion
of the satellite. It is the half value of the mean motion in revolutions per day squared
and is caused by atmospheric drag pulling a satellite into a lower orbit and accelerates
it downward towards the Earth.

**2nd Derivative of Mean Motion:** This term is the 2nd second derivative of the
mean motion divided by six, in units of revolutions per day cubed and is usually set
to zero since it is not used because the orbit model only considers the force of Earth’s
gravity acting on the satellite for estimation.

**Drag Term:** A drag term or radiation pressure coefficient consists of a coefficient
describing the effect of drag on a satellite. It is based on a satellite surface and mass.

**Mean Motion:** Mean motion describes the number of revolution a satellite com-
pletes in day.

**Revolution Number at Epoch:** This parameter gives the number of orbit at the
Epoch time when TLE was taken.

**2.3.3 Julian Date**

A very useful and common representation of time which simplifies astronomical
calculation and satellite orbit propagation is the Julian date. It is counted in day plus a
fraction of the day beginning at noon universal time. It has been counted in days since 1st of January 4713 BC at noon universal time (Wertz, 1978; Danby, 1988; Sinnott, 1991).

The Julian date used in this research will not use the entire Julian date from 1st January 4713BC noon but instead will use the Julian date starting from noon UTC on the 1st January 2000. This will offset the Julian date by 2451545 days subtracted from the ordinary Julian date.

2.3.4 Orbit Model

Having known the orbit of a satellite, at a certain point and time from a TLE, it is also required to determine the position and motion of a satellite in order to determine the attitude of a satellite. A simple two body Keplerian orbit propagator model developed by the AAUSAT team will be used (Krogh et al., 2002). This orbit model will use the orbital and time parameters from a given TLE.

The first step of the orbit model is to determine the current mean anomaly of the satellites orbital position. The time since epoch, $T_{se}$ is used here. It is the current time in Julian date minus the Julian date at Epoch given in the TLE. The current Mean Anomaly, $M_a$ is calculated in degrees using Equation 2.6, which includes mean anomaly at epoch, $M_{Epoch}$ and the mean motion, $n_{rev}$ from the TLE.

$$M_a = M_{Epoch} + 360n_{rev}T_{se}$$ (2.6)
The semi major axis, \( a_s \) which represent the largest radius of an eccentric orbit as shown in Figure 2.3 is given by

\[
a_s = \left( \frac{m_g}{\left( \frac{2n_0 r_{earth}}{86400} \right)^2} \right)^{1/3}
\]  

(2.7)

Next the daily changes of Argument of Perigee, \( \dot{\omega}_s \) and daily changes of the Right Ascension of Ascending Node, \( \dot{\Omega}_s \) is determined since the Argument of Perigee, \( \omega_s \) and \( \Omega_s \) changes with a constant speed relative to the ECI frame is determined. Parameters used in the determination are orbital Inclination, \( i_s \), the orbital Eccentricity, \( e_s \), Semi Major Axis of the orbit, \( a_s \) and the Earth’s Equatorial Radius, \( r_{earth} \) (Krogh et al., 2002; Wertz, 1978).

\[
\dot{\omega}_s = 4.98204 \left( \frac{r_{earth}}{a_s} \right)^{3.5} \left( 5 \cos(i_s)^2 - 1 \right) \left( (1 - e_s^2)^2 \right)^{-1}
\]

(2.8)

\[
\dot{\Omega}_s = 9.9641 \left( \frac{r_{earth}}{a_s} \right)^{3.5} \cos(i_s) \left( (1 - e_s^2)^2 \right)^{-1}
\]

(2.9)

Accordingly the current Argument of Perigee, \( \omega_s \) and the \( \Omega_s \) can now be determined by updating the same parameters given in the TLE parameters, \( \omega_{s,TLE} \) and \( \Omega_{s,TLE} \) with \( \dot{\omega}_s \) and \( \dot{\Omega}_s \) resulting in

\[
\omega_s = \omega_{s,TLE} + T_{sc} \dot{\omega}_s
\]

(2.10)
\[ \Omega_s = \Omega_s^{TLE} - T_{se}\Omega_s \]  

(2.11)

Figure 2.7: Argument of Latitude as the sum of Argument of Perigee and True Anomaly

After that, the Argument of Latitude, \( lat \) is determined. It represents the angle between the ascending node and the current satellite position with respect to the center of the Earth or in the ECI frame as seen in Figure 2.7. The angle is actually the sum of Argument of Perigee, \( \omega_s \) and the True Anomaly, \( \nu_s \) from Equation 2.5 thus producing

\[ lat = \omega_s + \nu_s \]  

(2.12)

Using the Argument of Latitude, \( lat \), Right Ascension of Ascending Node, \( \Omega_s \) and the satellite orbital inclination, \( i_s \), the position of the satellite in unit vector in the ECI frame is determined by
\[ x_{\text{sat}} = \cos(\text{lat}) \cos(\Omega_s) - \sin(\text{lat}) \sin(\Omega_s) \cos(i_s) \]
\[ y_{\text{sat}} = \cos(\text{lat}) \sin(\Omega_s) + \sin(\text{lat}) \cos(\Omega_s) \cos(i_s) \]
\[ z_{\text{sat}} = \sin(\text{lat}) \sin(i_s) \]

(2.13)

To give a measurable figure to the orbital position of the satellite position in kilometers, the position vector in Equation 2.13 is multiplied with the radius of the satellites orbital position, \( r_{\text{sat}} \) (in kilometers) as below

\[ r_{\text{sat}} = a_s - e_s^2 \frac{1}{1 + e_s \cos(\nu_s)} \]

(2.14)

### 2.4 Attitude Representation

Rotation of reference frames expresses the orientation of a rigid body satellite, relative to some reference coordinate frame for example reference frames from Figure 2.10. The fundamental quantity specifying the orientation of a satellite is the Direction Cosine Matrix (DCM). Referring to Figure 2.8, the orthogonal, satellite right handed triad "uvw" is in the vicinity of the reference frame "xyz". Thus the satellite triad \( \hat{u}, \hat{v} \) and \( \hat{w} \) can be specified and fixed to the reference coordinate frame which creates a 3x3 matrix having 9 parameters. This 3x3 matrix is known as the, attitude matrix, \( \textbf{A} \) which
is

\[
A \equiv \begin{bmatrix}
    u_x & u_y & u_z \\
    v_x & v_y & v_z \\
    w_x & w_y & w_z
\end{bmatrix}
\]

(2.15)

with \( \hat{u} = (u_x, u_y, u_z)^T \), \( \hat{v} = (v_x, v_y, v_z)^T \) and \( \hat{w} = (w_x, w_y, w_z)^T \). Each of the elements is a cosine of the angle between a body unit vector and a reference axis. For example, the cosine of the angle between \( \hat{v} \) and reference y-axis is represented by \( v_y \). Due to this, \( A \) is referred to as DCM.

A DCM is a coordinate transformation (Wertz, 1978) that maps vectors from a reference frame to the body frame where if \( \mathbf{a} \) is a vector with components \( a_1, a_2 \) and \( a_3 \) along the reference axes, then

\[
Aa = \begin{bmatrix}
    u_x & u_y & u_z \\
    v_x & v_y & v_z \\
    w_x & w_y & w_z
\end{bmatrix}
\begin{bmatrix}
    a_1 \\
    a_2 \\
    a_3
\end{bmatrix} = \begin{bmatrix}
    \hat{u} \cdot \mathbf{a} \\
    \hat{v} \cdot \mathbf{a} \\
    \hat{w} \cdot \mathbf{a}
\end{bmatrix} \equiv \begin{bmatrix}
    a_u \\
    a_v \\
    a_w
\end{bmatrix}
\]

(2.16)

The components of \( Aa \) are the components of the vector \( \mathbf{a} \) along the body triad \( \hat{u}, \hat{v}, \) and \( \hat{w} \). Other than the DCM, there are other parameterizations that specifies the orientation of a rigid. However, in this thesis, the Euler angles and the Euler Symmetric Parameters normally known as quaternion are used. The Euler axis/angle (Wertz, 1978) based on the right-hand rule is a simple anti-clockwise rotation in the positive sense of the third axis by the angle, \( \Phi_a \) as in Figure 2.9. The DCM for this rotation is given by
The DCM rotation in the first and second axis by the angle, $\Phi_a$ is represented by $A_1(\Phi_a)$ and $A_2(\Phi_a)$, respectively and they are

$$A_1(\Phi_a) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \Phi_a & \sin \Phi_a \\ 0 & -\sin \Phi_a & \cos \Phi_a \end{bmatrix}$$ (2.18)$$

$$A_2(\Phi_a) = \begin{bmatrix} \cos \Phi_a & 0 & -\sin \Phi_a \\ 0 & 1 & 0 \\ \sin \Phi_a & 0 & \cos \Phi_a \end{bmatrix}$$ (2.19)$$

All three matrices have the trace

$$\text{tr}(A(\Phi_a)) = 1 + 2 \cos \Phi_a$$ (2.20)
The trace of a DCM representing a rotation by an angle, $\Phi_a$ about an arbitrary axis takes the same value. Generally, the axis rotation from one reference frame to another reference frame might not be one of the ‘xyz’ orthogonal axes but might be a unit vector, $\hat{e}$ and an angle of rotation, $\Phi_a$ which results with a general DCM as

$$A = \begin{bmatrix}
\cos \Phi_a + e_1^2 (1 - \cos \Phi_a) & e_1 e_2 (1 - \cos \Phi_a) + e_3 \sin \Phi_a & e_1 e_3 (1 - \cos \Phi_a) - e_2 \sin \Phi_a \\
e_1 e_2 (1 - \cos \Phi_a) - e_3 \sin \Phi_a & \cos \Phi_a + e_2^2 (1 - \cos \Phi_a) & e_2 e_3 (1 - \cos \Phi_a) + e_1 \sin \Phi_a \\
e_1 e_3 (1 - \cos \Phi_a) + e_2 \sin \Phi_a & e_2 e_3 (1 - \cos \Phi_a) - e_1 \sin \Phi_a & \cos \Phi_a + e_3^2 (1 - \cos \Phi_a)
\end{bmatrix}$$

(Wertz, 1978)

Using the same principal with a unit vector along rotation axis, $\hat{e}$ and an angle of rotation, $\Phi_a$, a set of four parameters known as the Euler symmetric parameter or quaternion can be introduced to represent rotation of a rigid body and they are (Wertz, 1978)

$$q_1 \equiv e_1 \sin \frac{\Phi_a}{2}$$

$$q_2 \equiv e_2 \sin \frac{\Phi_a}{2}$$

$$q_3 \equiv e_3 \sin \frac{\Phi_a}{2}$$

$$q_4 \equiv \cos \frac{\Phi_a}{2}$$

These four parameters are not independent but satisfy the constraint equation (Wertz, 1978)

$$q_1^2 + q_2^2 + q_3^2 + q_4^2 = 1$$
The quaternion DCM is given by

\[
A(q) = \begin{bmatrix}
q_1^2 - q_2^2 - q_3^2 + q_4^2 & 2(q_1 q_3 + q_2 q_4) & 2(q_1 q_3 - q_2 q_4) \\
2(q_1 q_2 - q_3 q_4) & -q_1^2 + q_2^2 - q_3^2 + q_4^2 & 2(q_2 q_3 + q_1 q_4) \\
2(q_1 q_3 + q_2 q_4) & 2(q_2 q_3 - q_1 q_4) & -q_1^2 - q_2^2 + q_3^2 + q_4^2
\end{bmatrix}
\] (2.27)

Quaternion is widely used because there are only four simple parameters for consideration and it is less burdening on the processor because the expression for the quaternion DCM does not involve trigonometric functions which require extensive computing. Even though the quaternion performs better at rigid body rotations, another type of rotation has more apparent geometrical significance of a rotation which known as the Euler angles (Wertz, 1978). Euler angles are usually used for analysis to find closed form solutions to the equation of motion in special cases particularly for small angle of rotations. Contrary to quaternion, Euler angles use three sets of rotation angle commonly known roll, pitch and yaw. For a satellite attitude, the angles rotates to the SBC frame about a given axis (Ose, 2004). The roll angle, \(\theta\) rotates about the SBC frame x-axis, pitch angle, \(\phi\) rotates about the SBC frame y-axis and finally, the yaw angle, \(\psi\) rotates about the SBC frame z-axis.

Since Euler angles provide better physical representation of attitude and the quaternion is useful for calculation (Wertz, 1978), the quaternion can be rotated using (Wertz, 1978)

\[
\begin{bmatrix}
\phi \\
\theta \\
\psi
\end{bmatrix} = \begin{bmatrix}
\arctan(\frac{2(q_1 q_3 + q_2 q_4)}{1-2(q_1^2+q_2^2)}) \\
\arctan(2(q_4 q_2 - q_1 q_3)) \\
\arctan(\frac{2(q_3 q_4 + q_2 q_1)}{1-2(q_2^2+q_3^2)})
\end{bmatrix}
\] (2.28)
2.5 Coordinate Frames

This section describes the frames used for determining the attitude in three dimensional space. The frames are important as the points on a rigid body is different depending on different coordinate frames thus the correct reference frame has to be known in all conditions. The method to rotate from the ECI frame to the Earth Centered Earth Fixed (ECEF) frame is also introduced in this section.

2.5.1 Reference Coordinate Frame

Since InnoSAT orbits the Earth, two specific Earth related coordinate system will be used. They are the ECI and ECEF coordinate frame which have their origin in the geographical center of Earth as shown in Figure 2.10.

The first coordinate frame, Earth Centered Inertial represents a coordinate system with its origin in the center of Earth and is fixed relative to the Earth rotation. The X-axis is parallel to the direction of Vernal Equinox. The Vernal Equinox is the point where the plane of the Earth’s orbit about the Sun, crosses the equator going from

![Figure 2.10: The ECI and ECEF coordinate frame](image-url)
South to North. The Z-axis is parallel to the Earth’s rotational axis.

The second Earth coordinate frame, the Earth Centered Earth Fixed frame has a similar origin and Z-axis as the ECI frame but with a different X-axis. The X-axis, stays and intersect the zero longitude of Earth designated the Greenwich Meridian which fixes the ECEF frame to Earth thus, the system rotates with it.

2.5.2 Spacecraft Coordinate Frame

The satellite itself requires a set of fixed coordinate frames for attitude determination of the satellite. The attitude and position of the satellite is accordingly given as a rotation between the satellite fixed coordinate frames and reference frames.

First is the SBC which is placed in the center of mass of the satellite and fixed to the satellite geometric axes. The axis representation can be seen in Figure 2.11.