
UNIVERSITI SAINS MALAYSIA

Second Semester Examination
Academic Session 2006/2007

April 2007

MAA 101 – Calculus for Science Students I
[Kalkulus untuk Pelajar Sains I]

Duration : 3 hours

[Masa : 3 jam]

Please check that this examination paper consists of NINE pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi SEMBILAN muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions: Answer **all eight** [8] questions.

Arahan: Jawab **semua lapan** [8] soalan.]

$$3. \quad (a) \quad \text{Let } m(x) = \begin{cases} \frac{4x^2 - 25}{2x - 5} & , \quad x \leq 1 \\ x + 6 & , \quad 1 < x < 2 \\ x^2 - 2x + 2 & , \quad x \geq 2 \end{cases}$$

- (i) Discuss whether the function is continuous at the point $x = 1$.
 (ii) Determine if the function is differentiable at the point $x = 2$.

(b) Find derivative of $A(x) = \frac{1}{(x-1)}$ by definition.

(c) (i) Differentiate $f(x) = (x^3 - x^2)5^x$

(ii) If $B(3) = 4$ and $B'(3) = -3$, find $\left. \frac{d}{dx} \left(\frac{B(x)}{8x^3} \right) \right|_{x=3}$

[30 marks]

4. (a) Find the limit:

(i) $\lim_{x \rightarrow 9} \frac{x^2 - 10x + 9}{\sqrt{x} - 3}$

(ii) $\lim_{x \rightarrow 0} \frac{e^{2x} - 1 - x}{3x^2}$

(b) (i) A function is given by $f(x) = \frac{x}{x^2 + 9}$ for $-1 \leq x \leq 3$.

Find the intervals of increase or decrease and hence find the local maximum and minimum points.

(ii) Sketch the graph of a function that satisfies all of the following conditions;

$$f'(1/2) = f'(2) = 0$$

$$f'(0) = -1$$

$$f'(x) > 0 \text{ if } 1/2 < x < 2$$

$$f'(x) < 0 \text{ if } 0 < x < 1/2 \text{ and } x > 2$$

$$f''(x) < 0 \text{ if } 1 < x < 4$$

$$f''(x) > 0 \text{ if } 0 < x < 1 \text{ and } x > 4$$

$$\lim_{x \rightarrow 0} f(x) = 1$$

$$f(-x) = -f(x) \text{ for all } x$$

(c) Car A is 100km northeast of car B. Car A travels south at 50 km/h while car B travels east at 60 km/h. At what rate is the distance between the cars increasing two hours later?

[30 marks]

$$3. \quad (a) \quad \text{Biar } m(x) = \begin{cases} \frac{4x^2 - 25}{2x - 5} & , \quad x \leq 1 \\ x + 6 & , \quad 1 < x < 2 \\ x^2 - 2x + 2 & , \quad x \geq 2 \end{cases}$$

(i) Bincang sekiranya fungsi adalah selanjar pada titik $x = 1$.

(ii) Tentukan sekiranya fungsi adalah terbezakan pada titik $x = 2$.

(b) Cari terbitan bagi $A(x) = \frac{1}{(x-1)}$ dengan menggunakan takrif.

(c) (i) Bezakan $f(x) = (x^3 - x^2)5^x$

(ii) Jika $B(3) = 4$ dan $B'(3) = -3$, cari $\left. \frac{d}{dx} \left(\frac{B(x)}{8x^3} \right) \right|_{x=3}$

[30 markah]

4. (a) Cari had:

(i) $\lim_{x \rightarrow 9} \frac{x^2 - 10x + 9}{\sqrt{x} - 3}$

(ii) $\lim_{x \rightarrow 0} \frac{e^{2x} - 1 - x}{3x^2}$

(b) (i) Suatu fungsi diberi sebagai $f(x) = \frac{x}{x^2 + 9}$ bagi $-1 \leq x \leq 3$.

Cari selang-selang menaik dan menyusut dan seterusnya cari titik maksimum dan minimum setempat.

(ii) Lakarkan graf bagi suatu fungsi yang memenuhi syarat-syarat berikut:

$$f'(1/2) = f'(2) = 0$$

$$f'(0) = -1$$

$$f'(x) > 0 \text{ jika } 1/2 < x < 2$$

$$f'(x) < 0 \text{ jika } 0 < x < 1/2 \text{ dan } x > 2$$

$$f''(x) < 0 \text{ jika } 1 < x < 4$$

$$f''(x) > 0 \text{ jika } 0 < x < 1 \text{ dan } x > 4$$

$$\lim_{x \rightarrow 0} f(x) = 1$$

$$f(-x) = -f(x) \text{ bagi semua } x$$

(c) Kereta A adalah 100km di timur laut kereta B. Kereta A bergerak ke selatan pada kelajuan 50 km/j sementara kereta B bergerak ke timur pada kelajuan 60 km/j. Apakah kadar peningkatan jarak di antara dua kereta tersebut selepas dua jam.

[30 markah]

...16-

5. (a) Sketch the graphs of $y = x^2$ and $y = \sqrt{x}$. Then find the area of the region bounded by these curves.
- (b) Find the volume of revolution when the region in (a) is rotated 360° about the x -axis.

[35 marks]

6. (a) Evaluate

$$(i) \int_0^{\frac{\pi}{2}} \frac{d}{dx} \left(\sin\left(\frac{x}{2}\right) \cos\left(\frac{x}{3}\right) \right) dx$$

$$(ii) \frac{d}{dx} \int_0^{\frac{\pi}{2}} \left(\sin\left(\frac{x}{2}\right) \cos\left(\frac{x}{3}\right) \right) dx$$

$$(iii) \frac{d}{dx} \int_x^{\frac{\pi}{2}} \left(\sin\left(\frac{t}{2}\right) \cos\left(\frac{t}{3}\right) \right) dt$$

- (b) A student calculate an integral as follows:

$$\int_{-1}^1 \frac{1}{x^2} dx = \left[-\frac{1}{x} \right]_{-1}^1 = (-1) - 1 = -2.$$

But the student knows that the graph of $y = \frac{1}{x^2}$ is always above the x -axis, which means the area under the curve is never negative. So the student gets confused. Can you help the student by describing the error in the "evaluation" above?

[16 marks]

7. (a) Use the substitution $u = \pi - x$ to show that

$$\int_0^{\pi} x f(\sin x) dx = \frac{\pi}{2} \int_0^{\pi} f(\sin x) dx.$$

- (b) Evaluate the integral.

$$(i) \int \cos x \ln(\sin x) dx$$

$$(ii) \int_1^2 \frac{3x^2 - 2x - 6}{x(x+2)(x-3)} dx$$

[30 marks]

.../7-

5. (a) Lakarkan graf untuk $y = x^2$ dan $y = \sqrt{x}$. Kemudian cari luas rantau yang dibatasi oleh lengkung-lengkung tersebut.
- (b) Cari isipadu kisanan apabila rantau dalam (a) dikisarkan 360° terhadap paksi x .

[35 markah]

6. (a) Nilaikan

$$(i) \int_0^{\frac{\pi}{2}} \frac{d}{dx} \left(\sin\left(\frac{x}{2}\right) \cos\left(\frac{x}{3}\right) \right) dx$$

$$(ii) \frac{d}{dx} \int_0^{\frac{\pi}{2}} \left(\sin\left(\frac{x}{2}\right) \cos\left(\frac{x}{3}\right) \right) dx$$

$$(iii) \frac{d}{dx} \int_x^{\frac{\pi}{2}} \left(\sin\left(\frac{t}{2}\right) \cos\left(\frac{t}{3}\right) \right) dt$$

- (b) Seorang pelajar mengira satu kamiran seperti berikut:

$$\int_{-1}^1 \frac{1}{x^2} dx = \left[-\frac{1}{x} \right]_{-1}^1 = (-1) - 1 = -2$$

Tetapi pelajar tersebut tahu bahawa graf bagi $y = \frac{1}{x^2}$ sentiasa berada di atas paksi x , bermaksud luas rantau di bawah graf tersebut adalah sentiasa bukan negatif. Jadi pelajar tersebut keliru. Bolehkan anda tolong pelajar tersebut dengan menyatakan kesilapan dalam "penilaian" di atas?

[16 markah]

7. (a) Gunakan gantian $u = \pi - x$ untuk menunjuk

$$\int_0^{\pi} x f(\sin x) dx = \frac{\pi}{2} \int_0^{\pi} f(\sin x) dx.$$

- (b) Nilaikan kamiran.

$$(i) \int \cos x \ln(\sin x) dx$$

$$(ii) \int_1^2 \frac{3x^2 - 2x - 6}{x(x+2)(x-3)} dx$$

[30 markah]