UNIVERSITI SAINS MALAYSIA

First Semester Examination
Academic Session 2004/2005

October 2004

CPT102 – Discrete Structures

Duration : 3 hours

INSTRUCTIONS TO CANDIDATE:

- Please ensure that this examination paper contains **FOUR** questions in **SIX** printed pages before you start the examination.
- Answer **ALL** questions.
- This is an "Open Book" Examination.
- You can choose to answer either in Bahasa Malaysia or English.

ENGLISH VERSION OF THE QUESTION PAPER

1. (a) Given is a sequence on binary numbers: 11_2, 1111_2, 111111_2, 11111111_2, 1111111111_2, ...

   (i) Change the given sequence to decimal representation (write only the first **five (5)** terms).

   (10/100)

   (ii) Based on answer in (i), find J_n where J_n is the implicit formula for the given sequence.

   (10/100)

   (iii) Based on answer in (ii), use substitution method, to find S_n where S_n is the explicit formula for the sequence.

   (10/100)

   (iv) Is $3 \mid S_n$ for $n \in \mathbb{Z}^+$? If true, prove it using mathematical induction.

   (10/100)

   (b) In a football league there are 4 clubs (club A, club B, club C, club D) competing.

   (i) If each club competing has 20 players (4 strikers, 8 midfielders, 8 defenders), how many ways are there to choose a national team of 15 players from the clubs if the 15 players consist of 4 strikers, 5 midfielders and 6 defenders.

   (10/100)

   (ii) A club has brought 10 balls to a field. The club buys balls from only 3 well-know ball manufacturers. How many combinations of balls are there that can be brought to the field.

   (10/100)

   (iii) Between club A and club B, club A has twice the chance to be a winner. Between club B and club C, club B has three times chance to be a winner. Club C and club D has the same chance to be a winner. What are the chances for each club to be a winner?

   (10/100)

(c) Mathematical Structure S = *(Integer matrices size* 1×2, ∇*)*, where

$$[\,x \quad y\,]\ \nabla\ [\,w \quad z\,] = [\,x+w \quad (y+z)/2\,]$$

(i) Show that S is closed

(10/100)

(ii) Based on S, shows that ∇ is commutative.

(10/100)

(iii) Based on S, shows that ∇ is not associative.

(10/100)

2. (a) In an examination schedule, there are only two types of exam, exam for graduate students and exam for undergraduate students. Only one exam will be conducted in a day. All undergraduate exams are not allowed to be conducted in two or more days in row. If there are n exam days,

(i) Find the implicit formula for the sequence that shows how many schedules that can be generated?

(5/100)

(ii) Write a recursive pseudocode based on answer in 2(a)(i).

(20/100)

(iii) Write an iterative (loop) pseudocode based on answer in 2(a)(i).

(20/100)

(b) Given the following function:

```
Function Goo(a,b)
a,b: integer.

Begin
   If (a = 0) then
      return (b)
   If (b = 0) then
      return (a)
   If (a < b)
         Goo(a, b mod a)
      else
         Goo(a mod b, b)
End
```

(i) Trace the given pseudocode with `Goo(15,3)` and `Goo(14,5)`.
(10/100)

(ii) What is the task of the given function?
(5/100)

(iii) Rewrite the pseudocode using loop.
(20/100)

(c) Given the *NAND* operation as follows:

p	***q***	***p NAND q***
0	0	1
0	1	1
1	0	1
1	1	0

(i) Show that $\neg p \Leftrightarrow p$ *NAND* q.
(5/100)

(i) Show that $p \vee q \Leftrightarrow (p$ *NAND* $p)$ *NAND* $(q$ *NAND* $q)$.
(5/100)

(iii) Using only *NAND* find the equivalent proposition to $p \wedge q$.
(10/100)

3. (a) If aRb is a relation of congruent modulo n, $a \equiv b \ (mod\ n)$. Show that R is:

(i) reflexive.
(10/100)

(ii) symmetric.
(10/100)

(iii) transitive.
(10/100)

(b) A is a set and $|A| = 8$. R is a relation on A, $R \subseteq A \times A$.

(i) How many different R can be produced?

(10/100)

(ii) How many R are reflexive?

(10/100)

(iii) How many R are symmetric?

(10/100)

(iv) How many R are reflexive and symmetric?

(10/100)

(c) A computer application consists of 9 modules. The given table shows the relation on the modules with time required to produce the modules.

Module	**Should be done after this module(s)**	**Time required (week)**
1	-	5
2	1	4
3	1	1
4	2	4
5	2,3	3
6	4	1
7	2,3	3
8	4,5	2
9	6,7,8	5

(i) Draw the Hasse diagram for this project.

(10/100)

(ii) How long it takes to complete the project? (Assume there is no constraint on human resources but each module should be done by one person).

(10/100)

(iii) Draw the matrix representation of the relation represented by the Hasse diagram in 3(c)(i).

(10/100)

4. (a) Based on question 2 above.

(i) Draw the simplest finite state machine which accepts only the defined schedule.

(15/100)

(ii) Write the simplest Phase Structured Grammar based on answer in 4(a)(i).

(15/100)

(b) For each of the following, draw the respective tree or explain why the tree cannot be produced.

(i) Complete binary tree with 5 internal nodes.

(5/100)

(ii) Complete binary tree with 5 internal nodes and 7 leaves.

(5/100)

(iii) Complete binary tree with 9 nodes.

(5/100)

(iv) Complete binary tree with height 3 and having 7 leaves.

(5/100)

(v) A binary tree with height 3 and 7 leaves.

(5/100)

(vi) Complete binary tree with height 3 and 6 leaves.

(5/100)

(c) For each of the given function *(f, g* and *h)*, determine if the function is 1-to-1. If the function is 1-to-1 then find its inverse, otherwise name the function type.

(i) $f = \{ (a,b) \mid b=a^{100}, a \in \mathbb{N}, b \in \mathbb{N} \}$

(10/100)

(ii) $g = \{ (a,b) \mid b=2^{a}, a \in \mathbb{N}, b \in \mathbb{N} \}$

(10/100)

(iii) $h = \{(a,b) \mid a \geq 5, b = a+3, a \in \mathbb{N}, b \in \mathbb{N} \}$

(10/100)

(iv) Show that $f(n) = O(g)$ and $g(n) \neq O(f)$

(10/100)