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UNIVERSITI SAINS MALAYSIA

Peperiksaan Kursus Semasa Cuti Panjang  
Sidang Akademik 2006/2007

Jun 2007

**MAA 102 – Calculus for Science Students II**  
***[Kalkulus untuk Pelajar Sains II]***

Duration : 3 hours  
*[Masa : 3 jam]*

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Please check that this examination paper consists of FIVE pages of printed material before you begin the examination.

*[Sila pastikan bahawa kertas peperiksaan ini mengandungi LIMA muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]*

**Instructions :** Answer **all ten** [10] questions.

**Arahan** : Jawab **semua sepuluh** [10] soalan.]

...2/-

1. Determine whether  $\{a_n\}$  converges or diverges. If it converges, find its limit.

(a)  $a_n = \frac{3^n}{3^n + 2^n}$

(b)  $a_n = \frac{\sin 2n}{1 + \sqrt{n}}$

[8 marks]

2. Test the convergence of the series:

(a)  $\sum_{n=1}^{\infty} \frac{(-3)^{n+1}}{2^{3n}}$

(b)  $\sum_{n=1}^{\infty} \frac{\sqrt{n^2 - 1}}{n^3 + 2n^2 + 5}$

[10 marks]

3. Find the radius and interval of convergence of the power series  $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{n!(n+1)! 2^{2n+1}}$ .

[8 marks]

4. Find the Maclaurin series for  $e^x$ . Hence, calculate  $e^{-0.2}$  correct to five decimal places.

[5 marks]

5. Determine whether the integral  $\int_{-\infty}^{\infty} x^2 e^{-x^3} dx$  is convergent or divergent.

[7 marks]

6. Determine whether  $u = x^3 + 3xy^2$  is a solution of Laplace's equation,  $u_{xx} + u_{yy} = 0$ .

[4 marks]

7. (a) Show that  $f'(u) = \frac{1}{2} \left( \frac{1}{a} \frac{\partial z}{\partial t} + \frac{\partial z}{\partial x} \right)$  if  $z = f(x+at) + g(x-at)$ , where  $f$  and  $g$  are differentiable.

- (b) A certain function  $f(x, y)$  has directional derivative 8 in the direction  $\mathbf{v}_1 = 3\mathbf{i} - 4\mathbf{j}$  and 1 in the direction  $\mathbf{v}_2 = 12\mathbf{i} + 5\mathbf{j}$  at the point  $P_0(-1, 2)$ . Find the directional derivative of  $f$  at  $P_0$  in the direction of  $\mathbf{v} = 3\mathbf{i} - 5\mathbf{j}$ .

[13 marks]

...3/-

1. Tentukan sama ada  $\{a_n\}$  menumpu atau mencapah. Jika menumpu, cari hadnya.

$$(a) \quad a_n = \frac{3^n}{3^n + 2^n}$$

$$(b) \quad a_n = \frac{\sin 2n}{1 + \sqrt{n}}$$

[8 markah]

2. Uji penumpuan siri:

$$(a) \quad \sum_{n=1}^{\infty} \frac{(-3)^{n+1}}{2^{3n}}$$

$$(b) \quad \sum_{n=1}^{\infty} \frac{\sqrt{n^2 - 1}}{n^3 + 2n^2 + 5}$$

[10 markah]

3. Cari jejari dan selang penumpuan siri kuasa  $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{n!(n+1)! 2^{2n+1}}$ .

[8 markah]

4. Cari siri Maclaurin bagi  $e^x$ . Seterusnya, hitung  $e^{-0.2}$  betul kepada lima titik perpuluhan.

[5 markah]

5. Tentukan sama ada kamiran  $\int_{-\infty}^{\infty} x^2 e^{-x^3} dx$  menumpu atau mencapah.

[7 markah]

6. Tentukan sama ada  $u = x^3 + 3xy^2$  merupakan penyelesaian persamaan Laplace,  $u_{xx} + u_{yy} = 0$ .

[4 markah]

7. (a) Tunjukkan bahawa  $f'(u) = \frac{1}{2} \left( \frac{1}{a} \frac{\partial z}{\partial t} + \frac{\partial z}{\partial x} \right)$  jika  $z = f(x+at) + g(x-at)$ ,  $f$  dan  $g$  merupakan fungsi yang terbezakan.

- (b) Suatu fungsi  $f(x, y)$  mempunyai terbitan berarah 8 dalam arah  $\mathbf{v}_1 = 3\mathbf{i} - 4\mathbf{j}$  dan 1 dalam arah  $\mathbf{v}_2 = 12\mathbf{i} + 5\mathbf{j}$  pada titik  $P_0(-1, 2)$ . Cari terbitan berarah  $f$  pada  $P_0$  dalam arah  $\mathbf{v} = 3\mathbf{i} - 5\mathbf{j}$ .

[13 markah]

...4/-