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UNIVERSITI SAINS MALAYSIA

Second Semester Examination  
2013/2014 Academic Session

June 2014

**ESA 380/3 – Orbital Mechanics**  
**[Mekanik Orbit]**

Duration : 3 hours  
[Masa : 3 jam]

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Please ensure that this paper contains **TWELVE (12)** printed pages and **FIVE (5)** questions before you begin examination.

*[Sila pastikan bahawa kertas soalan ini mengandungi **DUA BELAS (12)** mukasurat bercetak dan **LIMA (5)** soalan sebelum anda memulakan peperiksaan].*

**Instructions** : Answer **FIVE (5)** questions.

**Arahan** : Jawab **LIMA (5)** soalan].

Student may answer the questions either in English or Bahasa Malaysia.

*[Pelajar boleh menjawab soalan dalam Bahasa Inggeris atau Bahasa Malaysia].*

Each questions must begin from a new page.

*[Setiap soalan mestilah dimulakan pada mukasurat yang baru].*

In the event of any discrepancies, the English version shall be used.

*[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai].*

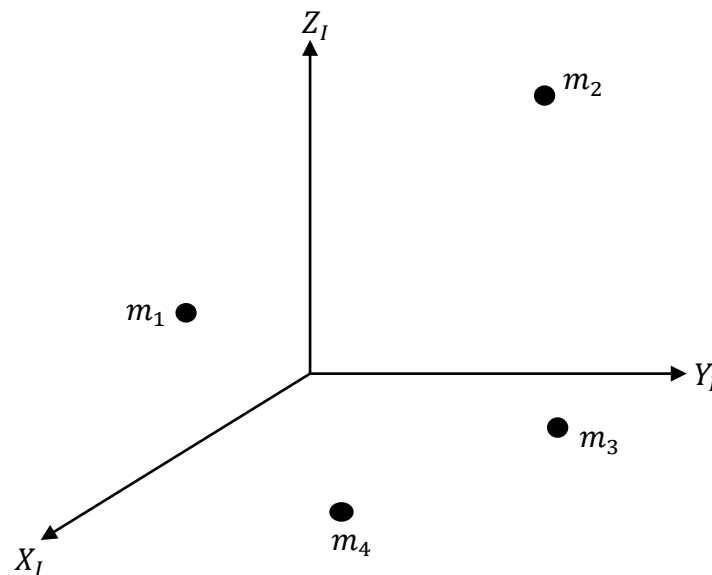
1. [a] Figure 1 shows a system of  $n$ -body problem. Assume that  $m_4$  is an earth satellite and that  $m_1$  is the Earth. While the remaining masses  $m_2$  and  $m_3$  are other planets. By considering only gravitational forces of these objects,

(i) Develop the equation of motion of the satellite with respect to the inertial frame.

**(30 marks)**

(ii) Develop the equation of motion of the satellite relative to the earth.

**(30 marks)**



**Figure 1**

- [b] A spacecraft on a parabolic trajectory around the earth has a perigee radius of 7500 km. Given the anomaly for parabola and its solution for true anomaly as follow

$$M_p = \frac{1}{2} \tan \frac{\theta}{2} + \frac{1}{6} \tan^3 \frac{\theta}{2}$$

$$\tan \frac{\theta}{2} = \left( 3M_p + \sqrt{(3M_p)^2 + 1} \right)^{\frac{1}{3}} - \left( 3M_p + \sqrt{(3M_p)^2 + 1} \right)^{-\frac{1}{3}}$$

(i) How long does it take to coast from  $\theta = -90$  degrees to  $\theta = +90$  degrees?

**(20 marks)**

(ii) How far is the spacecraft from the center of the earth 24 hours after passing through perigee?

**(20 marks)**

2. [a] Given that, with respect to the geocentric equatorial frame

$$\mathbf{r} = -6634.2\hat{\mathbf{i}} - 1261.8\hat{\mathbf{j}} - 5230.9\hat{\mathbf{k}} \text{ (km)},$$

$$\mathbf{v} = 5.7644\hat{\mathbf{i}} - 7.2005\hat{\mathbf{j}} - 1.8106\hat{\mathbf{k}} \text{ (km/s)}$$

and the eccentricity vector is

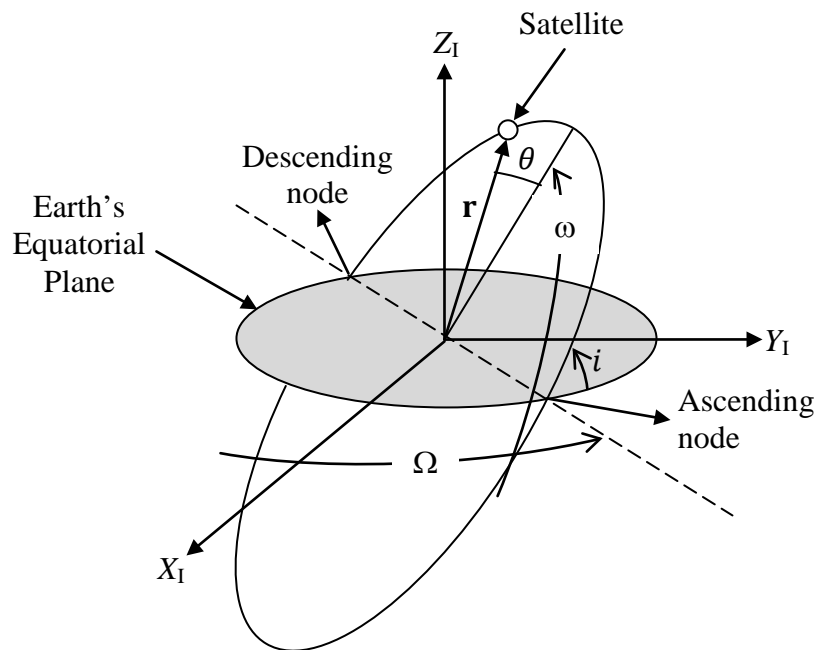
$$\mathbf{e} = -0.40907\hat{\mathbf{i}} - 0.48751\hat{\mathbf{j}} - 0.63640\hat{\mathbf{k}},$$

Calculate the true anomaly  $\theta$  of the earth-orbiting satellite.

(30 marks)

- [b] Given the 6 classical orbital parameters as in **Figure 2**, define the vector  $\mathbf{r}$  in the Earth-Centred-Inertial (ECI) coordinate system.

(50 marks)



**Figure 2**

- [c] Discuss and draw selection of the coordinate systems to be used for controlling the attitude of the Earth Observation Satellite.

(20 marks)

3. [a] A space vehicle in a circular orbit at an altitude of 500km above the earth as in Figure 3 execute a Hohmann transfer to 1000 km circular orbit. Calculate total delta-v requirement.

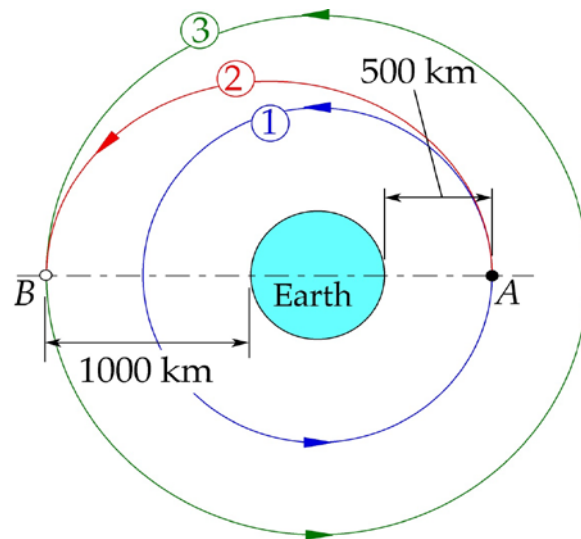


Figure 3

(40 marks)

- [b] Two geocentric elliptical orbits have common apse lines and their perigees are on the same side of the earth. The first orbit has a perigee radius of  $r_p = 7000\text{km}$  and  $e = 0.3$ , whereas for the second orbit  $r_p = 32\,000\text{km}$  and  $e = 0.5$
- Find the minimum total delta-v and the time of flight for a transfer from the perigee of the inner orbit to the apogee of outer orbit
  - Do part (i) for a transfer from the apogee of the inner orbit to the perigee of outer orbit.

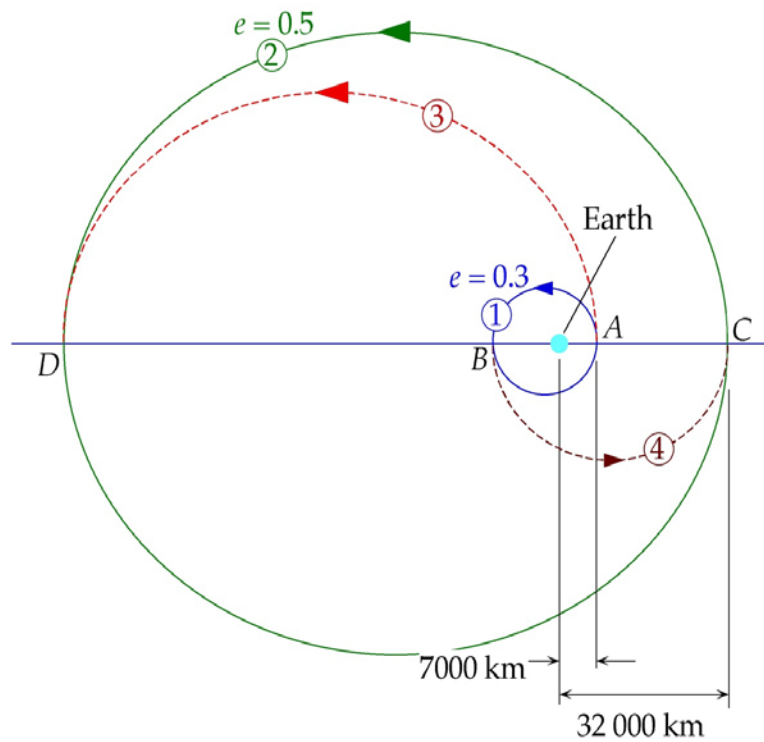


Figure 4

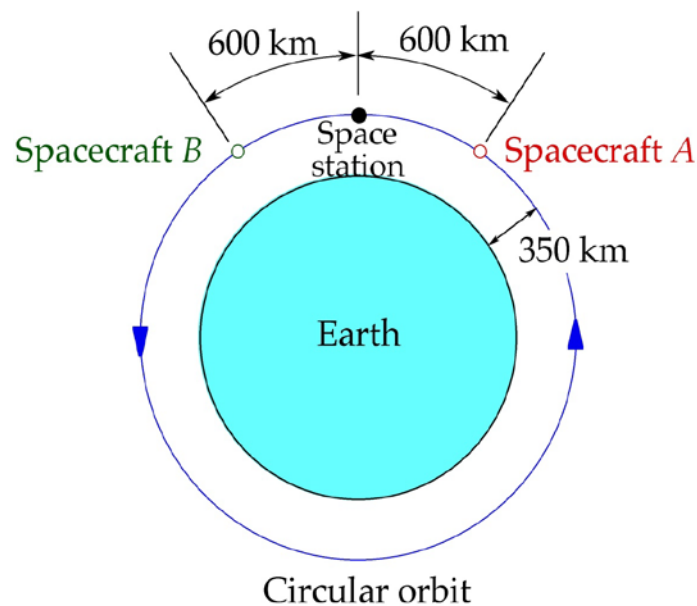
(60 marks)

4. [a] With a single impulsive maneuver, an earth satellite changes from 400 km circular orbit inclined at 60 deg to an elliptical orbit of eccentricity  $e = 0.5$  with an inclination of 40 deg. Calculate the minimum required delta-v.

**(50 marks)**

- [b] The space station and spacecraft A and B are all in the same circular orbit of 350 km altitude. Spacecraft A is 600 km behind the space station and Spacecraft B is 600 km ahead of the space station. At the same instant, both spacecraft apply a  $\Delta v_{\perp}$  so as to arrive at the space station in one revolution of their phasing orbits.

- (i) Calculate the times required for each spacecraft to reach the space station.
- (ii) Calculate the total delta-v requirement for each spacecraft.



**Figure 5**

**(50 marks)**

5. A 2000kg spacecraft will be launched to Saturn for scientific mission to study the characteristic of that planet. Assuming the propulsion system has a specific impulse of 300s, calculate :
- [a] the time required for the mission **( 50 marks)**
- [b] the propellant mass required to launch this spacecraft from 180km circular earth orbit on a Hohmann transfer trajectory to orbit of Saturn. **(50 marks)**

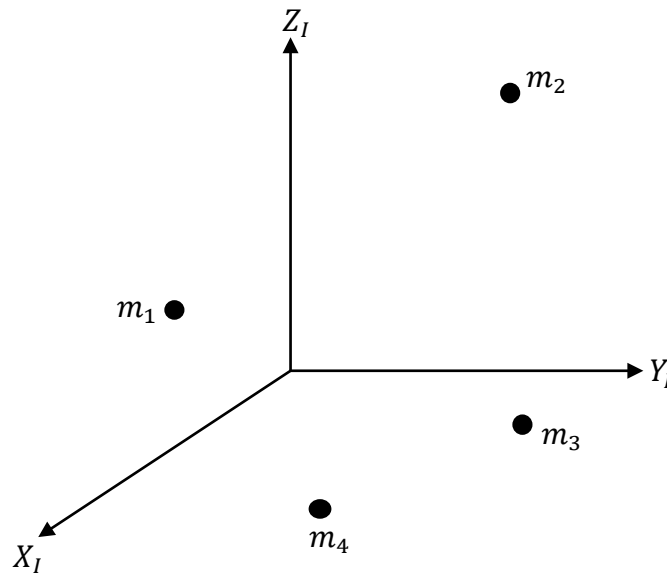
1. [a] *Rajah 1 menunjukkan sebuah sistem masalah 4-jasad. Anggap  $m_4$  sebagai sebuah satelit bumi dan  $m_1$  sebagai bumi. Sementara jasad  $m_2$  dan  $m_3$  adalah bulan dan planet lain. Dengan hanya mempertimbangkan tarikan graviti jasad-jasad tersebut,*

(i) *Hasilkan persamaan gerakan satelit relatif kepada bingkai tegar.*

**(30 markah)**

(ii) *Hasilkan persamaan gerakan satelit relatif kepada bumi.*

**(30 markah)**



**Rajah 1**

- [b] *Sebuah kapal angkasa pada trajektori parabola sekitar bumi mempunyai radius perigee 7500 km. Diberikan anomaly untuk parabola dan penyelesaiannya anomaly benar seperti berikut*

$$M_p = \frac{1}{2} \tan \frac{\theta}{2} + \frac{1}{6} \tan^3 \frac{\theta}{2}$$

$$\tan \frac{\theta}{2} = \left( 3M_p + \sqrt{(3M_p)^2 + 1} \right)^{\frac{1}{3}} - \left( 3M_p + \sqrt{(3M_p)^2 + 1} \right)^{-\frac{1}{3}}$$

(i) *Berapa lamakah masa yang diambil untuk bergerak melalui trajektori dari sudut  $\theta = -90$  darjah ke sudut  $\theta = 90$  darjah?*

**(20 markah)**

(ii) *Sejauh manakah kapal angkasa berada dari pusat bumi selepas 24 jam melalui perigee?*

**(20 markah)**



2. [a] Diberi, merujuk kepada rangka equatorial geocentrik

$$\mathbf{r} = -6634.2\hat{\mathbf{i}} - 1261.8\hat{\mathbf{j}} - 5230.9\hat{\mathbf{k}} \text{ (km)},$$

$$\mathbf{v} = 5.7644\hat{\mathbf{i}} - 7.2005\hat{\mathbf{j}} - 1.8106\hat{\mathbf{k}} \text{ (km/s)}$$

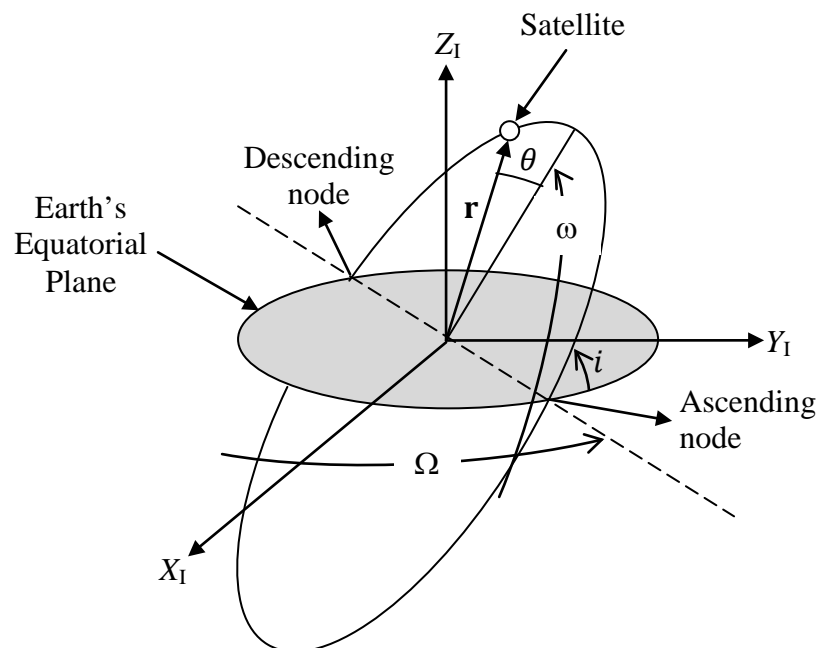
dan vektor keeksentrikan

$$\mathbf{e} = -0.40907\hat{\mathbf{i}} - 0.48751\hat{\mathbf{j}} - 0.63640\hat{\mathbf{k}},$$

(30 markah)

- [b] Diberi 6 orbit parameter klasik seperti **Gambarajah 2**, terbitkan vektor  $\mathbf{r}$  pada ECI sistem koordinat.

(50 markah)

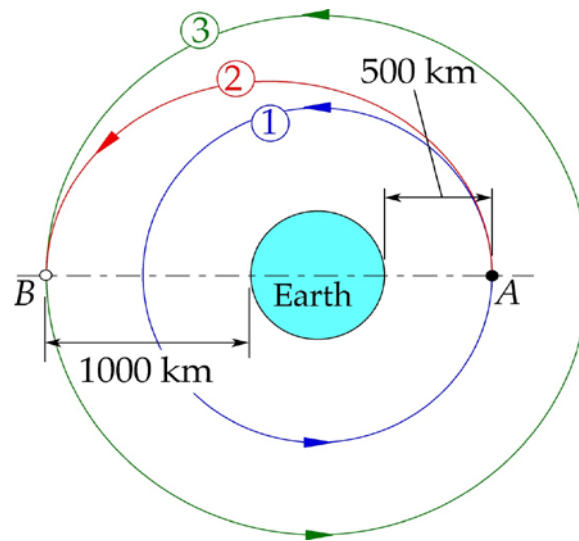


**Rajah 2**

- [c] Bincang dan lukiskan pemilihan sistem koordinat untuk kegunaan mengawal sikap Satelite Pemerhati Bumi.

(20 markah)

3. [a] Sebuah kendaraan angkasa dalam sebuah orbit bulat pada ketinggian 500km di atas bumi seperti Rajah 3 melaksanakan orbit pindah Hohmann ke 1000 km orbit bulat. Kira jumlah keperluan delta-v.

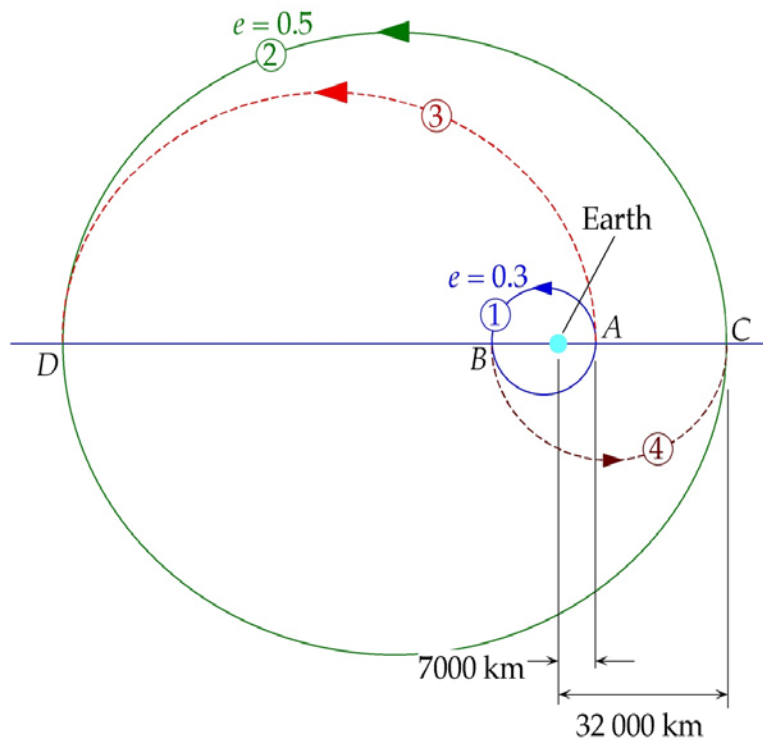


**Rajah 3**

**(40 markah)**

[b] Dua orbit ellips geosentrik mempunyai garis apsis sepunya dan kedua-dua perigi berada di belah bumi yang sama. Orbit yang pertama mempunyai jejari perigi  $r_p = 7000\text{km}$  dan  $e = 0.3$ . Manakala orbit yang kedua  $r_p = 32\,000\text{km}$  dan  $e = 0.5$

- (i) Cari jumlah minimum delta- $v$  dan masa penerbangan untuk pemindahan orbit daripada perigi orbit dalam ke apogi orbit luar.
- (ii) Lakukan bahagian (i) daripada apogi orbit dalam ke perigi orbit luar.



**Rajah 4**

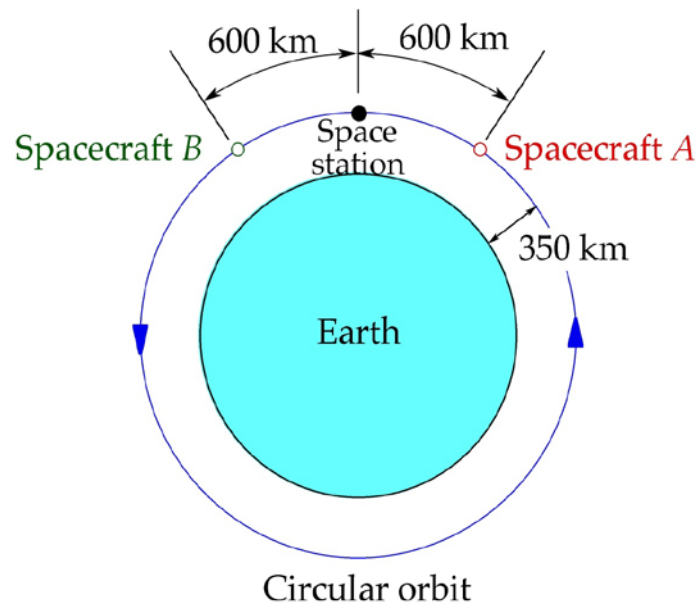
**(60 markah)**

4. [a] Dengan menggunakan olah gerak impulsive, sebuah satelitte bumi berubah daripada 400km orbit bumi berkecondongan 60 darjah kepada orbit elips dengan kesipian  $e = 0.5$  dan berkecondongan 40 darjah. Kirakan delta-v minimum yang diperlukan.

(50 markah)

- [b] Stesen angkasa dan kapal angkasa A dan B kedua-duanya dalam orbit bulat yang sama pada ketinggian 350 km. Kapal angkasa A adalah 600 km di belakang stesen angkasa dan Kapal angkasa B berada 600 km dihadapan stesen angkasa. Pada masa yang sama, kedua-dua kapal angkasa mengenakan  $\Delta v_{\perp}$  supaya tiba di stesen angkasa dalam satu kitaran orbit fasa berperingkat mereka.

- (i) Kira masa yang diperlukan untuk setiap kapal angkasa untuk sampai di stesen angkasa.
- (ii) Kira jumlah keperluan delta-v bagi setiap kapal angkasa.



Rajah 5

(50 markah)

5. Sebuah kapal angkasa yang akan dilancarkan ke Zuhal untuk misi saintifik mengkaji ciri-ciri planet tersebut. Dianggarkan sistem pendorong mempunyai impuls spesifik sebanyak 300s, kirakan :

- [a] masa yang diperlukan untuk misi tersebut

( 50 markah)

- [b] Bahan api yang diperlukan untuk melancarkan kapal angkasa daripada 180 km orbit bulat untuk laluan orbit pindah Hohmann ke Zuhal.

(50 markah)

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