UNIVERSITI SAINS MALAYSIA

Second Semester Examination
Academic Session 2006/2007

April 2007

MAA 102 – Calculus For Science Students II
[Kalkulus Untuk Pelajar Sains II]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of FIVE pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi LIMA muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions : Answer **all ten** [10] questions.

[Arahan *:* *Jawab* ***semua sepuluh*** *[10] soalan.]*

...2/-

1. Determine whether $\left\{\left(\frac{n+1}{2n}\right)\left(1-\frac{1}{n}\right)\right\}$ converges or diverges.

[3 marks]

2. Test the convergence of the series:

(a) $\sum_{n=1}^{\infty}\frac{2^n 3^n}{n^n}$ (b) $\sum_{n=1}^{\infty}\frac{\ln n}{n^3}$

[10 marks]

3. Find the interval of convergence of the power series $\sum_{n=1}^{\infty}\frac{x^n}{\sqrt{n}}$.

[10 marks]

4. Estimate the maximum error in using Taylor polynomial $T_3(x)$ about $a=1$ to approximate $f(x)=\frac{1}{x+2}$ when $0.9\le x\le 1.1$.

[7 marks]

5. Determine whether the integral $\int_{-2}^{\infty}\frac{d\theta}{(\theta+1)^{3/5}}$ is convergent or divergent.

[8 marks]

6. By using the definition of partial derivative, find $f_x(0,y)$ and $f_y(x,0)$ if

$$f(x,y)=\begin{cases} xy\,\dfrac{x^2-y^2}{x^2+y^2} & , \ (x,y)\neq(0,0) \\ 0 & , \ (x,y)=(0,0) \end{cases}.$$

[6 marks]

7. (a) If f is differentiable and $z=xy+f\left(x^2+y^2\right)$,find $y\frac{\partial z}{\partial x}-x\frac{\partial z}{\partial y}$.

(b) A certain function $f(x,y)$ has directional derivative 8 in the direction $\boldsymbol{v}_1=3\boldsymbol{i}-4\boldsymbol{j}$ and 1 in the direction $\boldsymbol{v}_2=12\boldsymbol{i}+5\boldsymbol{j}$ at the point $\mathrm{P}_0(-1,2)$. Find the directional derivative of f at P_0 in the direction of $\boldsymbol{v}=3\boldsymbol{i}-5\boldsymbol{j}$.

[13 marks]

...3/-

1. *Tentukan sama ada* $\left\{\left(\frac{n+1}{2n}\right)\left(1-\frac{1}{n}\right)\right\}$ *menumpu atau mencapah.*

[3 markah]

2. *Uji penumpuan siri:*

(a) $\sum_{n=1}^{\infty}\frac{2^n 3^n}{n^n}$ *(b)* $\sum_{n=1}^{\infty}\frac{\ln n}{n^3}$

[10 markah]

3. *Cari selang penumpuan siri kuasa* $\sum_{n=1}^{\infty}\frac{x^n}{\sqrt{n}}$.

[10 markah]

4. *Anggarkan ralat maksimum apabila menggunakan polinomial Taylor* $T_3(x)$ *sekitar* $a=1$ *untuk penghampiran* $f(x)=\frac{1}{x+2}$ *bila* $0.9\le x\le 1.1$.

[7 markah]

5. *Tentukan sama ada kamiran* $\int_{-2}^{\infty}\frac{d\theta}{(\theta+1)^{3/5}}$ *menumpu atau mencapah.*

[8 markah]

6. *Dengan menggunakan takrif terbitkan separa, cari* $f_x(0,y)$ and $f_y(x,0)$ *jika*

$$f(x,y)=\begin{cases} xy\,\dfrac{x^2-y^2}{x^2+y^2} & , \quad (x,y)\neq(0,0) \\ 0 & , \quad (x,y)=(0,0) \end{cases},$$

[6 markah]

7. *(a) Jika* f *terbezakan dan* $z=xy+f\left(x^2+y^2\right)$ *, cari* $y\frac{\partial z}{\partial x}-x\frac{\partial z}{\partial y}$.

(b) Suatu fungsi $f(x,y)$ *mempunyai terbitan berarah 8 dalam arah* $\mathbf{v}_1=3\mathbf{i}-4\mathbf{j}$ *dan 1 dalam arah* $\mathbf{v}_2=12\mathbf{i}+5\mathbf{j}$ *pada titik* $P_0(-1,2)$. *Cari terbitan berarah* f *pada* P_0 *dalam arah* $\mathbf{v}=3\mathbf{i}-5\mathbf{j}$.

[13 markah]

...4/-

8. Find the largest and smallest values of the function $f(x,y)=x^2+y^2-3y$ on the disk $x^2+y^2\le 4$.

[15 marks]

9. Evaluate the integrals.

(a) $\int_0^8 \int_{\sqrt[3]{x}}^2 \frac{1}{y^4+1}\,dy\,dx.$

(b) $\iint_D \frac{\ln\left(\sqrt{x^2+y^2}\right)}{x^2+y^2}\,dA$, where D is the region in the first quadrant bounded by the circles, $x^2+y^2=1$ and $x^2+y^2=e^2$.

[10 marks]

10. (a) Solve the differential equation $y^2dy-\left(x^2+\frac{y^3}{x}\right)dx=0$ by suitable substitution.

(b) A bacteria culture starts with 1000 bacteria and the growth rate is proportional to the number of bacteria. After 2 hours the population is 9000. Find the number of bacteria after 3 hours and how long it take to double the number of bacteria?

[18 marks]

...5/-

8. *Cari nilai terbesar dan terkecil bagi fungsi* $f(x,y)=x^2+y^2-3y$ *di atas cakera* $x^2+y^2\le 4$.

[15 markah]

9. *Nilaikan kamiran.*

(a) $\int_0^8 \int_{\sqrt[3]{x}}^2 \frac{1}{y^4+1}\, dy\, dx.$

(b) $\iint_D \frac{\ln\left(\sqrt{x^2+y^2}\right)}{x^2+y^2} dA$, D *merupakan rantau dalam sukuan pertama yang dibatasi oleh bulatan* $x^2+y^2=1$ *dan* $x^2+y^2=e^2$.

[10 markah]

10. (a) *Selesaikan persamaan pembezaan* $y^2dy-\left(x^2+\frac{y^3}{x}\right)dx=0$ *dengan penggantian yang sesuai.*

(b) *Pembiakan bakteria bermula dengan 1000 dan kadar pertambahan berkadaran dengan bilangannya. Selepas 2 jam populasinya 9000. Cari bilangan bakteria selepas 3 jam dan masa yang diperlukan untuk menjadi dua kali ganda bilangannya.*

[18 markah]

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