UNIVERSITI SAINS MALAYSIA

Peperiksaan Semester Pertama
Sidang Akademik 1996/97

Oktober/November 1996

EKC 440 - Fenomena Pengangkutan

Masa: [3 jam]

ARAHAN KEPADA CALON:

Sila pastikan kertas soalan ini mengandungi **LAPAN (8)** mukasurat dan **ENAMBELAS (16)** Lampiran bercetak sebelum anda memulakan peperiksaan.

Kertas soalan ini mengandungi **ENAM (6)** soalan.

Jawab hanya **EMPAT (4)** soalan.

Soalan No. 1 MESTI dijawab dalam Bahasa Malaysia. Anda dibolehkan menjawab soalan-soalan lain dalam Bahasa Inggeris.

Soalan terjemahan Bahasa Inggeris ditaip dalam bentuk tulisan **Italic**.

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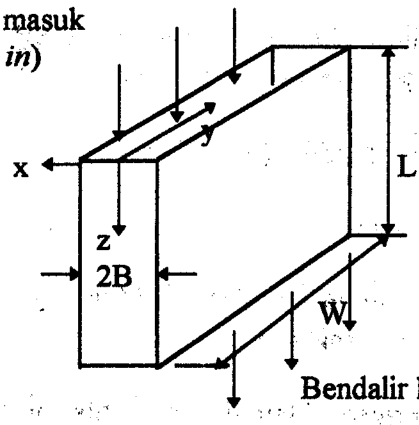
1. Satu bendalir pekat dalam aliran lamina dipisahkan melalui dua dinding yang selari para jarak 2B. Bergantung pada maklumat yang diberi dalam rajah Q1, buatkan satu kebezaan keseimbangan momentum dan dapatkan ungkapan-ungkapan untuk fluk momentum dan agihan-agihan halaju. Apakah nisbah purata halaju $< V_z >$ kepada halaju maksima $V_{z\,max}$ dalam lekah?

A viscous fluid is in laminar flow in a split formed by two parallel walls a distance 2B apart. Depending on the information given in figure Q1, make a differential momentum balance and obtain the expressions for the momentum flux and velocity distributions. What is the ratio of the average velocity $< V_z >$ *to the maximum velocity* $V_{z\,max}$ *in the slit?*

(25 markah)



Rajah Q1

2. [a] Berdasarkan keseimbangan haba pada petala, terbitkan persamaan kebezaan yang boleh memperihalkan agihan suhu jejari di dalam sirip anulus yang ditunjukkan dalam rajah Q2. Dengan menganggapkan sirip itu dalam keadaan mantap, perihalkan keadaan-keadaan sempadan yang diperlukan untuk menyelesaikan persamaan kebezaan itu. Anggap suhu pada permukaan tiub pada T_o.

Based on shell heat balance, derive the differential equation that describes the radial temperature distribution in the annular fin as shown in figure Q2. Assuming the fin is at steady-state, describe the boundary conditions required in solving the differential equation. Assume the tube surface temperature is T_o.

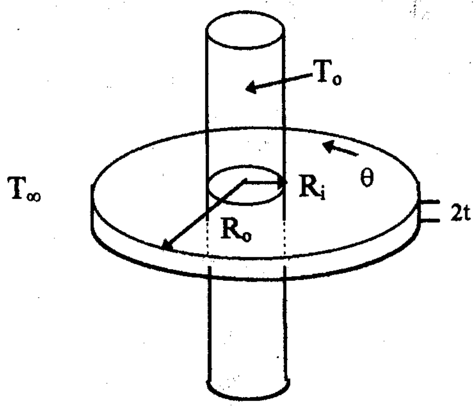
(12 markah)

...3/-

[b] Jika haba yang hilang melalui olakan daripada permukaan sirip itu tidak seragam dalam arah θ, kemudian terbitkan persamaan separa kebezaan untuk agihan suhu.

If the convection loss from the surface of the fin is not uniform in the θ - direction, then derive the partial diffential equation for the temperature distribution.

(13 markah)



Rajah Q2

3. Dalam mempelajari kadar pengurasan untuk bahan A daripada zarah-zarah pepejal oleh pelarut B, kadar langkah kawalan ialah resapan oleh A daripada permukaan zarah melalui filem cecair keluar ke dalam aliran cecair utama. Rajah Q3 menunjukkan susuk kepekatan untuk proses itu. Kebolehlarutan A dalam B ialah CA_o g.mol/sm^3 , dan kepekatan dalam aliran utama-selepas filem cecair itu dengan ketebalan δ ialah CA_δ

In studying the rate of leaching of a substance A from solid particles by a solvent B, the rate controlling step is the diffusion of A from the particle surface through a liquid film out into the main liquid stream. Fig. Q3 shows concentration profile of this process. The solubility of A in B is CA_o g.mole/cm^3, and the concentration in the main stream-beyond the liquid film of thickness δ is CA_δ .

[a] Dapatkan persamaan kebezaan untuk C_A sebagai satu fungsi dalam Z dengan membuat keseimbangan jisim pada A di atas papak nipis dengan ketebalan ΔZ. Anggap D_{AB} adalah malar dan A hanya larut sedikit dalam B. Abaikan kelengkungan pada zarah tersebut. Dan tunjukkan juga susuk kepekatan adalah lelurus, bila tiada tindakbalas berlaku.

...4/-

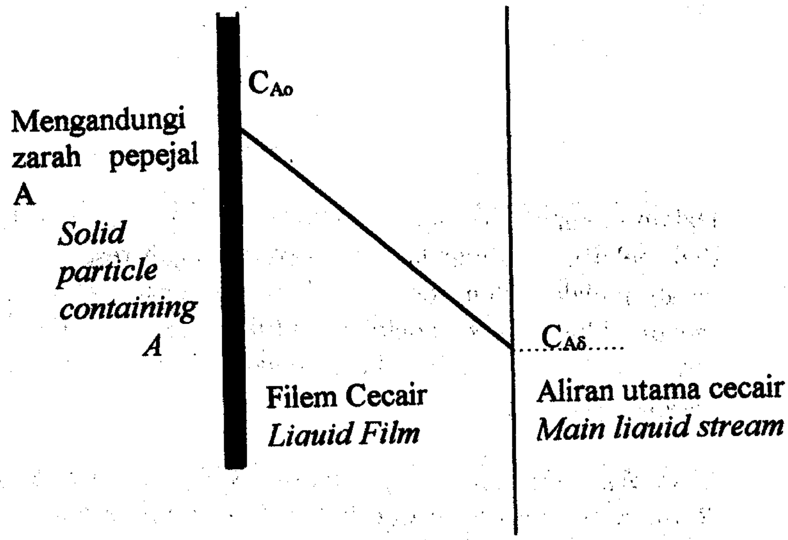
Obtain a differential equation for C_A as a function of Z by making a mass balance on A over a thin slab of thickness ΔZ. Assuming D_{AB} is constant and that A is only slighty soluble in B. Neglect the curvature of the particle. Also show that the concentration profile is linear; when no reaction is taking place.

[17 markah]

[b] Tunjukkan kadar pengurasan boleh diberi sebagai
Show that the rate of leaching is given by

$$N_A = \frac{D_{AB}(C_{Ao} - C_{A\delta})}{\delta}$$

[8 markah]



Rajah Q3.

4. Untuk aliran upaya dalam keadaan mantap sekeliling sebuah sfera dengan halaju tuju V_∞ (Rajah Q4), aliran fungsi dan halaju upaya adalah

For the steady potential flow around a sphere with approach velocity V_∞ (Fig. Q4) the stream function and velocity potential are

$$\Psi = \frac{V_\infty R^3}{2r}\sin^2\theta - \frac{V_\infty r^2}{2}\sin^2\theta$$

$$\phi = -\frac{V_\infty R^3}{2r^2}\cos\theta - V_\infty r\cos\theta$$

Aliran fungsi boleh dikaitkan dengan komponen-komponen halaju seperti yang ditunjukkan dalam carta yang dikepilkan, dan halaju upaya boleh dikaitkan dengan komponen-komponen oleh

The stream function is related to the velocity components as shown in the attached table, and the velocity potential is related to the velocity components by

$$V = -\nabla\phi$$

atau khas untuk koordinat-koordinat sfera ialah,
or specifically in spherical coordinates

$$V_r = -\frac{\partial\phi}{\partial r}$$

$$V_\theta = -\frac{1}{r}\frac{\partial\phi}{\partial\theta}$$

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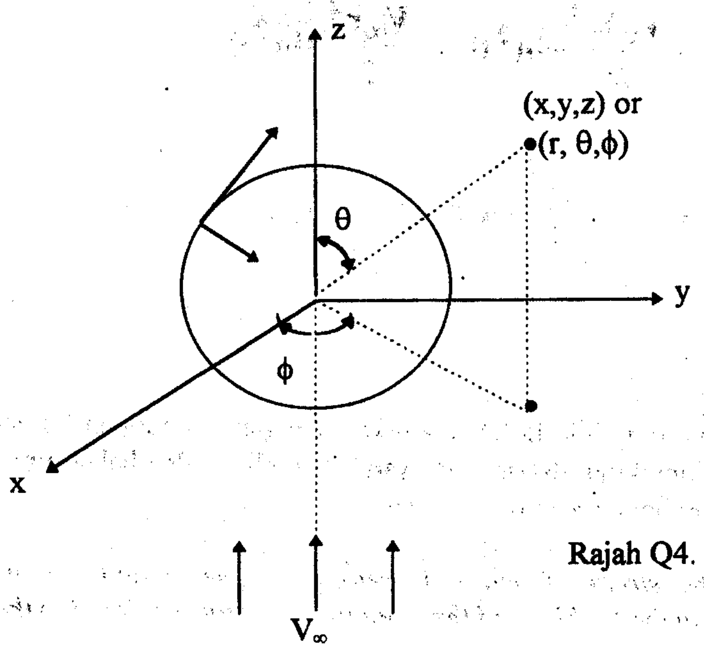

Rajah Q4.

[a] Dengan menggunakan persamaan aliran fungsi dan halaju upaya, tunjukkan bahawa halaju $V_Z = V_\infty$ pada sebarang titik jauh daripada sfera. (Petunjuk: $V_Z = V_r \text{ kos } \theta - V_\theta \sin \theta$).

Using the equation of stream function and the velocity potential, show that the velocity $V_z = V_\infty$ *at any point far from the sphere (Hint:* $V_z = V_r \cos \theta - V_\theta \sin \theta$*).*

(13 markah)

[b] Tunjukkan bahawa halaju pada sebarang titik pada permukaan sfera itu ialah:

Show that the velocity at any point on the surface of the sphere is

$$V_\theta = -\frac{2}{3} V_\infty \sin\theta$$

(12 markah)

5. Bahagian dalam dua buah silinder sepusat adalah dalam keadaan pegun (Rajah Q5), manakala bahagian luarnya berputar pada halaju sudut Ω radian/s. Bahagian dalam dan luar silinder-silinder itu ditetapkan pada suhu T_o dan T_b setiap satu dimana ($T_b > T_a$). Bendalir di antara dua silinder itu tidak boleh dimampatkan, Newtonan dan melalui pemanasan likat disebabkan putarannya.

The inner cylinder of two concentric cylinders is stationary (Figure Q5), while the outer one rotates at angular velocity Ω radian/s. The inner and outer cylinders are kept at the temperatures T_o and T_b respectively ($T_b > T_a$). The fluid between the two cylinders is incompressible, Newtonian and undergoes viscous heating due to its rotation.

[a] Anggap bahawa b adalah kecil jika dibandingkan dengan R, kemudian dua silinder itu boleh digambarkan sebagai dua permukaan rata, satu adalah tetap dan yang satu lagi bergerak keluar dengan halaju lelurus $V = R\Omega$, seperti yang ditunjukkan dalam lakaran. Gunakan persamaan-persamaan momentum dan tenaga daripada carta-carta yang dikepilkan untuk menganggarkan agihan halaju dan suhu dalam bendalir itu. Nyatakan sebarang anggapan yang telah anda buat untuk meringkaskan persamaan-persamaan itu. Apakah agihan suhu jika pemanasan likat diabaikan.

Assume that b is small compared to R, then the two cylinders may be visualized as two plate surfaces, one is fixed and the other is moving out linear velocity $V = R\Omega$, as shown in the sketch. Use the appropriate momentum and energy equations from the attached tables to evaluate the velocity and temperature distribution of the fluid. State any assumption you make in the simplification of the equations. What will be the temperature distribution if the viscous heating is ignored.

(17 markah)

[b] Tuliskan persamaan-persamaan momentum dan tenaga jika kelengkungan tidak boleh diabaikan. Anda tidak dikehendaki untuk menyelesaikan persamaan-persamaan itu.

Write down the momentum and the energy equations if the carvature cannot be ignored. You are not required to solve the equations.

(8 markah)

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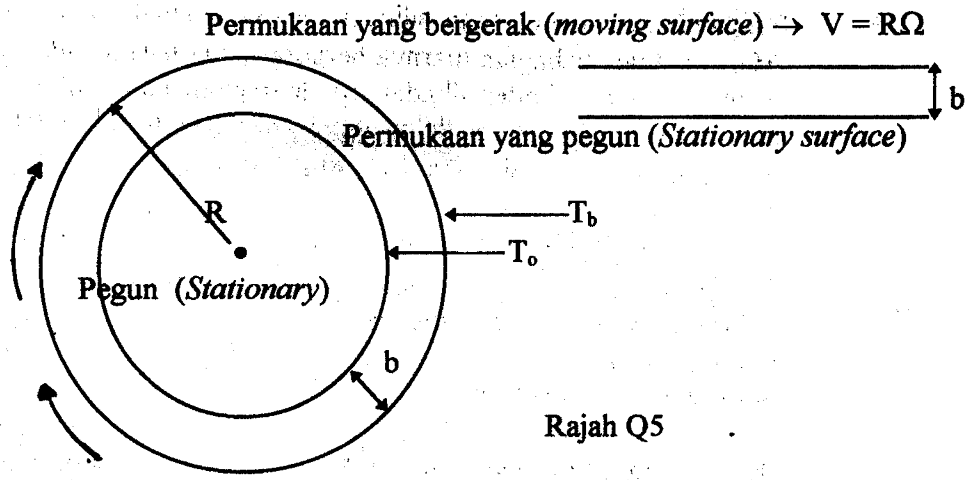

Rajah Q5

6. Gunakan persamaan-persamaan Navier-Stokes pada ketumpatan malar untuk mendapatkan persamaan-persamaan kebezaan untuk agihan halaju.

Use the Navier-Stokes equations for constant density to obtain the differential equations for velocity distribution.

[a] Untuk aliran isoterma filem dengan ketebalan δ dan mengalir dalam permukaan cerun dengan sudut β kepada permukaan mendatar.

For the flow of an isothermal film of thickness δ and flowing in an inclined surface with an angle β to the horizontal.

(13 markah)

[b] Bendalir yang sama mengalir dalam lekah mendatar seperti dalam soalan Q1.

The same fluid flowing in a horizontal slit as of problem Q1.

(12 markah)

Nyatakan keadaan-keadaan sempadan yang perlu dan gunakan persamaan-persamaan daripada carta yang dikepilkan.

State the necessary boundary conditions and use the equations from the attached tables.

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APPENDIX

THE EQUATION OF CONTINUITY IN SEVERAL COORDINATE SYSTEMS

Rectangular coordinates (x, y, z):

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho v_x) + \frac{\partial}{\partial y}(\rho v_y) + \frac{\partial}{\partial z}(\rho v_z) = 0 \qquad (A)$$

Cylindrical coordinates (r, θ, z):

$$\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(\rho r v_r) + \frac{1}{r}\frac{\partial}{\partial \theta}(\rho v_\theta) + \frac{\partial}{\partial z}(\rho v_z) = 0 \qquad (B)$$

Spherical coordinates (r, θ, ϕ):

$$\frac{\partial \rho}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}(\rho r^2 v_r) + \frac{1}{r \sin \theta}\frac{\partial}{\partial \theta}(\rho v_\theta \sin \theta) + \frac{1}{r \sin \theta}\frac{\partial}{\partial \phi}(\rho v_\phi) = 0 \qquad (C)$$

THE EQUATION OF MOTION IN RECTANGULAR COORDINATES (x, y, z)

In terms of τ:

x-component
$$\rho\left(\frac{\partial v_x}{\partial t} + v_x\frac{\partial v_x}{\partial x} + v_y\frac{\partial v_x}{\partial y} + v_z\frac{\partial v_x}{\partial z}\right) = -\frac{\partial p}{\partial x} - \left(\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z}\right) + \rho g_x \qquad (A)$$

y-component
$$\rho\left(\frac{\partial v_y}{\partial t} + v_x\frac{\partial v_y}{\partial x} + v_y\frac{\partial v_y}{\partial y} + v_z\frac{\partial v_y}{\partial z}\right) = -\frac{\partial p}{\partial y} - \left(\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} + \frac{\partial \tau_{zy}}{\partial z}\right) + \rho g_y \qquad (B)$$

z-component
$$\rho\left(\frac{\partial v_z}{\partial t} + v_x\frac{\partial v_z}{\partial x} + v_y\frac{\partial v_z}{\partial y} + v_z\frac{\partial v_z}{\partial z}\right) = -\frac{\partial p}{\partial z} - \left(\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \tau_{zz}}{\partial z}\right) + \rho g_z \qquad (C)$$

In terms of velocity gradients for a Newtonian fluid with constant ρ and μ:

x-component
$$\rho\left(\frac{\partial v_x}{\partial t} + v_x\frac{\partial v_x}{\partial x} + v_y\frac{\partial v_x}{\partial y} + v_z\frac{\partial v_x}{\partial z}\right) = -\frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial z^2}\right) + \rho g_x \qquad (D)$$

y-component
$$\rho\left(\frac{\partial v_y}{\partial t} + v_x\frac{\partial v_y}{\partial x} + v_y\frac{\partial v_y}{\partial y} + v_z\frac{\partial v_y}{\partial z}\right) = -\frac{\partial p}{\partial y} + \mu\left(\frac{\partial^2 v_y}{\partial x^2} + \frac{\partial^2 v_y}{\partial y^2} + \frac{\partial^2 v_y}{\partial z^2}\right) + \rho g_y \qquad (E)$$

z-component
$$\rho\left(\frac{\partial v_z}{\partial t} + v_x\frac{\partial v_z}{\partial x} + v_y\frac{\partial v_z}{\partial y} + v_z\frac{\partial v_z}{\partial z}\right) = -\frac{\partial p}{\partial z} + \mu\left(\frac{\partial^2 v_z}{\partial x^2} + \frac{\partial^2 v_z}{\partial y^2} + \frac{\partial^2 v_z}{\partial z^2}\right) + \rho g_z \qquad (F)$$

...3/-

THE EQUATION OF MOTION IN CYLINDRICAL COORDINATES (r, θ, z)

In terms of τ:

r-component[a]
$$\rho\left(\frac{\partial v_r}{\partial t} + v_r \frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} + v_z \frac{\partial v_r}{\partial z}\right) = -\frac{\partial p}{\partial r} - \left(\frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rr}) + \frac{1}{r}\frac{\partial \tau_{r\theta}}{\partial \theta} - \frac{\tau_{\theta\theta}}{r} + \frac{\partial \tau_{rz}}{\partial z}\right) + \rho g_r \quad (A)$$

θ-component[b]
$$\rho\left(\frac{\partial v_\theta}{\partial t} + v_r \frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r v_\theta}{r} + v_z \frac{\partial v_\theta}{\partial z}\right) = -\frac{1}{r}\frac{\partial p}{\partial \theta} - \left(\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\tau_{r\theta}) + \frac{1}{r}\frac{\partial \tau_{\theta\theta}}{\partial \theta} + \frac{\partial \tau_{\theta z}}{\partial z}\right) + \rho g_\theta \quad (B)$$

z-component
$$\rho\left(\frac{\partial v_z}{\partial t} + v_r \frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z \frac{\partial v_z}{\partial z}\right) = -\frac{\partial p}{\partial z} - \left(\frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rz}) + \frac{1}{r}\frac{\partial \tau_{\theta z}}{\partial \theta} + \frac{\partial \tau_{zz}}{\partial z}\right) + \rho g_z \quad (C)$$

In terms of velocity gradients for a Newtonian fluid with constant ρ and μ:

r-component[a]
$$\rho\left(\frac{\partial v_r}{\partial t} + v_r \frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} + v_z \frac{\partial v_r}{\partial z}\right) = -\frac{\partial p}{\partial r} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rv_r)\right) + \frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2} - \frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial^2 v_r}{\partial z^2}\right] + \rho g_r \quad (D)$$

θ-component[b]
$$\rho\left(\frac{\partial v_\theta}{\partial t} + v_r \frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r v_\theta}{r} + v_z \frac{\partial v_\theta}{\partial z}\right) = -\frac{1}{r}\frac{\partial p}{\partial \theta} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rv_\theta)\right) + \frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2} + \frac{2}{r^2}\frac{\partial v_r}{\partial \theta} + \frac{\partial^2 v_\theta}{\partial z^2}\right] + \rho g_\theta \quad (E)$$

z-component
$$\rho\left(\frac{\partial v_z}{\partial t} + v_r \frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z \frac{\partial v_z}{\partial z}\right) = -\frac{\partial p}{\partial z} + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial^2 v_z}{\partial z^2}\right] + \rho g_z \quad (F)$$

THE EQUATION OF MOTION IN SPHERICAL COORDINATES (r, θ, ϕ)

In terms of τ:

r-component $\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_r}{\partial \phi} - \frac{v_\theta^2 + v_\phi^2}{r}\right)$

$$= -\frac{\partial p}{\partial r} - \left(\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\tau_{rr}) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\tau_{r\theta}\sin\theta) + \frac{1}{r\sin\theta}\frac{\partial\tau_{r\phi}}{\partial\phi} - \frac{\tau_{\theta\theta}+\tau_{\phi\phi}}{r}\right) + \rho g_r \qquad (A)$$

θ-component $\rho\left(\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_\theta}{\partial \phi} + \frac{v_r v_\theta}{r} - \frac{v_\phi^2\cot\theta}{r}\right)$

$$= -\frac{1}{r}\frac{\partial p}{\partial \theta} - \left(\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\tau_{r\theta}) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\tau_{\theta\theta}\sin\theta) + \frac{1}{r\sin\theta}\frac{\partial\tau_{\theta\phi}}{\partial\phi} + \frac{\tau_{r\theta}}{r} - \frac{\cot\theta}{r}\tau_{\phi\phi}\right) + \rho g_\theta \qquad (B)$$

φ-component $\rho\left(\frac{\partial v_\phi}{\partial t} + v_r\frac{\partial v_\phi}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\phi}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi} + \frac{v_\phi v_r}{r} + \frac{v_\theta v_\phi}{r}\cot\theta\right)$

$$= -\frac{1}{r\sin\theta}\frac{\partial p}{\partial \phi} - \left(\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\tau_{r\phi}) + \frac{1}{r}\frac{\partial\tau_{\theta\phi}}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial\tau_{\phi\phi}}{\partial\phi} + \frac{\tau_{r\phi}}{r} + \frac{2\cot\theta}{r}\tau_{\theta\phi}\right) + \rho g_\phi \qquad (C)$$

In terms of velocity gradients for a Newtonian fluid with constant ρ and μ:

r-component
$$\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_r}{\partial \phi} - \frac{v_\theta^2 + v_\phi^2}{r}\right)$$
$$= -\frac{\partial p}{\partial r} + \mu\left(\frac{1}{r^2}\frac{\partial^2}{\partial r^2}(r^2 v_r) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial v_r}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 v_r}{\partial\phi^2}\right)$$
$$+ \rho g_r \qquad (D)$$

θ-component
$$\rho\left(\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_\theta}{\partial \phi} + \frac{v_r v_\theta}{r} - \frac{v_\phi^2\cot\theta}{r}\right)$$
$$= -\frac{1}{r}\frac{\partial p}{\partial \theta} + \mu\left(\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial v_\theta}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial\theta}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}(v_\theta\sin\theta)\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 v_\theta}{\partial\phi^2}\right.$$
$$\left. + \frac{2}{r^2}\frac{\partial v_r}{\partial\theta} - \frac{2\cos\theta}{r^2\sin^2\theta}\frac{\partial v_\phi}{\partial\phi}\right) + \rho g_\theta \qquad (E)$$

ϕ-component
$$\rho\left(\frac{\partial v_\phi}{\partial t} + v_r\frac{\partial v_\phi}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\phi}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi} + \frac{v_\phi v_r}{r} + \frac{v_\theta v_\phi}{r}\cot\theta\right)$$
$$= -\frac{1}{r\sin\theta}\frac{\partial p}{\partial \phi} + \mu\left(\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial v_\phi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial\theta}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}(v_\phi\sin\theta)\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 v_\phi}{\partial\phi^2}\right.$$
$$\left. + \frac{2}{r^2\sin\theta}\frac{\partial v_r}{\partial\phi} + \frac{2\cos\theta}{r^2\sin^2\theta}\frac{\partial v_\theta}{\partial\phi}\right) + \rho g_\phi \qquad (F)$$

COMPONENTS OF THE STRESS TENSOR FOR NEWTONIAN FLUIDS IN RECTANGULAR COORDINATES (x, y, z)

$$\tau_{xx} = -\mu\left[2\frac{\partial v_x}{\partial x} - \tfrac{2}{3}(\nabla \cdot v)\right] \qquad (A)$$

$$\tau_{yy} = -\mu\left[2\frac{\partial v_y}{\partial y} - \tfrac{2}{3}(\nabla \cdot v)\right] \qquad (B)$$

$$\tau_{zz} = -\mu\left[2\frac{\partial v_z}{\partial z} - \tfrac{2}{3}(\nabla \cdot v)\right] \qquad (C)$$

$$\tau_{xy} = \tau_{yx} = -\mu\left[\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x}\right] \qquad (D)$$

$$\tau_{yz} = \tau_{zy} = -\mu\left[\frac{\partial v_y}{\partial z} + \frac{\partial v_z}{\partial y}\right] \qquad (E)$$

$$\tau_{zx} = \tau_{xz} = -\mu\left[\frac{\partial v_z}{\partial x} + \frac{\partial v_x}{\partial z}\right] \qquad (F)$$

$$(\nabla \cdot v) = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} \qquad (G)$$

COMPONENTS OF THE STRESS TENSOR FOR NEWTONIAN FLUIDS IN CYLINDRICAL COORDINATES (r, θ, z)

$$\tau_{rr} = -\mu\left[2\frac{\partial v_r}{\partial r} - \tfrac{2}{3}(\nabla \cdot v)\right] \qquad (A)$$

$$\tau_{\theta\theta} = -\mu\left[2\left(\frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r}\right) - \tfrac{2}{3}(\nabla \cdot v)\right] \qquad (B)$$

$$\tau_{zz} = -\mu\left[2\frac{\partial v_z}{\partial z} - \tfrac{2}{3}(\nabla \cdot v)\right] \qquad (C)$$

$$\tau_{r\theta} = \tau_{\theta r} = -\mu\left[r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{1}{r}\frac{\partial v_r}{\partial \theta}\right] \qquad (D)$$

$$\tau_{\theta z} = \tau_{z\theta} = -\mu\left[\frac{\partial v_\theta}{\partial z} + \frac{1}{r}\frac{\partial v_z}{\partial \theta}\right] \qquad (E)$$

$$\tau_{zr} = \tau_{rz} = -\mu\left[\frac{\partial v_z}{\partial r} + \frac{\partial v_r}{\partial z}\right] \qquad (F)$$

$$(\nabla \cdot v) = \frac{1}{r}\frac{\partial}{\partial r}(r v_r) + \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial v_z}{\partial z} \qquad (G)$$

COMPONENTS OF THE STRESS TENSOR FOR NEWTONIAN FLUIDS IN SPHERICAL COORDINATES (r, θ, ϕ)

$$\tau_{rr} = -\mu\left[2\frac{\partial v_r}{\partial r} - \tfrac{2}{3}(\nabla \cdot v)\right] \qquad (A)$$

$$\tau_{\theta\theta} = -\mu\left[2\left(\frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r}\right) - \tfrac{2}{3}(\nabla \cdot v)\right] \qquad (B)$$

$$\tau_{\phi\phi} = -\mu\left[2\left(\frac{1}{r \sin\theta}\frac{\partial v_\phi}{\partial \phi} + \frac{v_r}{r} + \frac{v_\theta \cot\theta}{r}\right) - \tfrac{2}{3}(\nabla \cdot v)\right] \qquad (C)$$

$$\tau_{r\theta} = \tau_{\theta r} = -\mu\left[r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{1}{r}\frac{\partial v_r}{\partial \theta}\right] \qquad (D)$$

$$\tau_{\theta\phi} = \tau_{\phi\theta} = -\mu\left[\frac{\sin\theta}{r}\frac{\partial}{\partial\theta}\left(\frac{v_\phi}{\sin\theta}\right) + \frac{1}{r\sin\theta}\frac{\partial v_\theta}{\partial\phi}\right] \qquad (E)$$

$$\tau_{\phi r} = \tau_{r\phi} = -\mu\left[\frac{1}{r\sin\theta}\frac{\partial v_r}{\partial\phi} + r\frac{\partial}{\partial r}\left(\frac{v_\phi}{r}\right)\right] \qquad (F)$$

$$(\nabla \cdot v) = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(v_\theta \sin\theta) + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial\phi} \qquad (G)$$

THE FUNCTION $-(\tau : \nabla v) = \mu\Phi_v$ FOR NEWTONIAN FLUIDS

Rectangular

$$\Phi_v = 2\left[\left(\frac{\partial v_x}{\partial x}\right)^2 + \left(\frac{\partial v_y}{\partial y}\right)^2 + \left(\frac{\partial v_z}{\partial z}\right)^2\right]$$
$$+ \left[\frac{\partial v_y}{\partial x} + \frac{\partial v_x}{\partial y}\right]^2 + \left[\frac{\partial v_z}{\partial y} + \frac{\partial v_y}{\partial z}\right]^2 + \left[\frac{\partial v_x}{\partial z} + \frac{\partial v_z}{\partial x}\right]^2$$
$$- \frac{2}{3}\left[\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}\right]^2 \qquad (A)$$

Cylindrical

$$\Phi_v = 2\left[\left(\frac{\partial v_r}{\partial r}\right)^2 + \left(\frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r}\right)^2 + \left(\frac{\partial v_z}{\partial z}\right)^2\right]$$
$$+ \left[r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{1}{r}\frac{\partial v_r}{\partial \theta}\right]^2 + \left[\frac{1}{r}\frac{\partial v_z}{\partial \theta} + \frac{\partial v_\theta}{\partial z}\right]^2$$
$$+ \left[\frac{\partial v_r}{\partial z} + \frac{\partial v_z}{\partial r}\right]^2$$
$$- \frac{2}{3}\left[\frac{1}{r}\frac{\partial}{\partial r}(rv_r) + \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial v_z}{\partial z}\right]^2 \qquad (B)$$

Spherical

$$\Phi_v = 2\left[\left(\frac{\partial v_r}{\partial r}\right)^2 + \left(\frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r}\right)^2\right.$$
$$\left. + \left(\frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi} + \frac{v_r}{r} + \frac{v_\theta \cot\theta}{r}\right)^2\right]$$
$$+ \left[r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{1}{r}\frac{\partial v_r}{\partial \theta}\right]^2$$
$$+ \left[\frac{\sin\theta}{r}\frac{\partial}{\partial \theta}\left(\frac{v_\phi}{\sin\theta}\right) + \frac{1}{r\sin\theta}\frac{\partial v_\theta}{\partial \phi}\right]^2$$
$$+ \left[\frac{1}{r\sin\theta}\frac{\partial v_r}{\partial \phi} + r\frac{\partial}{\partial r}\left(\frac{v_\phi}{r}\right)\right]^2$$
$$- \frac{2}{3}\left[\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(v_\theta \sin\theta) + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi}\right]^2 \qquad (C)$$

EQUATIONS FOR THE STREAM FUNCTION[a]

Type of Motion	Coordinate System	Velocity Components	Differential Equations for ψ Which Are Equivalent to the Navier-Stokes Equation[b]	Expressions for Operators
Two-dimensional (Planar)	Rectangular with $v_z = 0$ and no z-dependence	$v_x = -\dfrac{\partial\psi}{\partial y}$ $v_y = +\dfrac{\partial\psi}{\partial x}$	$\dfrac{\partial}{\partial t}(\nabla^2\psi) + \dfrac{\partial(\psi, \nabla^2\psi)}{\partial(x, y)} = \nu\nabla^4\psi$ (*A*)	$\nabla^2 \equiv \dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2}$ $\nabla^4\psi \equiv \nabla^2(\nabla^2\psi)$ $\equiv \left(\dfrac{\partial^4}{\partial x^4} + 2\dfrac{\partial^4}{\partial x^2\,\partial y^2} + \dfrac{\partial^4}{\partial y^4}\right)\psi$
	Cylindrical with $v_z = 0$ and no z-dependence	$v_r = -\dfrac{1}{r}\dfrac{\partial\psi}{\partial\theta}$ $v_\theta = +\dfrac{\partial\psi}{\partial r}$	$\dfrac{\partial}{\partial t}(\nabla^2\psi) + \dfrac{1}{r}\dfrac{\partial(\psi, \nabla^2\psi)}{\partial(r, \theta)} = \nu\nabla^4\psi$ (*B*)	$\nabla^2 \equiv \dfrac{\partial^2}{\partial r^2} + \dfrac{1}{r}\dfrac{\partial}{\partial r} + \dfrac{1}{r^2}\dfrac{\partial^2}{\partial\theta^2}$
Axisymmetrical	Cylindrical with $v_\theta = 0$ and no θ-dependence	$v_z = -\dfrac{1}{r}\dfrac{\partial\psi}{\partial r}$ $v_r = +\dfrac{1}{r}\dfrac{\partial\psi}{\partial z}$	$\dfrac{\partial}{\partial t}(E^2\psi) - \dfrac{1}{r}\dfrac{\partial(\psi, E^2\psi)}{\partial(r, z)} - \dfrac{2}{r^2}\dfrac{\partial\psi}{\partial z}E^2\psi = \nu E^4\psi$ (*C*)	$E^2 \equiv \dfrac{\partial^2}{\partial r^2} - \dfrac{1}{r}\dfrac{\partial}{\partial r} + \dfrac{\partial^2}{\partial z^2}$ $E^4\psi \equiv E^2(E^2\psi)$
	Spherical with $v_\phi = 0$ and no ϕ-dependence	$v_r = -\dfrac{1}{r^2\sin\theta}\dfrac{\partial\psi}{\partial\theta}$ $v_\theta = +\dfrac{1}{r\sin\theta}\dfrac{\partial\psi}{\partial r}$	$\dfrac{\partial}{\partial t}(E^2\psi) + \dfrac{1}{r^2\sin\theta}\dfrac{\partial(\psi, E^2\psi)}{\partial(r, \theta)}$ $- \dfrac{2E^2\psi}{r^2\sin^2\theta}\left(\dfrac{\partial\psi}{\partial r}\cos\theta - \dfrac{1}{r}\dfrac{\partial\psi}{\partial\theta}\sin\theta\right) = \nu E^4\psi$ (*D*)	$E^2 \equiv \dfrac{\partial^2}{\partial r^2} + \dfrac{\sin\theta}{r^2}\dfrac{\partial}{\partial\theta}\left(\dfrac{1}{\sin\theta}\dfrac{\partial}{\partial\theta}\right)$

COMPONENTS OF THE ENERGY FLUX q

Rectangular		Cylindrical		Spherical	
$q_x = -k \frac{\partial T}{\partial x}$	(A)	$q_r = -k \frac{\partial T}{\partial r}$	(D)	$q_r = -k \frac{\partial T}{\partial r}$	(G)
$q_y = -k \frac{\partial T}{\partial y}$	(B)	$q_\theta = -k \frac{1}{r} \frac{\partial T}{\partial \theta}$	(E)	$q_\theta = -k \frac{1}{r} \frac{\partial T}{\partial \theta}$	(H)
$q_z = -k \frac{\partial T}{\partial z}$	(C)	$q_z = -k \frac{\partial T}{\partial z}$	(F)	$q_\phi = -k \frac{1}{r \sin \theta} \frac{\partial T}{\partial \phi}$	(I)

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THE EQUATION OF ENERGY IN TERMS OF ENERGY AND MOMENTUM FLUXES (Eq. 10.1–19)

Rectangular coordinates:

$$\rho \hat{C}_v \left(\frac{\partial T}{\partial t} + v_x \frac{\partial T}{\partial x} + v_y \frac{\partial T}{\partial y} + v_z \frac{\partial T}{\partial z} \right) = -\left[\frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} + \frac{\partial q_z}{\partial z} \right]$$

$$- T \left(\frac{\partial p}{\partial T} \right)_\rho \left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} \right) - \left\{ \tau_{xx} \frac{\partial v_x}{\partial x} + \tau_{yy} \frac{\partial v_y}{\partial y} + \tau_{zz} \frac{\partial v_z}{\partial z} \right\}$$

$$- \left\{ \tau_{xy} \left(\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x} \right) + \tau_{xz} \left(\frac{\partial v_x}{\partial z} + \frac{\partial v_z}{\partial x} \right) + \tau_{yz} \left(\frac{\partial v_y}{\partial z} + \frac{\partial v_z}{\partial y} \right) \right\} \quad (A)$$

Cylindrical coordinates:

$$\rho \hat{C}_v \left(\frac{\partial T}{\partial t} + v_r \frac{\partial T}{\partial r} + \frac{v_\theta}{r} \frac{\partial T}{\partial \theta} + v_z \frac{\partial T}{\partial z} \right) = -\left[\frac{1}{r} \frac{\partial}{\partial r} (r q_r) + \frac{1}{r} \frac{\partial q_\theta}{\partial \theta} + \frac{\partial q_z}{\partial z} \right]$$

$$- T \left(\frac{\partial p}{\partial T} \right)_\rho \left(\frac{1}{r} \frac{\partial}{\partial r} (r v_r) + \frac{1}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{\partial v_z}{\partial z} \right) - \left\{ \tau_{rr} \frac{\partial v_r}{\partial r} + \tau_{\theta\theta} \frac{1}{r} \left(\frac{\partial v_\theta}{\partial \theta} + v_r \right) \right.$$

$$\left. + \tau_{zz} \frac{\partial v_z}{\partial z} \right\} - \left\{ \tau_{r\theta} \left[r \frac{\partial}{\partial r} \left(\frac{v_\theta}{r} \right) + \frac{1}{r} \frac{\partial v_r}{\partial \theta} \right] + \tau_{rz} \left(\frac{\partial v_z}{\partial r} + \frac{\partial v_r}{\partial z} \right) \right.$$

$$\left. + \tau_{\theta z} \left(\frac{1}{r} \frac{\partial v_z}{\partial \theta} + \frac{\partial v_\theta}{\partial z} \right) \right\} \quad (B)$$

Spherical coordinates:

$$\rho \hat{C}_v \left(\frac{\partial T}{\partial t} + v_r \frac{\partial T}{\partial r} + \frac{v_\theta}{r} \frac{\partial T}{\partial \theta} + \frac{v_\phi}{r \sin \theta} \frac{\partial T}{\partial \phi} \right) = -\left[\frac{1}{r^2} \frac{\partial}{\partial r} (r^2 q_r) \right.$$

$$\left. + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (q_\theta \sin \theta) + \frac{1}{r \sin \theta} \frac{\partial q_\phi}{\partial \phi} \right] - T \left(\frac{\partial p}{\partial T} \right)_\rho \left(\frac{1}{r^2} \frac{\partial}{\partial r} (r^2 v_r) \right.$$

$$\left. + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (v_\theta \sin \theta) + \frac{1}{r \sin \theta} \frac{\partial v_\phi}{\partial \phi} \right) - \left\{ \tau_{rr} \frac{\partial v_r}{\partial r} + \tau_{\theta\theta} \left(\frac{1}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r} \right) \right.$$

$$\left. + \tau_{\phi\phi} \left(\frac{1}{r \sin \theta} \frac{\partial v_\phi}{\partial \phi} + \frac{v_r}{r} + \frac{v_\theta \cot \theta}{r} \right) \right\} - \left\{ \tau_{r\theta} \left(\frac{\partial v_\theta}{\partial r} + \frac{1}{r} \frac{\partial v_r}{\partial \theta} - \frac{v_\theta}{r} \right) \right.$$

$$\left. + \tau_{r\phi} \left(\frac{\partial v_\phi}{\partial r} + \frac{1}{r \sin \theta} \frac{\partial v_r}{\partial \phi} - \frac{v_\phi}{r} \right) + \tau_{\theta\phi} \left(\frac{1}{r} \frac{\partial v_\phi}{\partial \theta} + \frac{1}{r \sin \theta} \frac{\partial v_\theta}{\partial \phi} - \frac{\cot \theta}{r} v_\phi \right) \right\}$$

$$(C)$$

Note: The terms contained in braces { } are associated with viscous dissipation and may usually be neglected, except for systems with large velocity gradients.

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THE EQUATION OF ENERGY IN TERMS OF THE TRANSPORT PROPERTIES

(for Newtonian fluids of constant ρ and k)

(Eq. 10.1–25 with viscous dissipation terms included)

Rectangular coordinates:

$$\rho\hat{C}_p\left(\frac{\partial T}{\partial t} + v_x\frac{\partial T}{\partial x} + v_y\frac{\partial T}{\partial y} + v_z\frac{\partial T}{\partial z}\right) = k\left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right]$$
$$+ 2\mu\left\{\left(\frac{\partial v_x}{\partial x}\right)^2 + \left(\frac{\partial v_y}{\partial y}\right)^2 + \left(\frac{\partial v_z}{\partial z}\right)^2\right\} + \mu\left\{\left(\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x}\right)^2\right.$$
$$\left.+ \left(\frac{\partial v_x}{\partial z} + \frac{\partial v_z}{\partial x}\right)^2 + \left(\frac{\partial v_y}{\partial z} + \frac{\partial v_z}{\partial y}\right)^2\right\} \qquad (A)$$

Cylindrical coordinates:

$$\rho\hat{C}_p\left(\frac{\partial T}{\partial t} + v_r\frac{\partial T}{\partial r} + \frac{v_\theta}{r}\frac{\partial T}{\partial \theta} + v_z\frac{\partial T}{\partial z}\right) = k\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2}\right]$$
$$+ 2\mu\left\{\left(\frac{\partial v_r}{\partial r}\right)^2 + \left[\frac{1}{r}\left(\frac{\partial v_\theta}{\partial \theta} + v_r\right)\right]^2 + \left(\frac{\partial v_z}{\partial z}\right)^2\right\} + \mu\left\{\left(\frac{\partial v_\theta}{\partial z} + \frac{1}{r}\frac{\partial v_z}{\partial \theta}\right)^2\right.$$
$$\left.+ \left(\frac{\partial v_z}{\partial r} + \frac{\partial v_r}{\partial z}\right)^2 + \left[\frac{1}{r}\frac{\partial v_r}{\partial \theta} + r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right)\right]^2\right\} \qquad (B)$$

Spherical coordinates:

$$\rho\hat{C}_p\left(\frac{\partial T}{\partial t} + v_r\frac{\partial T}{\partial r} + \frac{v_\theta}{r}\frac{\partial T}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial T}{\partial \phi}\right) = k\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right)\right.$$
$$\left.+ \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial T}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 T}{\partial \phi^2}\right] + 2\mu\left\{\left(\frac{\partial v_r}{\partial r}\right)^2\right.$$
$$\left.+ \left(\frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r}\right)^2 + \left(\frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi} + \frac{v_r}{r} + \frac{v_\theta\cot\theta}{r}\right)^2\right\}$$
$$+ \mu\left\{\left[r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{1}{r}\frac{\partial v_r}{\partial \theta}\right]^2 + \left[\frac{1}{r\sin\theta}\frac{\partial v_r}{\partial \phi} + r\frac{\partial}{\partial r}\left(\frac{v_\phi}{r}\right)\right]^2\right.$$
$$\left.+ \left[\frac{\sin\theta}{r}\frac{\partial}{\partial \theta}\left(\frac{v_\phi}{\sin\theta}\right) + \frac{1}{r\sin\theta}\frac{\partial v_\theta}{\partial \phi}\right]^2\right\} \qquad (C)$$

Note: The terms contained in braces { } are associated with viscous dissipation and may usually be neglected, except for systems with large velocity gradients.

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EQUATIONS OF CHANGE FOR PURE FLUIDS IN TERMS OF THE FLUXES

Eq.	Special Form	In Terms of D/Dt		Comments
Cont.	—	$\frac{D\rho}{Dt} = -\rho(\nabla \cdot v)$	3.1–6 (*A*)	For $D\rho/Dt = 0$ simplifies to $(\nabla \cdot v) = 0$
Motion	Forced convection	$\rho \frac{Dv}{Dt} = -\nabla p - [\nabla \cdot \tau] + \rho g$	3.2–10 (*B*)	For $\tau = 0$ this becomes Euler's equation
	Free convection	$\rho \frac{Dv}{Dt} = -[\nabla \cdot \tau] - \rho\beta g(T - \bar{T})$	10.3–3 (*C*)	Approximate
Energy	In terms of $\hat{E} = \hat{U} + \hat{K} + \hat{\Phi}$	$\rho \frac{D\hat{E}}{Dt} = -(\nabla \cdot q) - (\nabla \cdot pv) - (\nabla \cdot [\tau \cdot v])$	10.1–15 (*D*)	Exact only for $\hat{\Phi}$ time independent
	In terms of $\hat{U} + \hat{K}$	$\rho \frac{D(\hat{U} + \hat{K})}{Dt} = -(\nabla \cdot q) - (\nabla \cdot pv) - (\nabla \cdot [\tau \cdot v]) + \rho(v \cdot g)$	10.1–11 (*E*)	
	In terms of $\hat{K} = \frac{1}{2}v^2$	$\rho \frac{D\hat{K}}{Dt} = -(v \cdot \nabla p) - (v \cdot [\nabla \cdot \tau]) + \rho(v \cdot g)$	3.3–1 (*F*)	
	In terms of $\hat{U}$	$\rho \frac{D\hat{U}}{Dt} = -(\nabla \cdot q) - p(\nabla \cdot v) - (\tau : \nabla v)$	10.1–13 (*G*)	Term containing p is zero for $D\rho/Dt = 0$
	In terms of $\hat{H}$	$\rho \frac{D\hat{H}}{Dt} = -(\nabla \cdot q) - (\tau : \nabla v) + \frac{Dp}{Dt}$	(*H*)	
	In terms of $\hat{C}_v$ and T	$\rho\hat{C}_v \frac{DT}{Dt} = -(\nabla \cdot q) - T\left(\frac{\partial p}{\partial T}\right)_\rho (\nabla \cdot v) - (\tau : \nabla v)$	10.1–19 (*I*)	For an ideal gas $T(\partial p/\partial T)_\rho = p$
	In terms of $\hat{C}_p$ and T	$\rho\hat{C}_p \frac{DT}{Dt} = -(\nabla \cdot q) + \left(\frac{\partial \ln \hat{V}}{\partial \ln T}\right)_p \frac{Dp}{Dt} - (\tau : \nabla v)$	(*J*)	For an ideal gas $(\partial \ln \hat{V}/\partial \ln T)_p = 1$

(*Continued*)

Eq.	Special Form	In Terms of $\partial/\partial t$	
Cont.	—	$\frac{\partial}{\partial t}\rho = -(\nabla \cdot \rho \boldsymbol{v})$	3.1-4 (*K*)
Motion	Forced convection	$\frac{\partial}{\partial t}\rho \boldsymbol{v} = -[\nabla \cdot \rho \boldsymbol{v}\boldsymbol{v}] - \nabla p - [\nabla \cdot \boldsymbol{\tau}] + \rho \boldsymbol{g}$	3.2-8 (*L*)
	Free convection	$\frac{\partial}{\partial t}\rho \boldsymbol{v} = -[\nabla \cdot \rho \boldsymbol{v}\boldsymbol{v}] - [\nabla \cdot \boldsymbol{\tau}] - \rho\beta \boldsymbol{g}(T - \bar{T})$ (Approximate)	(*M*)
Energy	In terms of $\hat{E} = \hat{U} + \hat{K} + \hat{\Phi}$	$\frac{\partial}{\partial t}\rho \hat{E} = -(\nabla \cdot \rho \hat{E}\boldsymbol{v}) - (\nabla \cdot \boldsymbol{q}) - (\nabla \cdot p\boldsymbol{v}) - (\nabla \cdot [\boldsymbol{\tau} \cdot \boldsymbol{v}])$	(*N*)
	In terms of $\hat{U} + \hat{K}$	$\frac{\partial}{\partial t}\rho(\hat{U} + \hat{K}) = -(\nabla \cdot \rho(\hat{U} + \hat{K})\boldsymbol{v}) - (\nabla \cdot \boldsymbol{q}) - (\nabla \cdot p\boldsymbol{v}) - (\nabla \cdot [\boldsymbol{\tau} \cdot \boldsymbol{v}]) + \rho(\boldsymbol{v} \cdot \boldsymbol{g})$	10.1-9 (*O*)
	In terms of $\hat{K} = \frac{1}{2}v^2$	$\frac{\partial}{\partial t}\rho \hat{K} = -(\nabla \cdot \rho \hat{K}\boldsymbol{v}) - (\boldsymbol{v} \cdot \nabla p) - (\boldsymbol{v} \cdot [\nabla \cdot \boldsymbol{\tau}]) + \rho(\boldsymbol{v} \cdot \boldsymbol{g})$	3.3-2 (*P*)
	In terms of $\hat{U}$	$\frac{\partial}{\partial t}\rho \hat{U} = -(\nabla \cdot \rho \hat{U}\boldsymbol{v}) - (\nabla \cdot \boldsymbol{q}) - p(\nabla \cdot \boldsymbol{v}) - (\boldsymbol{\tau} : \nabla \boldsymbol{v})$	(*Q*)
	In terms of $\hat{H}$	$\frac{\partial}{\partial t}\rho \hat{H} = -(\nabla \cdot \rho \hat{H}\boldsymbol{v}) - (\nabla \cdot \boldsymbol{q}) - (\boldsymbol{\tau} : \nabla \boldsymbol{v}) + \frac{Dp}{Dt}$	(*R*)
	In terms of $\hat{C}_v$ and T	$\frac{\partial}{\partial t}\rho \hat{C}_v T = -(\nabla \cdot \rho \hat{C}_v T\boldsymbol{v}) - (\nabla \cdot \boldsymbol{q}) - T\left(\frac{\partial p}{\partial T}\right)_\rho(\nabla \cdot \boldsymbol{v}) - (\boldsymbol{\tau} : \nabla \boldsymbol{v}) + \rho T \frac{D\hat{C}_v}{Dt}$	(*S*)
	In terms of $\hat{C}_p$ and T	$\frac{\partial}{\partial t}\rho \hat{C}_p T = -(\nabla \cdot \rho \hat{C}_p T\boldsymbol{v}) - (\nabla \cdot \boldsymbol{q}) - (\boldsymbol{\tau} : \nabla \boldsymbol{v}) + \left(\frac{\partial \ln \hat{V}}{\partial \ln T}\right)_p \frac{Dp}{Dt} + \rho T \frac{D\hat{C}_p}{Dt}$	(*T*)

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EQUATIONS OF CHANGE FOR PURE FLUIDS OF CONSTANT ρ, μ, AND k

Eq.	Special Form	Equation in Symbolic Form		Coordinate System: Rect.	Cyl.	Sph.
Cont.	—	$(\nabla \cdot v) = 0$	(A)	Table 3.4-1 Eq. A	Table 3.4-1 Eq. B	Table 3.4-1 Eq. C
Motion	Forced convection	$\rho \frac{Dv}{Dt} = -\nabla p + \mu \nabla^2 v + \rho g$	(B)	Table 3.4-2 Eqs. D, E, F	Table 3.4-3 Eqs. D, E, F	Table 3.4-4 Eqs. D, E, F
	Free convection	$\rho \frac{Dv}{Dt} = \mu \nabla^2 v - \rho \beta g (T - \bar{T})$	(C)	—	—	—
Energy	In terms of $\hat{U}$	$\rho \frac{D\hat{U}}{Dt} = k \nabla^2 T + \mu \Phi_v$	(D)	—	—	—
	In terms of $\hat{C}_p$ and T	$\rho \hat{C}_p \frac{DT}{Dt} = k \nabla^2 T + \mu \Phi_v$	(E)	Table 10.2-3 Eq. A	Table 10.2-3 Eq. B	Table 10.2-3 Eq. C