
UNIVERSITI SAINS MALAYSIA

Second Semester Examination
Academic Session 2005/2006

April/May 2006

MAA 102E – Calculus for Science Students II
[Kalkulus untuk Pelajar Sains II]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of FIVE pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi LIMA muka surat yang bercetak sebelum anda memulakan peperiksaan ini].

Instructions: Answer **all ten** [10] questions.

[Arahan: Jawab **semua sepuluh** [10] soalan].

1. Determine whether $\{a_n\}$ converges or diverges. If it converges, find its limit.

(a) $a_n = \frac{9^{n+1}}{10^n}$

(b) $a_n = \frac{(-1)^n n^2}{n^3 + 1}$

[8 marks]

1. Tentukan sama ada $\{a_n\}$ menumpu atau mencapah. Jika menumpu, cari hadnya.

(a) $a_n = \frac{9^{n+1}}{10^n}$

(b) $a_n = \frac{(-1)^n n^2}{n^3 + 1}$

[8 markah]

2. Test the convergence of the series:

(a) $\sum_{n=1}^{\infty} \frac{1}{2^n + n}$

(b) $\sum_{n=1}^{\infty} \frac{(n+1)(n^2-1)}{4n^3-2n+1}$

[10 marks]

2. Uji penumpuan siri:

(a) $\sum_{n=1}^{\infty} \frac{1}{2^n + n}$

(b) $\sum_{n=1}^{\infty} \frac{(n+1)(n^2-1)}{4n^3-2n+1}$

[10 markah]

3. Find the interval of convergence of the power series $\sum_{n=0}^{\infty} \frac{x^n}{n^5}$.

[13 marks]

3. Cari selang penumpuan siri kuasa $\sum_{n=0}^{\infty} \frac{x^n}{n^5}$.

[13 markah]

4. Find the power series representation of $\frac{1}{1+x}$. Hence, find the power series representation of $f(x) = x \ln(1+x)$ which holds for $|x| < 1$.

[6 marks]

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4. Cari perwakilan siri kuasa $\frac{1}{1+x}$. Seterusnya, cari perwakilan siri kuasa $f(x) = x \ln(1+x)$ yang memenuhi $|x| < 1$.

[6 markah]

5. Determine whether the integral $\int_{-1}^1 \frac{dx}{(2x+1)^{1/3}}$ is convergent or divergent.

[9 marks]

5. Tentukan sama ada kamiran $\int_{-1}^1 \frac{dx}{(2x+1)^{1/3}}$ menumpu atau mencapah.

[9 markah]

6. If $\frac{1}{w} = \frac{1}{x} + \frac{1}{y} + \frac{1}{z}$, find $\frac{\partial w}{\partial y}$ when $x = 30$, $y = 45$ and $z = 90$.

[5 marks]

6. Jika $\frac{1}{w} = \frac{1}{x} + \frac{1}{y} + \frac{1}{z}$, cari $\frac{\partial w}{\partial y}$ bila $x = 30$, $y = 45$ dan $z = 90$.

[5 markah]

7. Let $z = f(x, y)$ where $x = r \cos \theta$ and $y = r \sin \theta$. Suppose $f_x(0, 2) = -1$ and $f_y(0, 2) = 3$. Find $\frac{\partial z}{\partial r}$ and $\frac{\partial z}{\partial \theta}$ at the point where $r = 2$ and $\theta = \pi/2$.

[7 marks]

7. Biar $z = f(x, y)$, $x = r \cos \theta$ dan $y = r \sin \theta$. Andaikan $f_x(0, 2) = -1$ dan $f_y(0, 2) = 3$. Cari $\frac{\partial z}{\partial r}$ dan $\frac{\partial z}{\partial \theta}$ pada titik di mana $r = 2$ dan $\theta = \pi/2$.

[7 markah]

8. Evaluate the integrals.

(a) $\int_0^3 \int_0^{9-x^2} \frac{xe^{\frac{y}{3}}}{9-y} dy dx.$

(b) $\int_D \int (x^2 + y^2) dA$, where D is the region in the first quadrant bounded by the lines $y = 0$, $y = \sqrt{3}x$ and the circle $x^2 + y^2 = 9$.

[13 marks]

8. *Nilaikan Kamiran.*

(a) $\int_0^3 \int_0^{9-x^2} \frac{xe^{\frac{y}{3}}}{9-y} dy dx.$

(b) $\int_D \int (x^2 + y^2)^{\frac{3}{2}} dA$, D merupakan rantau dalam sukuan pertama yang dibatasi oleh garis $y = 0$, $y = \sqrt{3}x$ dan bulatan $x^2 + y^2 = 9$.

[13 markah]

9. (a) Solve the differential equation $(1 - xy + x^2y^2)dx + (x^3y - x^2)dy = 0$ by suitable substitution.

(b) Find the integrating factor for the equation $(y + x^3 + 3x^2 - 2x)dx - x dy = 0$. Hence, find its particular solution when $y(1) = 1$.

[16 marks]

9. (a) *Selesaikan persamaan pembezaan $(1 - xy + x^2y^2)dx + (x^3y - x^2)dy = 0$ dengan penggantian yang sesuai.*

(b) *Cari faktor pengamir bagi persamaan $(y + x^3 + 3x^2 - 2x)dx - x dy = 0$. Seterusnya, cari penyelesaian khususnya bila $y(1) = 1$.*

[16 markah]

10. In a certain culture of bacteria, the rate of increase is proportional to the number present.

- (a) If the number doubles in 4 hours, how many will be after 12 hours?
- (b) If the numbers are 10^4 after 3 hours and $4 \cdot 10^4$ after 5 hours, find the initial population.

[13 marks]

10. Dalam pembiakan bakteria tertentu, kadar pertambahan berkadaran dengan bilangan yang ada.

- (a) Jika bilangannya menjadi dua kali ganda dalam masa 4 jam, berapakah bilangan yang ada selepas 12 jam?
- (b) Jika bilangannya 10^4 selepas 3 jam dan $4 \cdot 10^4$ selepas 5 jam, cari populasi awal.

[13 markah]

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