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UNIVERSITI SAINS MALAYSIA

Second Semester Examination  
Academic Session 2005/2006

April/May 2006

**MAA 101E – Calculus for Science Students I**  
**[Kalkulus Untuk Pelajar Sains I]**

Duration : 3 hours  
[Masa : 3 jam]

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Please check that this examination paper consists of EIGHT pages of printed material before you begin the examination.

*[Sila pastikan bahawa kertas peperiksaan ini mengandungi LAPAN muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]*

**Instructions** : Answer **all ten** [10] questions.

**[Arahan** : Jawab **semua sepuluh** [10] soalan].

1. (a) Solve the inequality  $-5 < \frac{|x|}{x-3} < 4$ .
- (b) Given a circle:  $x^2 + y^2 + 2y = 9$ . A straight line that passes the center of the circle and point  $P(-1,2)$  intersect the x-axis and y-axis at point  $Q$  and  $R$  respectively. Find  $Q$  and  $R$ .

[20 marks]

1. (a) Selesaikan ketaksamaan  $-5 < \frac{|x|}{x-3} < 4$ .
- (b) Diberi suatu bulatan:  $x^2 + y^2 + 2y = 9$ . Satu garis lurus yang melalui pusat bulatan dan titik  $P(-1,2)$  memotong paksi-x dan paksi-y pada titik  $Q$  dan  $R$ . Cari  $Q$  dan  $R$ .

[20 markah]

2. (a) Below are values of three functions

$x$	-3	-2	-1	0	1	2	3
$f(x)$	-1	-1	-1	-1	0	1	2
$g(x)$	3	1	-1	-3	1	-1	-3
$h(x)$	2	-5	8	-2	8	-5	2

- (i) Show that  $g(x)$  is an odd function and  $h(x)$  is an even function.
- (ii) Construct a table of  $s(x) = (f \circ g)(x)$ .
- (b) Given  $f(x) = 2x - 1$ ,  $g(x) = \ln(x+3)$  and  $h(x) = e^{4x+7}$ . Find the value  $m$  such that:

$$m(g(h(m)-3)) = f(m)$$

[20 marks]

2. (a) Di bawah adalah nilai-nilai bagi tiga fungsi

$x$	-3	-2	-1	0	1	2	3
$f(x)$	-1	-1	-1	-1	0	1	2
$g(x)$	3	1	-1	-3	1	-1	-3
$h(x)$	2	-5	8	-2	8	-5	2

...3/-

- (i) Tunjukkan bahawa  $g(x)$  adalah suatu fungsi ganjil dan  $h(x)$  adalah suatu fungsi genap.  
 (ii) Bina satu jadual bagi  $s(x) = (f \circ g)(x)$ .

- (b) Diberi  $f(x) = 2x - 1$ ,  $g(x) = \ln(x + 3)$  dan  $h(x) = e^{4x+7}$ . Cari nilai  $m$  supaya:

$$m(g(h(m) - 3)) = f(m)$$

[20 markah]

3. (a) Consider the function  $f(x) = \begin{cases} 2x - 1 & x \leq 0 \\ x^2 - 1 & x > 0 \end{cases}$ . Find  $f^{-1}(x)$  and its domain.

- (b) Given that  $k(x) = \begin{cases} x^2 - 1 & x \leq 1 \\ p(x - 1) & x > 1 \end{cases}$ . Find the value of  $p$  so that  $k$  is differentiable.

[20 marks]

3. (a) Pertimbangkan fungsi  $f(x) = \begin{cases} 2x - 1 & x \leq 0 \\ x^2 - 1 & x > 0 \end{cases}$ . Cari  $f^{-1}(x)$  dan domainnya.

- (b) Diberi bahawa  $k(x) = \begin{cases} x^2 - 1 & x \leq 1 \\ p(x - 1) & x > 1 \end{cases}$ . Cari nilai  $p$  supaya  $k$  adalah terbezakan.

[20 markah]

4. (a) Use the first principle of differentiation to obtain the derivative of :

$$A(x) = \sqrt{2}x^2.$$

- (b) Differentiate the following:

(i)  $g(w) = e^{2 \sin x} (x^2 + 5^x)$

(ii)  $\ln(\cos y) = x^3 \sec^{-1}(\sqrt{y})$

- (c) Find the limit:

(i)  $\lim_{x \rightarrow 2} \frac{\sqrt{x+2} - \sqrt{2x}}{4 - x^2}$

(ii)  $\lim_{x \rightarrow 0} \frac{x - \tan^{-1} x}{x^2}$

[20 marks]

...4/-

4. (a) Guna prinsip pertama pembezaan untuk mendapatkan terbitan bagi:  
 $A(x) = \sqrt{2}x^2$ .

(b) Bezakan yang berikut:

(i)  $g(w) = e^{2\sin x} (x^2 + 5^x)$       (ii)  $\ln(\cos y) = x^3 \sec^{-1}(\sqrt{y})$

(c) Cari had:

(i) had  $\frac{\sqrt{x+2} - \sqrt{2x}}{4-x^2}$       (ii) had  $\frac{x - \tan^{-1} x}{x^2}$

[20 markah]

5. A function is given by  $z(x) = x^2 e^{1-x}$ . Find:

- (a) the intervals of increase or decrease;  
 (b) the local maximum and minimum points;  
 (c) the intervals of concavity and the inflection points.

Use the information obtained above to sketch the graph of  $z(x)$

[20 marks]

5. Suatu fungsi diberi sebagai  $z(x) = x^2 e^{1-x}$ . Cari:

- (a) selang-selang menaik dan menyusut;  
 (b) titik maksimum dan minimum setempat;  
 (c) selang-selang kecekungan dan titik-titik lengkungbalas;

Gunakan maklumat yang diperolehi untuk melakarkan graf bagi  $z(x)$ .

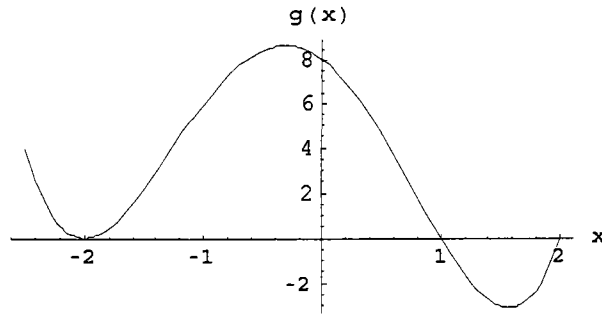
[20 markah]

6. (a) Integrate:

(i)  $\int \frac{4e^x(2+e^x)}{e^{2x} + 4e^x + 3} dx$       (ii)  $\int (\ln x)^2 dx$

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(b)



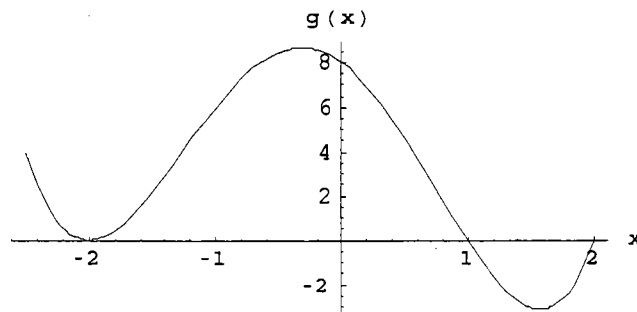
The figure shows the graph of a function  $g(x)$ . For which value of  $c$  in the interval  $[-2, 2]$  will  $\int_{-2}^c g(x) dx$  be the largest? Justify your answer.

[20 marks]

6. (a) *Kamirkan:*

$$(i) \int \frac{4e^x(2+e^x)}{e^{2x}+4e^x+3} dx \quad (ii) \int (\ln x)^2 dx$$

(b)



Rajah di atas menunjukkan graf bagi suatu fungsi  $g(x)$ . Beri nilai  $c$  dalam selang  $[-2, 2]$  yang mana  $\int_{-2}^c g(x) dx$  adalah terbesar. Beri alasan untuk jawapan anda.

[20 markah]

7. (a) Let  $u = x^3 + 1$ , show that 
$$\int_{-1}^0 x^2 \sqrt{x^3 + 1} dx = \int_0^1 \frac{1}{3} \sqrt{u} du.$$

Hence find the area under the curve  $y = 3x^2 \sqrt{x^3 + 1}$  from  $x = -1$  to  $x = 0$ .

...6/-

- (b) From the table below evaluate  $\int_8^{10} f'(t) dt$ , where  $f$  is differentiable.

$t$	$f(t)$
8	8.253
9	8.372
10	9.245
11	10.612

- (c) Integrate  $\int \frac{2+x}{x^2+5x} dx$ .

[20 marks]

7. (a) Biarkan  $u = x^3 + 1$ . Tunjukkan bahawa  $\int_{-1}^0 x^2 \sqrt{x^3 + 1} dx = \int_0^1 \frac{1}{3} du$ .

Seterusnya cari luas di bawah lengkung  $y = 3x^2 \sqrt{x^3 + 1}$  dari  $x = -1$  hingga  $x = 0$ .

- (b) Daripada jadual di bawah nilaikan  $\int_8^{10} f'(t) dt$ , yang mana  $f$  adalah terbezakan.

$t$	$f(t)$
8	8.253
9	8.372
10	9.245
11	10.612

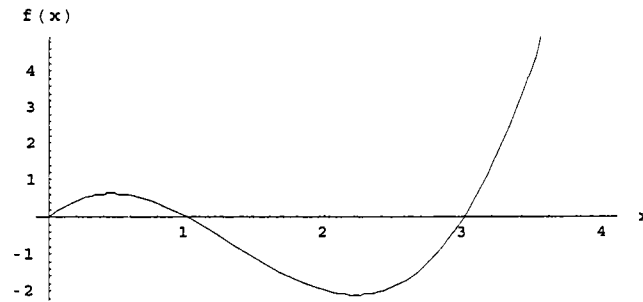
- (c) Kamirkan  $\int \frac{2+x}{x^2+5x} dx$ .

[20 markah]

8. (a) Evaluate  $\int_0^{\frac{\pi}{2}} \frac{1}{1+\sin x} dx$ .

- (b) Below is a graph of a function  $f(x)$  defined on the interval  $[0,4]$ . Find the interval for the function  $g(x) = \int_0^x f(t) dt$  to be decreasing. Also determine the type of critical points  $(1, g(1))$  and  $(3, g(3))$ .

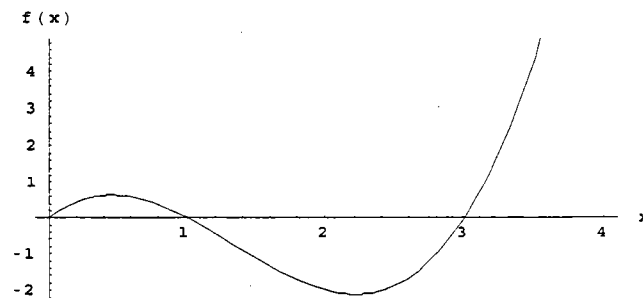
...7/-



[20 marks]

8. (a) Nilaikan  $\int_0^{\frac{\pi}{2}} \frac{1}{1+\sin x} dx$ .

- (b) Di bawah adalah graf suatu fungsi  $f(x)$  yang tertakrif pada selang  $[0,4]$ . Cari selang yang mana  $g(x) = \int_0^x f(t)dt$  adalah menyusut. Cari nombor genting bagi  $g$ . Tentukan jenis titik genting bagi titik-titik  $(1, g(1))$  and  $(3, g(3))$ .



[20 markah]

9. (a) Show that  $F(x) = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)$ ,  $a > 0$ , is an anti derivative of  $f(x) = \sqrt{a^2 - x^2}$ . Hence evaluate  $\int_{-a}^a \sqrt{a^2 - x^2} dx$ .

(b) Evaluate  $\int_{-1}^2 |x-1| dx$ .

[20 marks]

.../8-

9. (a) Tunjukkan bahawa  $F(x) = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)$ ,  $a > 0$ , adalah anti terbitan bagi  $f(x) = \sqrt{a^2 - x^2}$ . Seterusnya nilaikan  $\int_{-a}^a \sqrt{a^2 - x^2} dx$ .

(b) Nilaikan  $\int_{-1}^2 |x-1| dx$ .

[20 markah]

10. Sketch the graph of  $y = \sin^2 x$  on the interval of  $x \in [-\frac{\pi}{2}, \frac{\pi}{2}]$ , then find the volume of revolution when the region enclosed by the curve and the x-axis for  $x \in [-\frac{\pi}{2}, \frac{\pi}{2}]$  is rotated through  $360^\circ$  about the x-axis.

[20 marks]

10. Lakarkan graf  $y = \sin^2 x$  pada selang  $x \in [-\frac{\pi}{2}, \frac{\pi}{2}]$ , kemudian cari isipadu kisanan apabila rantau yang dibendung of lengkung  $y$  dan paksi-x pada  $x \in [-\frac{\pi}{2}, \frac{\pi}{2}]$  dikisarkan  $360^\circ$  terhadap paksi-x.

[20 markah]

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