GENERATING AVIAN EGG USING RATIONAL BÉZIER QUADRATIC CURVES

by

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**Dissertation submitted in partial fulfillment
of the requirement for the degree
of Master of Science in Mathematics**

JUNE 2010

ACKNOWLEDGEMENTS

All praise is due to Allah the giver of all knowledge. I am especially grateful to my supervisor, Associate Professor Dr. Jamaludin Md. Ali for his encouragement, guidance, help and support during my study of this dissertation. A very special note of appreciation to Associate Professor Dr. Ahmad Izani Md. Ismail, Associate Professor Dr. Norhashidah Hj Mohd. Ali, Associate Professor Dr. Abd.Rahni Mt.Piah, Associate Professor Dr. Ahmad Abd. Majid and Dr. Md. Fazlul Karim for teaching me during my master study. The knowledge that they taught was very useful and important in this study.

I also wish to acknowledge with appreciation the invaluable assistance I received from the technical staffs of School of Mathematical Sciences, University Science of Malaysia especially to Pn. Faridah Hashim, who in charge post graduate students.

To all my colleagues in Penang Matriculation Centre, also I sincerely thanked for their help, support and useful discussions.

Finally yet important, I would like to thank my family for their patient support and encouragement during my studies.

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JANAAN TELUR AVIAN MENGGUNAKAN LENGKUNG NISBAH BÉZIER KUADRATIK

ABSTRAK

Tujuan utama kajian ini adalah untuk merekabentuk telur dari spesies burung (avian) menggunakan pendekatan Rekabentuk Geometri Berbantukan Komputer (RGBK) dan menganggarkan isipadu telur menggunakan pendekatan yang sama. Kaedah Rekabentuk Geometri Berbantukan Komputer yang digunakan dalam kajian ini adalah Lengkung Nisbah Bézier Kuadratik. Lengkung Nisbah Bézier bergantung kepada pemberat dalam menentukan bentuk grafnya, jadi fokus utama dalam kajian ini adalah untuk mencari pemberat yang paling sesuai dengan bentuk telur yang ingin dibentuk. Dalam kajian ini, rekabentuk telur dianggap sebagai kombinasi antara separuh bulatan dan separuh elips. Hasil daripada kajian yang dijalankan, pemberat yang diperolehi bagi bahagian bulatan adalah $w_0 = w_1 = 1$ dan $w_2 = 2$ manakala pemberat bagi bahagian elips adalah $w_0 = w_2 = 1$ and $w_1 = 0.6$. Secara umumnya, bentuk telur yang terhasil menggunakan kaedah Lengkung Nisbah Bézier Kuadratik adalah berkeselanjaran G^1 bagi setiap titik modal.

Dengan menggunakan pemberat dan data yang diperolehi daripada eksperimen, kajian diteruskan untuk mencari isipadu telur menggunakan kaedah kamiran iaitu menjana isipadu putaran lengkung 360° terhadap paksi yang membujur. Ralat yang terhasil daripada isipadu janaan Lengkung Nisbah Bézier Kuadratik menggunakan nilai-nilai pemberat yang dinyatakan di atas adalah tidak melebihi 3% daripada purata nilai isipadu yang diperolehi daripada eksperimen.

Data-data daripada eksperimen yang diperlukan untuk membentuk Lengkung Nisbah Bézier Kuadratik supaya menepati rupabentuk telur adalah panjang maksima (L), lebar maksima (B) dan panjang sebiji telur daripada hujung yang paling tajam hingga ke titik persilangan di antara panjang maksima dan lebar maksima telur tersebut (L_1).

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ABSTRACT

The main purpose of this research is to design an avian egg geometrically and to estimate the egg volume by using Computer Aided Geometric Design (CAGD) approach. One of the methods of Computer Aided Geometric Design is Rational Bézier Quadratic Curves which will be used to model the egg curve and will be discussed further in this study. Since the Rational Bézier is depending on their weighting, thus the best values of weighting need to be studied and will be discovered in this research. In this study, the shape of an egg was assumed to be a combination of semicircle and half ellipse. From the result, the best weighting for circle part were $w_0 = w_1 = 1$ dan $w_2 = 2$ while for ellipse $w_0 = w_2 = 1$ and $w_1 = 0.6$ were chosen. This study has proven that the curves of an avian egg generated from Rational Bézier Quadratics were G^1 for all of their joining paths.

By using the weighting and the experimental data, the volumes of egg were generated using integration method by revolving the curve 360° about its longitudinal axis. For this method, the calculations errors for volume did not exceed 3% from the average of experimental volumes.

The experimental data that used to design a closest curve to an avian egg curvature by using Rational Bézier Quadratic Curves were maximum length (L), maximum breadth (B) and length of longitudinal axis from sharp pole to a crossing point of longitudinal axis with the axis corresponding to the maximum diameter (L_1).

CHAPTER 1

INTRODUCTION

1.1 Background

Avian egg size is an extremely useful parameter in studies of avian reproduction. However, it can be difficult to collect egg size data. Methods for measuring egg size range from taking weights and linear measures to observe water displacement and all have their advantages and disadvantages. Appropriate approximation models or equations of an avian egg are needed in order to facilitate data collecting and to reduce time taken for data collection.

It is well known that the shape of an avian egg shell is an ovoid. Many explanations were made in order to illustrate the shape of an avian egg. Shott and Preston (1973) proposed an egg curve is an ellipse and the surface revolution of an egg is a prolate spheroid. Itou and Yamamoto (2008) suggested that an egg profile is a curve obtained by intersecting a Pseudo-sphere by means of inclined plane. Preston (1974), Jukuchou and Yamamoto (2009a, 2009b), Itou and Yamamoto (2008) and Yamamoto (2009a) presented an egg curve in parametric equation while Narushin (1993, 1997, 2001, 2005), Yamamoto (2007, 2009b) and Jukuchou and Yamamoto (2009b) expressed an egg description by mathematical equations. Jukuchou and

Yamamoto (2009b) also proposed a simple method to connect two ellipsoids into an egg shaped curve.

Paganelli *et al.* (1974) discovered the first estimated outlines of an egg using photographic techniques to calculate volume and surface area. Bridge et al. (2007) then modified the methods of Paganelli et al. (1974) and used an automated computer analysis procedure to read in data directly from digital photographs. Zhou et al. (2009) developed a Machine Vision technology that gave definition of volume and surface area in pixels.

1.2 Problem Statement

During the last five to six decades, many equations were modelled mathematically by scientist to estimate the avian eggs size. Expression of an avian egg curve geometrically and estimations volume using mathematical equations are important in agriculture field especially in poultry industry. However in Malaysia, there is no research yet to express an avian egg into mathematical equations. So in order to help biologist in data collection, mathematical expression of an avian egg should be done.

In this research, our focus is to geometrically design egg curvatures using Computer Aided Geometric Design approach. One of the methods of Computer Aided Geometric Design is rational Bézier curve which will be used to model the egg curve and will be discussed further in this study.

1.3 Scope of Dissertation

This study, as well as other studies that were undertaken to provide an avian egg profile geometrically and to estimate the volume, is intended to address two sets of questions. Firstly, how to construct egg profile? Secondly, how to estimate the volume of an egg using the curves obtained?

Several methods of Computer Aided Geometric Design have been revised to address these questions. Rational Bézier quadratic curve was chosen in this study of an avian egg profile. Bézier methods have been the basis of modern field of Computer Aided Geometric Design was developed by Dr. Pierre Etienne Bézier, an engineer of the car manufacturer Renault in the early 1960's (Solomon, 2006). Since the rational Bézier is depending on their weighting, thus the best values of weighting need to be studied and will be discovered in this research.

1.4 Objectives of Dissertation

The aims of this study are:

i. To construct the profile equation of an avian egg using Rational Bézier Quadratic Curves and discuss the continuity of joining of the paths.

ii. To determine the best weighting for the curve of Rational Bézier Quadratic Approximation using curves plotting.

iii. To estimate the volume of avian egg using the curves obtained.

iv. To compare the data obtained from experiments, the curve of Rational Bézier Quadratic Approximation and some of equations from literature review.

1.5 Structure of Dissertation

There are six chapters in this study.

Chapter 1 is an introduction of the study and gives a summary of background and the objectives of the study.

Chapter 2 is the literature review. Related researches were reviewed and these researches can be used as a guideline to this study.

Chapter 3 is the theory and formulae. This chapter describes about the basic concept of of Bézier approximation, Rational Bézier Quadratic Curves, continuity of joining Bézier curves and some statistics formulae used in this study.

Chapter 4 is the methodology. This chapter discusses the basic idea of an avian egg curve, how to construct two dimensional egg shape using Rational Bézier Quadratic Curves and how to generate the volume of an egg.

Chapter 5 is result and discussion. By using the experimental data, the two dimensional egg profile and volumes of eggs were run using MATLAB 6.7 and Wolfram Mathematica 7.0. The best weightings of Rational Bézier Quadratic Curves that gave the closest shape to an avian egg and the best estimation of volume were summarized.

Chapter 6 is the conclusion. A brief discussion on the contributions of this study to industries is also given. Besides that, areas for further research are also being suggested in this chapter.

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

The aim of this chapter is to extensively review on equations of avian egg curve and volume. The flow of study was summarized and conclusion was made. These previous studies were used to guide this study.

2.2 Equations of Avian Egg Curves

Todd and Smart (1984) claimed that the curve of an avian is axisymmetric. They applied the method of transformed co-ordinates quantitatively to the shape of avian eggs. Started from equation of circle $X^2+Y^2=1$ (with center at origin and radius is 1), they allowed co-ordinate transformations of the form $X=x/a$ and $Y=y/a\ f(X)$ and yield a transformed circle equation as

$$y=a\ f(X)\sqrt{1-\frac{x^2}{a^2}} \tag{2.1}$$

They found that the cubic term $f(X)=k+cX+dX^2+eX^3$ is the closest to an avian egg profile where *k, c, d* and *e* are constants.

Preston (1974) discovered the equation of an egg curve in parametric form as

$$\left.\begin{aligned} x &= b\sin\theta \\ y &= a\cos\theta\left(1 + c_1\sin\theta + c_2\sin^2\theta\right) \end{aligned}\right\} \tag{2.2}$$

where c_1 and c_2 are coefficients representing the departure of the oval from an ellipse. In particular, c_1 represents a departure from symmetric being zero for a symmetric egg.

Yamamoto (2007) presented the equation of an egg curve as

$$y = \pm\frac{(a-b) - 2x + \sqrt{4bx + (a-b)^2}}{\sqrt{2}}\sqrt{x} \tag{2.3}$$

He used a = 4 and discovered that the curve approached to the egg shape when $b = 0.7a$.

Itou and Yamamoto (2008) described the egg equation in parametric form as below

$$\left.\begin{aligned} x &= a\cos\theta \\ y &= b\cos\frac{\theta}{4}\sin\theta \\ \theta &\le |\pi| \end{aligned}\right\} \tag{2.4}$$

By choosing the value $a = 0.5$ and varying several values of b, they found that the curve of $b = 0.37$ was the closest to the profile of an egg.

Yamamoto (2009a) found equation of egg shaped curve was written as:

$$\left.\begin{aligned} x &= a\cos\theta \\ y &= (b + \cos\theta)\sin\theta \end{aligned}\right\} \tag{2.5}$$

In his study, he found that the values of a = 1, b = 0.72 and c = 0.08 were the closest curve of an egg shape.

Yamamoto (2009b) transformed the equation from the concave circle $k_2 e^{-b^2x^2} + e^{-c^2y^2} = k_1 e^{-a^2} + k_3 e^{-dx}$ into an egg shaped curve. The obtained equation was:

$$y = \frac{1}{c}\sqrt{-\log\left(k_1 e^{-a^2} + k_3 e^{-dx} - k_2 e^{-b^2x^2}\right)} \tag{2.6}$$

with the condition $0 < k_1 e^{-a^2} + k_3 e^{-dx} - k_2 e^{-b^2x^2} \leq 1$ to ensure that equation (2.6) exist. For this case, when $a = 0$, $b = 2.0$, $c = 2.65$ and $d = k_1 = k_2 = k_3 = 1$ the curve obtained was the closest shape of an avian egg.

Jukuchou and Yamamoto (2009a) proposed the egg equation in the form of parametric equation as shown below:

$$\left.\begin{aligned} x &= a\{(c-a)\cos\theta + c + 2\}(\cos\theta + 1)/4 \\ y &= a\sin\theta \end{aligned}\right\} \tag{2.7}$$

They plotted several curves by varying the value of c and discovered that the curve in the case that a = 1 and c = 2.9 was the closest shape of an actual egg.

Jukuchou and Yamamoto (2009b) described two types of egg equations. First type of equation was defined as:

$$y = \pm\frac{1}{b}\left(1 + \frac{xa}{c}\right)\sqrt{1 - \left(\frac{x}{a}\right)^2} \tag{2.8}$$

In this case, the curve in the case that a = 1.5, b = 1.05 and c = 11 gave the closest shape of an avian egg. Second type of equation proposed by Jukuchou and Yamamoto (2009b) was:

$$\left.\begin{aligned} & x=\frac{1}{2}b\cos\theta+r(\theta)\cos\theta \\ & y=r(\theta)\sin\theta \\ & \text{where } r(\theta)=\frac{1}{2}\{a+c+(c-a)\cos\theta\} \end{aligned}\right\} \tag{2.9}$$

The nearest shape to an actual egg when a = 1.35, b = 0.9 and c = 0.5.

Narushin (1997) considered an egg as a solid revolution about the longitudinal axis and developed his idea by considering the contour of half-projection of an egg. Narushin (1997) presented mathematically a profile of an egg by the given equation:

$$y=\frac{\pm 1.5396B\sqrt{L^{1/2}x^{3/2}-x^2}}{L} \tag{2.10}$$

Where L is the length of egg, B is its breadth, x the coordinate along the longitudinal axis and y is the transverse distance to the egg profile.

Narushin (2001) considered his previous study Narushin (1997) basic equation of the contour of an egg , $r=L\cos^n\theta$ and substituted the equations of transformation from the polar coordinates to Cartesian coordinates by Pogorelov (1980), $y=r\sin\theta$ and $x=r\cos\theta$; Narushin (2001) deduced the equation of an avian egg as:

$$y=\pm\sqrt{L^{2/(n+1)}x^{2n/(n+1)}-x^2} \tag{2.11}$$

In which L is egg length and B is maximum breadth, whilst x and y have been explained above in the review of Narushin (1997). To estimate y_{max}, he equated the derivative of equation 2.5 to zero and obtained $x = L\left(\frac{n}{n+1}\right)^{(n+1)/2}$ and then found the best fit to express n as a function of the shape index or breadth-to-length ratio by using the method of approximation which is $n = 1.057\left(\frac{L}{B}\right)^{2.372}$.

2.3 Equations of Avian Egg Volumes

Narushin (1997) substituted equation 2.10 into formulae of the solid revolution about the long axis $V = \pi \int_0^L y^2 dx$ to calculate the estimation volume of avian egg. The result of the volume of the solid generated was:

$$V = 0.49645LB^2 \quad (2.12)$$

To minimize the error of equation 2.12, Narushin (1997) transformed the equation using the longitudinal axis, L and the long circumference, C. This transformation formula will be discussed in this study and was defined as:

$$V = 1.849L^3\left(\log_e \frac{C}{1.84L}\right)^2 \quad (2.13)$$

Narushin (2001) calculated the egg volume by revolving equation 2.11 about long axis and resulting formula for an avian egg volume was obtained:

$$V = \frac{2\pi L^3}{3(3n+1)} \quad (2.14)$$

where L is the egg length and n is defined in section 2.2. In this study, we will be discussed the transformation of equation 2.14 into theoretical contour which was defined as:

$$V=\frac{L^3}{1339.848\left(\frac{L}{C}\right)^{6.024}+0.478} \qquad (2.15)$$

Narushin (2005) transformed equation 2.13 into the form of $V=k_v LB^2$ by approximation of the data from Narushin (2001) by a function $f(L/B)$. The best fit approximation is given by $n=1.687(L/B)^2-0.661(L/B)$ with $R^2=0.999$. By substituting this new equation n into equation 2.14, a theoretical formula obtained however he transformed the formula to make it a better fit for practical needs. As a result, the new equation was obtained and defined as:

$$V=(0.6057-0.0018B)LB^2 \qquad (2.16)$$

Tatum (1975) modified the volume equation given by Preston (1974) to second order in c_1 and c_2. The equation was:

$$V=\frac{\pi LB^2}{6}\left(1+\frac{2}{5}c_2+\frac{1}{5}c_1^2+\frac{3}{35}c_2^2\right) \qquad (2.17)$$

Zhou *et al.* (2009) considered the equation 2.8 defined by Narushin (2005) to develop a Machine Vision technology that gave definition of volume in pixels. Then, with aid of SAS V9.0 programming, they found the equation to transform their volume in pixels (V_p) to predict volume. The equation was:

$$V=3.447E-0.5V_p+23.296991 \qquad (2.18)$$

2.4 The Surface-Volume Relationship of Avian Egg

Panganelli (1974) used a photographic technique in conjunction with hybrid analog-digital computer to measure the surface area and volume of 225 eggs of 29 species of avian egg. The surface area (S) and volume (V) of every single egg was plotted and they discovered that the relationship between surface areas and volumes by calculating the correlation lines using the method of least square is $S = 4.951 \times V^{0.666}$ (correlation coefficient, $r = 0.99997$).

Hoyt (1976) used water displacement methods to collect accurate data of volumes of 29 eggs from some of the families of North American Birds. He then used mathematical approaches with the aid of computer to estimate the surfaces and volumes. Hoyt (1976) defined the relationship between surface area and volumes as $S = 4.928 \times V^{0.668}$ with $r = 0.9999$ and standard error of the estimation is 0.0061.

González *et al.* (1982) determined the volumes and surface areas of 111 eggs from 15 different species of avian with more than two order of magnitude span in size and made comparison of the obtained data. The result presented the relationship between surface area and volume of those eggs by using the method of least squares is given by $S = 4.689 \times V^{0.673}$ $(r = 0.983)$ for all type of egg studied, however for large eggs from the species of *Anser anser* (Graylag goose) the best exponential fitted is $S = 3.877 \times V^{0.727}$ $(r = 0.983)$.

2.5 Conclusion

The previous studies of avian egg were proven that the mathematical expression of an egg curvature and volumes were often used especially in biological studies and poultry industry. Many formulae have been proposed and widely used by researchers. In this study, the main objectives are to design an avian egg curve using rational Bézier approximation and to estimate the volume of an avian egg from the curve obtained.

CHAPTER 3

THEORIES AND FORMULAE

3.1 Introduction

This chapter is divided into 2 major sections. Section 3.2 discusses the basic concept of Bézier approximation, Rational Bézier Quadratic Curves and the continuity of the joining Bézier curves while section 3.3 discusses the statistics formulae used in this study.

3.2 Bézier Approximation

The Bézier curve is a parametric curve defined as

$$P(t)=\sum_{i=0}^{n} p_i B_i^n \ , \ 0 \le x \le 1 \tag{3.1}$$

where p_i are control points with $i=0,1,\ldots,n$ and B_i^n Bézier blending functions or Bernstein polynomials. The definition of Bernstein polynomial was derived by Sergeĭ Natanovich Bernshteĭn as (Solomon, 2006):

$$B_i^n(t)=\binom{n}{i} t^i (1-t)^{n-i} \text{, where } \binom{n}{i}=\frac{n!}{i!(n-i)!} \tag{3.2}$$

A Bézier curve can be fitted to any number of control points (Bézier, 1974). The degree of polynomial depends on the number of points used to define the curve (Hearn, 1997).

In general the rational Bézier approximation is given as

$$P(t)=\frac{\sum_{i=0}^{n} w_i p_i B_i^n}{\sum_{i=0}^{n} w_i B_i^n}\ ,\quad 0\le t\le 1 \tag{3.3}$$

where p_i are control points, B_i^n Bézier blending functions or Bernstein polynomials w_i is the weighting with $i=0,1,\ldots,n$ (Solomon, 2006).

3.2.1 Rational Bézier Quadratic Curves

The Rational Bézier Quadratic Curves are generated with three control points $p_i=(x_i,y_i)$ with i varying from 0 to 2. These coordinate points can be blended to produce the following position vector which described the path of an approximation Bézier polynomial function between the control points p_0 and p_2:

$$P(t)=\frac{w_0 p_0 B_0^2 + w_1 p_1 B_1^2 + w_2 p_2 B_2^2}{w_0 B_0^2 + w_1 B_1^2 + w_2 B_2^2} \quad 0\le t\le 1 \tag{3.4}$$

Position vector $P(t)$ depends on weights (w_0, w_1 and w_2) that acted as additional parameters that control the shape of the curve. Nonnegative weights are normally used to ensure that the denominator never lead to zero.

The three Bernstein polynomials B_0^2, B_1^2 and B_2^2 can be given as

$$\left.\begin{aligned} B_0^2(t) &= (1-t)^2 \\ B_1^2(t) &= 2t(1-t) \\ B_2^2(t) &= t^2 \end{aligned}\right\} \tag{3.5}$$

The plot of Bernstein polynomials of quadratic Bézier is given in Figure 3.1.

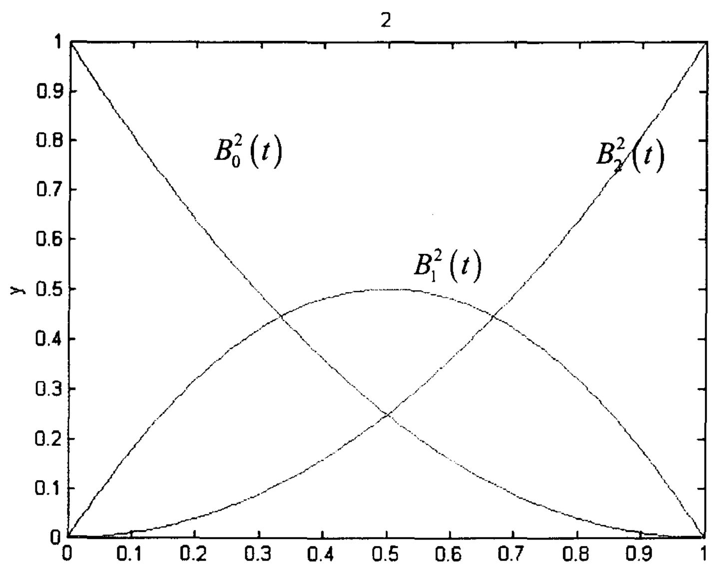

Figure 3.1: Plot of Bézier blending functions or Bernstein polynomials of quadratic Bézier.

Some of the useful properties and advantages of using rational Bézier curves to obtain an approximation curve:

i. A rational Bézier curve starts at the first control point p_0 and stops at the last control point p_2. So, this property is very useful when we want to join 2 curves together.

ii. The rational Bézier curve always lies within the convex hull of its control points.

iii. The value of blending function / Bernstein polynomials does not depend upon the position of any control poins.

iv. The rational Bézier blending function / Bernstein polynomials are all positive and their sum is always 1.

$$\sum_{i=0}^{2} B_i^n(t) = 1 \tag{3.6}$$

v. The rational Bézier curve provides for accurate control of curve shape.

3.2.2 Rational Bézier Quadratics and Conics Section

By substituting the Bernstein polynomial into equation 3.4, we get:

$$P(t) = \frac{(1-t)^2 w_0 p_0 + t(1-t) w_1 p_1 + t^2 w_2 p_2}{(1-t)^2 w_0 + t(1-t) w_1 + t^2 w_2} \quad 0 \le t \le 1 \tag{3.7}$$

Three control points was chosen such as $p_0 = (1,0)$, $p_1 = (1,1)$ and $p_2 = (0,1)$ (Solomon, 2006). When the values of weights $w_0 = w_1 = 1$ and $w_2 = 2$, the results obtained is a circle:

$$P(t) = \frac{(1-t)^2 p_0 + 2t(1-t) p_1 + 2t^2 p_2}{(1-t)^2 + 2t(1-t) + 2t^2} = \left(\frac{1-t^2}{1+t^2}, \frac{2t}{1+t^2} \right) \tag{3.8}$$

If values of $w_0 = w_2 = 1$ were fixed, by choosing the value of $w_1 = 1$ will give us a curve of a parabola. If we choose $w_1 < 1$ the curve for equation 3.7 turned to a shape of an ellipse and for $w_1 > 1$, it produced a hyperbola curve. The curves of conic sections by varying the weights that discussed here were illustrated in Figure 3.2.

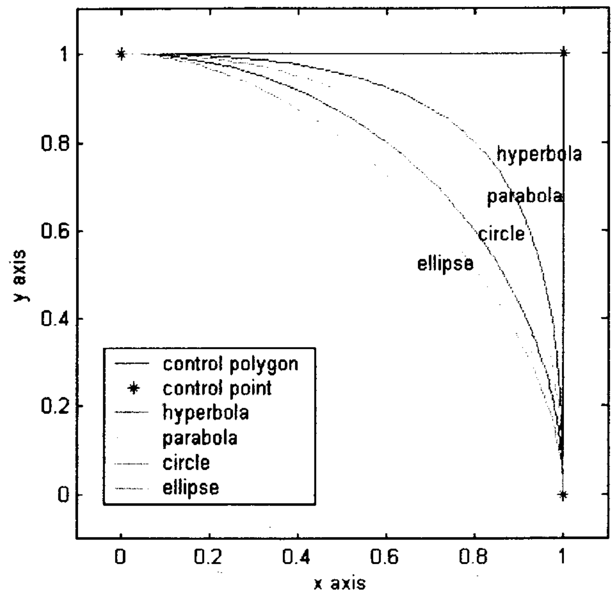

Figure 3.2: The curves of conic sections by varying the weights

3.2.3 Continuity of The Joining Bézier Curves

i. Parametric Continuity

The two curves have at least C^0 if their end points are joined ($p_3 = q_0$). It also appears that they are C^1 if line segments $p_3 - p_2$ and $q_1 - q_0$ are collinear and of equal length (first derivative is equal). Furthermore, they are C^2 if first and second derivatives are equal which you can verify by sketching the second derivative curves.

ii. Geometric Continuity

The two curves obviously G^0 if the curves touch at joining point. For G^1, we only require line segments $p_3 - p_2$ and $q_1 - q_0$ are collinear or they share a common tangent direction at the joining point. G^2 means that the two neighbouring curves have the same tangent line (G^1) and also share a common centre of curvature at the joining point.

The summarization of parametric and geometric continuity for piecewise Bézier Curves was shown below in Table 3.1:

Table 3.1: Summary of parametric and geometric continuity piecewise Bézier curves

Continuity	**Summary of joining points**
C^0 and G^0 continuity	$p_3 = q_0$
C^1 continuity	$p_3 = q_0$
	$p_3 - p_2 = q_1 - q_0$
G^1 continuity	$p_3 = q_0$
	$p_3 - p_2 = \alpha\ (q_1 - q_0)$
C^2 continuity	$p_3 = q_0$
	$p_3 - p_2 = q_1 - q_0$
	$p_3 - 2p_2 + p_1 = q_2 - 2q_1 + q_0$
G^2 continuity	Geometric interpretation

Geometric interpretation for G^2 continuity was shown in Figure 3.3. Two piecewise Bézier curves were G^2 if $\frac{h_1}{h_2} = \frac{d_1^2}{d_2^2}$.

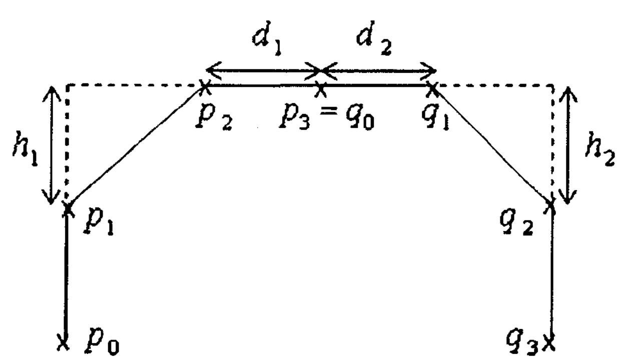

Figure 3.3 : Geometric interpretation for G^2 continuity

3.3 Statistics

3.3.1 Mean

Mean is one of the examples of measure of central tendency. In Mathematics, an average or central tendency of a set data refers to a measure of the middle or expected value of the data set. Mean is defined as the sum of measurements divided by the total number of measurements. Let $x_1, x_2, x_3, \ldots, x_n$ denoted by the measurements observed in a sample of size n as shown in equation 3.9.

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} \tag{3.9}$$

where $\sum_{i=1}^{n} x_i = x_1 + x_2 + x_3 + \ldots + x_n$.

3.3.2 Variance and Standard Deviation

A measure of spread or dispersion for sample, variance is constructed by adding the squared deviations and dividing the total by the number of observations minus one. The variance of n observations is given in equation 3.10:

$$s^2 = \frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n-1} \tag{3.10}$$

where $\sum_{i=1}^{n} (x_i - \bar{x})^2 = (x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + \ldots + (x_n - \bar{x})^2$.

Because the variance involves a sum of squares, its unit is the square of unit in which measurement are expressed. To obtain a measure of variability in the same unit as the data, standard deviation is used. Standard deviation is the positive square root of the variance and is given by:

$$s=\sqrt{\frac{\sum_{i=1}^{n}(x_i-\overline{x})^2}{n-1}} \qquad (3.11)$$

3.3.3 Coefficient of Variation

Whilst the mean and standard deviation can be used to measure location and dispersion within a data set, they also can be used to measure dispersion between data sets. To compare the dispersion between data sets, coefficient of variation is used. Coefficient of variation is the expression of the standard deviation as a percentage of what is being measured. The formula is given as:

$$\text{Coefficient of variation}, CV=\frac{s}{\overline{x}}\times 100\% \qquad (3.12)$$

Since coefficient of variation is a percentage and does not have unit, therefore it can be used to make comparisons between data sets.

3.4 Conclusion

The theories and formulae that have been discussed in this chapter will be used in Chapter 4 and Chapter 5.

CHAPTER 4

METHODOLOGY

4.1 Introduction

In this study, our focus is to generate two dimensional avian egg profile. If the shape of an egg is half ellipse and semicircle and also is symmetrical about the longitudinal axis, we can easily deduce the curves of an egg using quadratic rational Bézier equation with suitable weighting for the curves.

4.2 Collecting Data of Egg

In this study, thirty fresh eggs from Malaysian domestic chickens (*Gallus domesticus*) were used. Then they were labeled in order as T1, T2, T3, ... T30. The volume of each egg was taken using water displacement methods and determined by weighting the water. The volumes reported in this study are average of three replicate measurements. The maximum length, maximum breadth and other related measurement were measured using Vanier caliper while the circumference of egg was measured using a measuring tape.

4.3 Introducing to An Avian Egg Shape

The picture of egg used in Figure 4.1 below was found using the web search engine, Google and unaltered the original shape. The red line was a circle while the line of an ellipse was coloured in blue. The centre of the circle and ellipse lies on the dotted line which is the maximum width or breadth of the egg. By referring to the shape of an egg given in Figure 4.1, it is very clear that the curve of an egg was half circle and half ellipse.

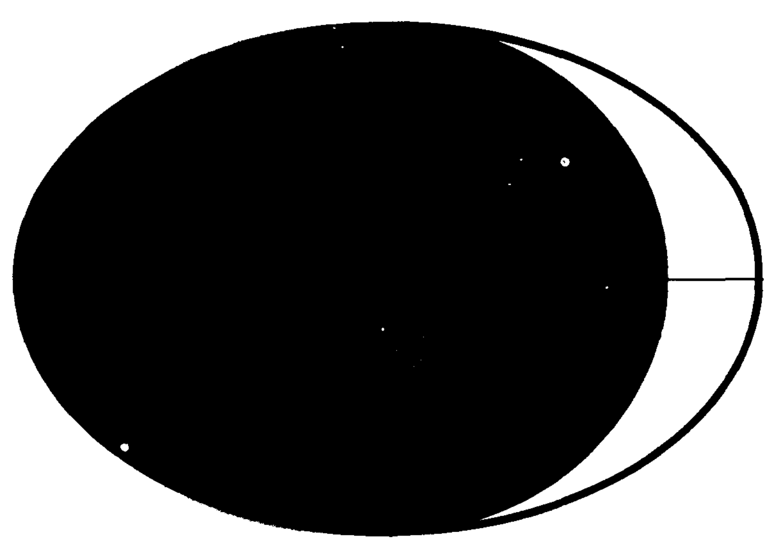

Figure 4.1: The shape of an egg

For easy understanding, Figure 4.2 illustrated the profile of an avian egg in Cartesian plane and the points that used in this study were stated below.

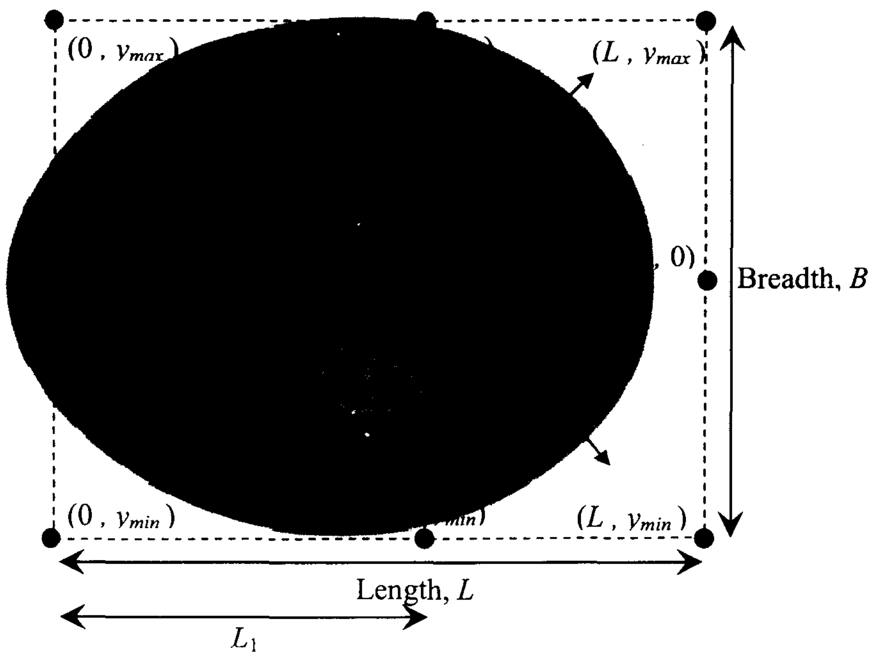

Figure 4.2: Interpret an egg profile in cartesian plane

L is the maximum length of egg, B is the maximum breadth (diameter), L_1 is the length of longitudinal axis from sharp pole to a crossing point of longitudinal axis with the axis corresponding to the maximum diameter, $y_{max} = B/2$ and $y_{min} = -B/2$. The shape of egg constructed above is formed from four paths of quadratic rational Bézier approximation, which is organized clockwise. Path I and path II are presenting half circle while path III and IV represents half ellipse.