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UNIVERSITI SAINS MALAYSIA

UNIVERSITY RESEARCH GRANT FINAL REPORT
Geran Penyelidikan Universiti Laporan Akhir

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A.	**TITLE OF RESEARCH:** *Tajuk penyelidikan:* A Neural Network Approach to Synthetic Control Chart for the Process Mean
B.	**PERSONAL PARTICULARS OF RESEARCHER** / *MAKLUMAT PENYELIDIK:*
(i)	**Name of Research Leader:** *Nama Ketua Penyelidik:* Prof. Michael Khoo Boon Chong
	Name of Co-Researcher *Nama Penyelidik Bersama:* Prof. Madya Low Heng Chin Dr. Ong Hong Choon
(ii)	**School/Institute/Centre/Unit :** *Pusat Pengajian /Institut/Pusat/Unit :* Pusat Pengajian Sains Matematik

C. Research Platform (Please tick (/) the appropriate box):
Pelantar Penyelidikan (Sila tanda (/) kotak berkenaan):

[ ]	**A. Life Sciences** *Sains Hayat*
[√]	**B. Fundamental** *Fundamental*
[ ]	**C. Engineering & Technology** *Kejuruteraan & Teknologi*
[ ]	**D. Social Transformation** *Transformasi Sosial*
[ ]	**E. Information & Communications Technology (ICT)** *Teknologi Maklumat & Komunikasi*
[ ]	**F. Clinical Sciences** *Sains Klinikal*
[ ]	**G. Biomedical & Health Sciences** *Bioperubatan Sains Kesihatan*

D.	**Duration of this research :** *Tempoh masa penyelidikan ini :* ***Duration :** 3 tahun *Tempoh :* **From** : 22 Oktober 2007 **To** : 30 September 2010 *Dari:* *Ke :*

Pejabat Pelantar Penyelidikan 2009

E. **ABSTRACT OF RESEARCH**

(An abstract of between 100 and 200 words must be prepared in **Bahasa Malaysia and in English**. This abstract will be included in the Annual Report of the Research and Innovation Section at a later date as a means of presenting the project findings of the researcher/s to the University and the community at large)

In this project, a multivariate synthetic control chart for monitoring the process mean vector of skewed populations using weighted standard deviations has been proposed. The proposed chart incorporates the weighted standard deviation (WSD) method of Chang and Bai (2004) into the standard multivariate synthetic chart of Ghute and Shirke (2008). The standard multivariate synthetic chart consists of the Hotelling's T^2 chart and the conforming run length (CRL) chart. The results show that the proposed chart performs better than the existing multivariate charts for skewed populations, in terms of false alarm rates and moderate and large mean shift detection rates based on the various degrees of skewnesses. This project also suggest a synthetic double sampling (DS) chart for the mean. The synthetic DS chart comprises a DS sub-chart and a CRL sub-chart. The DS chart was proposed by Daudin (1992) while the CRL chart was suggested by Bourke (1991). The synthetic DS chart outperforms both of its standard counterparts, namely the synthetic chart of Wu and Spedding (2000) and the DS chart of Daudin (1992). In addition, a comparison between statistical charts and neural network based charts is made, where the results indicate that neural network based charts perform better.

References

1. Bourke, P. D. (1991). Detecting a Shift in Fraction Nonconforming Using Run-Length Control Charts with 100% Inspection. *Journal of Quality Technology* 23, 225-238.

2. Chang, Y. S., Bai, D. S. (2004). A multivariate T^2 control chart for skewed populations using weighted standard deviations. *Quality and Reliability Engineering International* 20, 31–46.

3. Daudin, J. J. (1992). Double Sampling $\bar{X}$ Charts. *Journal of Quality Technology* 24, 78-87.

4. Ghute VB and Shirke DT. (2008). A multivariate synthetic control chart for monitoring process mean vector. *Communications in Statistics – Theory and Methods* 37, 2136 − 2148.

5. Wu, Z. and Spedding, T. A. (2000). A Synthetic Control Chart for Detecting Small Shifts in the process Mean. *Journal of Quality Technology* 32, 32-38.

Abstrak Penyelidikan

(Perlu disediakan di antara 100 - 200 perkataan di dalam **Bahasa Malaysia dan juga Bahasa Inggeris**. Abstrak ini akan dimuatkan dalam Laporan Tahunan Bahagian Penyelidikan & Inovasi sebagai satu cara untuk menyampaikan dapatan projek tuan/puan kepada pihak Universiti & masyarakat luar).

Dalam projek ini, suatu carta kawalan sintetik multivariat untuk pemantauan vektor min proses bagi taburan terpencong dengan menggunakan sisihan piawai berpemberat telah dicadangkan. Carta yang dicadangkan menggabungkan kaedah sisihan piawai berpemberat (WSD) Chang dan Bai (2004) dengan carta sintetik multivariat asas Ghute dan Shirke (2008). Carta sintetik multivariat asas terdiri daripada carta T^2 Hotelling dan carta *conforming run length* (CRL). Keputusan menunjukkan bahawa carta yang dicadangkan memberi prestasi yang lebih baik daripada carta multivariat sedia ada untuk populasi terpencong, daripada segi kadar isyarat palsu dan kadar pengesanan anjakan sederhana dan besar dalam min, berdasarkan pelbagai darjah kepencongan. Projek ini juga mencadangkan suatu carta pensampelan berganda (DS) sintetik untuk min. Carta DS sintetik terdiri daripada carta DS dan carta CRL. Carta DS dicadangkan oleh Daudin (1992) manakala carta CRL dicadangkan oleh Bourke (1991). Carta DS sintetik mempunyai prestasi yang lebih baik daripada kedua-dua carta asasnya, iaitu carta sintetik Wu dan Spedding (2000) dan carta DS Daudin (1992). Tambahan pula, suatu perbandingan antara carta berstatistik dan carta berasaskan rangkaian neural telah dilakukan, yang mana keputusan menunjukkan bahawa carta berasaskan rangkaian neural mempunyai prestasi yang lebih baik.

Rujukan

1. Bourke, P. D. (1991). Detecting a Shift in Fraction Nonconforming Using Run-Length Control Charts with 100% Inspection. *Journal of Quality Technology* 23, 225-238.

2. Chang, Y. S., Bai, D. S. (2004). A multivariate T^2 control chart for skewed populations using weighted standard deviations. *Quality and Reliability Engineering International* 20, 31–46.

3. Daudin, J. J. (1992). Double Sampling $\overline{X}$ Charts. *Journal of Quality Technology* 24, 78-87.

4. Ghute VB and Shirke DT. (2008). A multivariate synthetic control chart for monitoring process mean vector. *Communications in Statistics – Theory and Methods* 37, 2136 – 2148.

5. Wu, Z. and Spedding, T. A. (2000). A Synthetic Control Chart for Detecting Small Shifts in the process Mean. *Journal of Quality Technology* 32, 32-38.

F. SUMMARY OF RESEARCH FINDINGS

Ringkasan dapatan Projek Penyelidikan

The main findings of this research are the proposed extensions on synthetic control charts that improve upon the performances of existing charts. Two papers have been published in ISI indexed journals and several others in local journals and conference proceedings.

G. COMPREHENSIVE TECHNICAL REPORT

Laporan Teknikal Lengkap

Applicants are required to prepare a comprehensive technical report explaining the project.
(This report must be attached separately)
Sila sediakan laporan teknikal lengkap yang menerangkan keseluruhan projek ini.
[Laporan ini mesti dikepilkan]

List the key words that reflect our research:

Senaraikan kata kunci yang mencerminkan penyelidikan anda:

English	Bahasa Malaysia
Synthetic control chart	Carta kawalan sintetik
Double sampling control chart	Carta kawalan berganda dua
Neural Network	Rangkaian Neural

H.

a) Results/Benefits of this research
Hasil Penyelidikan

No. *Bil:*	Category/Number: *Kategori/ Bilangan:*	Promised	Achieved
1.	**Research Publications (Specify target journals)** *Penerbitan Penyelidikan (Nyatakan sasaran jurnal)*	8	10
2.	**Human Capital Development**		
	a. Ph. D Students	1	1
	b. Masters Students	2	1
	c. Undergraduates (Final Year Project)	0	4
	d. Research Officers	2	2
	e. Research Assisstants	2	2
	f. Other: Please specify		
3.	**Patents** *Paten*		
4.	**Specific / Potential Applications** *Spesifik/Potensi aplikasin*		
5.	**Networking & Linkages** *Jaringan & Jalinan*	2	2
6.	**Possible External Research Grants to be Acquired** *Jangkaan Geran Penyelidikan Luar Diperoleh*		

- Kindly provide copies/evidence for Category 1 to 6.

b) Equipment used for this research.
Peralatan yang telah digunakan dalam penyelidikan ini.

Items *Perkara*	Approved Equipment	Approved Requested Equipment	Location
Specialized Equipment Peralatan khusus			
Facility Kemudahan			
Infrastructure Infrastruktur			

- Please attach appendix if necessary.

I. **BUDGET** / *BAJET*

Perbelanjaan :Expenditure

Project Account No.	: 1001 / PMATHS / 811024
Total Approved Budget	: RM 122759
Total Additional Budget	: RM 0
Grand Total of Approved Budget	: RM 122759

Yearly Budget Distributed

Year 1	: RM 25343
Year 2	: RM 47590
Year 3	: RM 49826

Additional Budget Approved

Year 1	: RM 0
Year 2	: RM 0
Year 3	: RM 0

Total Expenditure	: RM 103,200.71
Balance	: RM 19,558.29 (as of Sept 30, 2010)

- **Please attach final account statement from Treasury**

Michael Khoo

Signature of Researcher
Tandatangan Penyelidik

Nov 16, 2010
Date
Tarikh

H. **COMMENTS OF PTJ'S RESEARCH COMMITTEE**
KOMEN JAWATANKUASA PENYELIDIKAN PERINGKAT PTJ

General Comments:
Ulasan Umum:

Telah dibincang & diperakukan.

Mesy Jkh PS & Pengetua ke PTSM 24/11/2010

PROF. MADYA AHMAD IZANI MD. ISMAIL
DEKAN
PUSAT PENGAJIAN SAINS MATEMATIK
UNIVERSITI SAINS MALAYSIA
11800 PULAU PINANG

.....

Signature and Stamp of Chairperson of PTJ's Evaluation Committee
Tandatangan dan Cop Pengerusi Jawatankuasa Penilaian PTJ

Date : 25/11/10
Tarikh :

PROF. MADYA AHMAD IZANI MD. ISMAIL
DEKAN
PUSAT PENGAJIAN SAINS MATEMATIK
UNIVERSITI SAINS MALAYSIA
11800 PULAU PINANG

Signature and Stamp of Dean/ Director of PTJ
Tandatangan dan Cop Dekan/ Pengarah PTJ

Date : 25/11/10
Tarikh :

TECHNICAL REPORT
RESEARCH UNIVERSITY GRANT
(Oct 22, 2007 - Sept 30, 2010)
Universiti Sains Malaysia

A Neural Network Approach to Synthetic Control Chart for the Process Mean

Research Leader:
Prof. Michael Khoo Boon Chong
Research Collaborators:
1) Assoc. Prof. Low Heng Chin
2) Dr. Ong Hong Choon

Technical Report (Nov 2010)

SCHOOL OF MATHEMATICAL SCIENCES
UNIVERSITI SAINS MALAYSIA

1. Introduction

This research proposes two main extensions of quality control charts. The first extension deals with a proposed multivariate synthetic control chart for monitoring the process mean vector of skewed populations using weighted standard deviations while the second involves the construction of a synthetic double sampling (DS) chart for the mean. This technical report is organized as follows : Section 2 explains the research background while Section 3 discusses the research objectives achieved. Section 4 gives an overview of the two proposed control charting methods and a discussion on the comparison of the proposed charts with existing charts. Section 5 describes the performance evaluation and analysis of results. Section 6 concludes the technical report and identifies some future works.

2. Research Background

There are many situations where the quality of a process is determined by at least two related quality characteristics (Chang, 2007). Since these quality characteristics are correlated, monitoring them independently is misleading. Thus, multivariate charts are developed to enable a simultaneous monitoring of several related variables. The commonly used multivariate charts are the Hotelling's T^2, multivariate cumulative sum (MCUSUM), and multivariate exponentially weighted moving average (MEWMA) charts. A potential disadvantage of multivariate charts is the multivariate normality assumption of the underlying process distribution. In practice, the normality assumption is usually violated. For example, measurements from chemical processes, filling processes, and semiconductor processes are often skewed (Chang, 2007). For a skewed population, the false alarm rate of a conventional chart increases with the skewness. Thus, this project proposes a multivariate synthetic chart, which comprises a WSD Hotelling's T^2 sub-chart and a CRL sub-chart, for monitoring the mean vector of skewed populations using the weighted standard deviation (WSD) method. The WSD T^2 chart was suggested by Chang and Bai (2004) and the CRL chart by Bourke (1991).

Another problem faced by a conventional control chart is its inability to detect small and moderate shifts promptly. Hence, adaptive charts, like the double sampling (DS) chart of Daudin (1992), which has warning limits that enable a quicker detection of an out-of-control signal without increasing the false alarm rate, have been suggested. This project shows that the out-of-control detection speed of the DS chart can be further enhanced by constructing a synthetic DS chart. A synthetic DS chart consists of the basic DS chart of Daudin (1992) and the CRL chart of Bourke (1991).

3. Research Objectives Achieved

The main objectives achieved in this research are as follows:

(i) A multivariate synthetic control chart for monitoring the process mean vector of skewed populations using weighted standard deviations has been proposed. This work was published in an ISI indexed journal, i.e., Communications in Statistics – Simulation and Computation, 38 (7), 1493 – 1518, in 2009.

(ii) A synthetic double sampling (DS) chart for the mean has been proposed. This work will be published in IIE Transactions, 43 (1), 1 – 16, in 2011. IIE Transactions is an ISI indexed journal.

4. Contributions of the Research

4.1. A proposed multivariate synthetic chart for monitoring the mean vector of skewed populations using WSDs

The purpose of this work is to improve the sensitivity of the T^2 chart based on the WSD approach, which is used in the monitoring of a multivariate process mean from a skewed population. In this work, a synthetic WSD–T^2 chart is developed by combining the standard WSD–T^2 sub-chart (see Chang and Bai, 2004) and the CRL sub-chart. The proposed synthetic WSD–T^2 chart is a multivariate version of the univariate chart suggested by Khoo et al. (2008).

The standard WSD T^2 statistic for the i^{th} sample is defined as (Khoo et al. (2009))

$$T^2_{W,i} = \left(\boldsymbol{Z}_i^W\right)' \rho^{-1} \boldsymbol{Z}_i^W, \text{ for } i = 1, 2, ..., \quad (1)$$

where $\boldsymbol{Z}_i^W = \left(Z_{1i}^W, Z_{2i}^W, ..., Z_{pi}^W\right)'$. Here,

$$Z_{ji}^W = \frac{\bar{X}_{ji} - \mu_j}{W_j \sigma_j / \sqrt{n}} = \begin{cases} \dfrac{1}{2P_j} Z_{ji}, & \text{if } X_j > \mu_j \\ \dfrac{1}{2(1-P_j)} Z_{ji}, & \text{otherwise} \end{cases}. \quad (2)$$

Note that $P_j = \Pr\left(\bar{X}_{ji} \le \mu_j\right)$, $Z_{ji} = \dfrac{\bar{X}_{ji} - \mu_j}{\sigma_j / \sqrt{n}}$ and ρ is the correlation matrix for $\boldsymbol{X}$. The WSD T^2 statistic is plotted on the WSD T^2 sub-chart, where a nonconforming sample is obtained when $T^2_{W,i}$ > UCL. The number of nonconforming samples between two conforming ones (inclusive of the last nonconforming sample) is computed as the CRL value of the CRL sub-chart. An out-of-control signal is detected if CRL < L. Here, the optimal limits (UCL, L) are computed using the approach discussed in Khoo et al. (2009).

4.2. A proposed synthetic double sampling (DS) chart for the mean

This work suggests a synthetic DS chart for the mean (Khoo et al., 2011). The synthetic DS chart integrates two basic charts, i.e., the DS sub-chart of Daudin (1992) and the CRL sub-chart of Bourke (1991). The synthetic DS chart was proposed to improve upon the performance of the DS chart of Daudin (1992) and synthetic chart of Wu and Spedding (2000). The DS sub-chart control charting process consists of two sampling stages. In sampling stage 1, a sample is taken and if $\bar{X}$ plots within $-L_1$ and L_1, the process is in-control. However, if $\bar{X}$ plots beyond the L/$-L$ limits, the process is out-of-control. A second sample is taken if $\bar{X}$ plots in the region (L_1, L) or ($-L$, $-L_1$). For this case, the grand sample mean, $\bar{\bar{X}}$ (average of the 1st and 2nd sample means) is computed and if $\bar{\bar{X}}$ falls within ($-L_2$, L_2), the process is in-control, otherwise the process is out-of-control.

5. Performance Evaluation and Analysis of Results

SAS (Statistical Analysis System) programs based on the Monte Carlo simulations are written to evaluate the performances of the proposed charts. Comparisons of performances between the proposed and existing charts have been made. The detailed results are discussed in Khoo et al. (2009) and Khoo et al. (2011). This section provides a summary of the results.

The synthetic WSD–T^2 chart gives favorable results by having the highest mean shift detection rates for moderate and large shifts when process parameters are known and unknown, based on most levels of skewnesses, in comparison to the Hotelling's T^2, WSD–T^2, WSD–MEWMA and WSD–MCUSUM charts. The synthetic WSD–T^2 chart also has lower false alarm rates than all the above mentioned charts, except the WSD–MCUSUM chart, when subgrouping is considered.

The proposed synthetic DS chart outperforms the synthetic chart of Wu and Spedding (2000) and the DS chart of Daudin (1992). It is also shown that the proposed synthetic DS chart have better zero-state average run length (ARL) and average number of observations to sample (ANOS) performances than the EWMA chart for detecting a shift in the mean of about 0.5σ onwards. When the size of a mean shift is small and the mean sample size $n = 5$, the proposed procedure reduces the out-of-control ARL and ANOS values by nearly half, compared with both the synthetic and DS charts. In terms of detection ability versus the EWMA chart, the synthetic DS chart is superior to the synthetic or even the DS chart, as the former outperforms the EWMA chart for a larger range of magnitudes of shifts compared to the latter.

6. Conclusions

The main works in this project deals with two types of proposed control charting schemes, i.e., the multivariate synthetic chart for monitoring the mean vector of skewed populations using WSDs (Khoo et al., 2009) and the synthetic double sampling (DS) chart (Khoo et al., 2011). The proposed charts address the shortcomings and enhance the performances of existing charts. The following are some potential future works in this area that can be carried out:

(i) A study on the performance of the synthetic WSD–T^2 chart under a steady state mode, where a mean shift occurs randomly at any time.

(ii) Constructions of Hotelling's T^2, MEWMA, MCUSUM and synthetic T^2 charts incorporating the weighted variance (WV) method.

(iii) Development of new multivariate control charting methods to monitor the process dispersion of skewed populations.

(iv) A study of the performance of the synthetic DS chart for the steady state mode.

References

1. Bourke, P. D. (1991). Detecting a shift in fraction nonconforming using run-length control charts with 100% inspection. *Journal of Quality Technology* 23 (3), 225-238.

2. Chang, Y. S. (2007). Multivariate CUSUM and EWMA control charts for skewed populations using weighted standard deviations. *Communications in Statistics – Simulation and Computation* 36 (4), 921–936.

3. Chang, Y. S. and Bai, D. S. (2004). A multivariate T^2 control chart for skewed populations using weighted standard deviations. *Quality and Reliability Engineering International* 20 (1), 31–46.

4. Daudin, J.J. (1992). Double sampling $\overline{X}$ charts. *Journal of Quality Technology*, 24 (2), 78–87.

5. Khoo M. B. C., Abdu M. A. Atta and Wu, Z. (2009). A multivariate synthetic control chart for monitoring the process mean vector of skewed populations using weighted standard deviations. *Communications in Statistics – Simulation and Computation*, 38 (7), 1493 – 1518.

6. Khoo M. B. C., Lee, H. C., Wu, Z., Chen, C-H. and Castagliola, P. (2011). A Synthetic Double Sampling Control Chart for the Process Mean. *IIE Transactions*, 43 (1), 1 – 16.

7. Khoo, M. B. C., Wu, Z. and Atta, A. M. A. (2008). A synthetic control chart for monitoring the process mean of skewed populations based on the weighted variance method. *International Journal of Reliability Quality and Safety Engineering*, 15 (3), 217 – 245.

8. Wu, Z. and Spedding, T. A. (2000). A synthetic control chart for detecting small shifts in the process mean. *Journal of Quality Technology*, 32 (1), 32 – 38.

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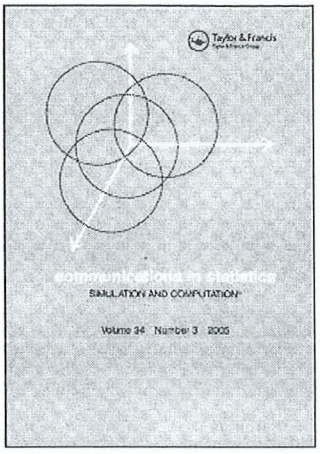

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A Multivariate Synthetic Control Chart for Monitoring the Process Mean Vector of Skewed Populations Using Weighted Standard Deviations

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[a] School of Mathematical Sciences, Univertiti Sains Malaysia, Penang, Malaysia [b] School of Mechanical and Aerospace Engineering, Nanyang Technological University, Singapore

Online Publication Date: 01 August 2009

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DOI: 10.1080/03610910903019905

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A Multivariate Synthetic Control Chart for Monitoring the Process Mean Vector of Skewed Populations Using Weighted Standard Deviations

MICHAEL B. C. KHOO[1], ABDU M. A. ATTA[1], AND ZHANG WU[2]

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[2]School of Mechanical and Aerospace Engineering, Nanyang Technological University, Singapore

This article proposes a multivariate synthetic control chart for skewed populations based on the weighted standard deviation method. The proposed chart incorporates the weighted standard deviation method into the standard multivariate synthetic control chart. The standard multivariate synthetic chart consists of the Hotelling's T^2 chart and the conforming run length chart. The weighted standard deviation method adjusts the variance–covariance matrix of the quality characteristics and approximates the probability density function using several multivariate normal distributions. The proposed chart reduces to the standard multivariate synthetic chart when the underlying distribution is symmetric. In general, the simulation results show that the proposed chart performs better than the existing multivariate charts for skewed populations and the standard T^2 chart, in terms of false alarm rates as well as moderate and large mean shift detection rates based on the various degrees of skewnesses.

Keywords Conforming run length; Multivariate control charts; Multivariate synthetic control chart; Skewed populations; Weighted standard deviation; Weighted variance.

Mathematics Subject Classification 62P30; 62E17.

1. Introduction

There are many situations where the quality of a process is determined by at least two related quality characteristics. For example, the quality of a chemical process may depend on the joint level of viscosity, concentration, molecular weight and pH (Chang, 2007). Since these quality characteristics are correlated, monitoring

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them independently is misleading. Thus, multivariate control charts are developed to enable a simultaneous monitoring of several related variables.

The commonly used multivariate control charts are the Hotelling's T^2, multivariate cumulative sum (MCUSUM), and multivariate exponentially weighted moving average (MEWMA) charts. To date, numerous works on multivariate charts have been made. Excellent summaries and discussions of most of these works were given in Bersimis et al. (2007).

A potential disadvantage of control charts is the normality assumption of the underlying process distribution. In practice, the normality assumption is usually violated. For example, measurements from chemical processes, filling processes, and semiconductor processes are often skewed (Chang, 2007). For a skewed population, the false alarm rate of a conventional chart increases with the skewness. Numerous univariate control charting approaches had been suggested in the literature to address this problem. These include the heuristic methods, such as the $\overline{X}$ and R charts based on the weighted variance (WV) method proposed by Bai and Choi (1995), the $\overline{X}$, cumulative sum (CUSUM), and exponentially weighted moving average (EWMA) charts based on the weighted standard deviation (WSD) method suggested by Chang and Bai (2001), and the $\overline{X}$ and R charts based on the skewness correction method presented by Chan and Cui (2003). Other works on skewed univariate charts include that by Yourstone and Zimmer (1992), Wu (1996), Castagliola (2000), Dou and Sa (2002), Chen (2004), Nichols and Padgett (2005), and Tsai (2007). For a multivariate process, the T^2 (Chang and Bai, 2004), MEWMA, and MCUSUM (Chang, 2007) charts based on the WSD method have been proposed to solve the problem of multivariate skewness.

One recent method developed in the literature to enhance the performances of univariate and multivariate charts is to combine the chart with a conforming run length (CRL) chart, leading to a synthetic control chart. The CRL chart is an attribute chart developed by Bourke (1991) to monitor the fraction nonconforming. Wu and Spedding (2000a) proposed a univariate synthetic $\overline{X}$ control chart as a combination of the $\overline{X}$ and CRL charts and show that the new chart outperforms the standard $\overline{X}$ chart for all magnitudes of shifts. It also surpasses the EWMA chart in terms of the detection power for a mean shift of greater than 0.8σ. Other research articles that deal with univariate synthetic charts were suggested by Wu and Spedding (2000b), Wu and Yeo (2001), Wu et al. (2001), Calzada and Scariano (2001), Davis and Woodall (2002), Scariano and Calzada (2003), Huang and Chen (2005), Chen and Huang (2005), and Costa and Rahim (2006). Ghute and Shirke (2008a) extended the synthetic $\overline{X}$ chart proposed by Wu and Spedding (2000a) to the multivariate case by combining the Hotelling's T^2 chart and the CRL chart. Ghute and Shirke (2008b) also suggested a multivariate synthetic chart for the process dispersion. A joint monitoring of both the multivariate process mean and variance using a combined scheme involving a simultaneous use of the synthetic T^2 and the synthetic $|S|$ charts was presented by Ghute and Shirke (2007). Costa and Machado (2007) considered a synthetic chart with two stage sampling to control bivariate processes. All the univariate and multivariate synthetic charts that are available today are based on the normality assumption of the underlying process distribution.

Khoo et al. (2008) proposed an univariate synthetic chart for monitoring the mean of skewed populations using the WV method. The purpose of this article is to improve the sensitivity of the T^2 chart based on the WSD approach, which is used in the monitoring of a multivariate process mean from a skewed population.

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In this article, a synthetic WSD–T^2 chart is developed by combining the standard WSD–T^2 chart (see Chang and Bai, 2004) and the CRL chart. The proposed synthetic WSD–T^2 chart is a multivariate extension of the chart suggested by Khoo et al. (2008).

The discussion hereafter is organized as follows. Section 2 reviews some related multivariate control charts. In Sec. 3, the synthetic WSD–T^2 chart is developed. A performance comparison is made in Sec. 4. In Sec. 5, a numerical example is given to illustrate how the proposed synthetic WSD–T^2 chart is constructed. Conclusions are drawn in Sec. 6.

2. Review of Related Multivariate Control Charts

Section 2.1 explains the Hotelling's T^2 chart, while Sec. 2.2 reviews the synthetic T^2 chart. The T^2, MEWMA, and MCUSUM charts using the WSD method for the monitoring of skewed populations are discussed in Sec. 2.3.

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2.1. *Hotelling's T^2 Chart*

Let $X_1, X_2, \ldots, X_n$ represent a sequence of vectors from a random sample having a p-variate normal distribution with in-control mean vector, μ_0, and covariance matrix, Σ_0. The T^2 chart signals a shift in the mean vector if the T^2 statistic obtained from the ith sample,

$$T_i^2 = n(\overline{X}_i - \mu_0)'\Sigma_0^{-1}(\overline{X}_i - \mu_0) = Z_i'\rho^{-1}Z_i, \quad \text{for } i = 1, 2, \ldots, \tag{1}$$

exceeds a predetermined control limit $h = \chi_\alpha^2(p)$, where α is a specified Type I error and $\chi_\alpha^2(p)$ is the $100(1-\alpha)$th percentile of the χ^2 distribution with p degrees of freedom (Chang and Bai, 2004). Note that $\overline{X}_i = (\overline{X}_{1i}, \overline{X}_{2i}, \ldots, \overline{X}_{pi})'$ is the ith sample mean vector, $Z_i = (Z_{1i}, Z_{2i}, \ldots, Z_{pi})'$ is the ith standardized mean vector and n is the sample size. Here, the jth element of Z_i is

$$Z_{ji} = \frac{\overline{X}_{ji} - \mu_j}{\sigma_j/\sqrt{n}}, \quad \text{for } j = 1, 2, \ldots, p. \tag{2}$$

Note also that ρ is the correlation matrix for X.

2.2. *Synthetic T^2 Chart*

The synthetic T^2 chart for monitoring the mean of a multivariate normally distributed process was suggested by Ghute and Shirke (2008a). It consists of a combination of the Hotelling's T^2 (a.k.a. the T^2 sub-chart) and the CRL (a.k.a. the CRL sub-chart) charts. Here, CRL is defined as the number of conforming units between two consecutive nonconforming units, including the end nonconforming unit. The control flow of the synthetic T^2 chart is as follows (Ghute and Shirke, 2008a):

Step 1. Determine the upper control limit (UCL) of the T^2 sub-chart and the lower control limit, L, of the CRL sub-chart. Ghute and Shirke (2008a) presented a procedure and formulae for computing the optimal values of UCL and L.

Step 2. Take a p-variate random sample of size n, at each inspection point and compute the statistic T^2 using Eq. (1), for the T^2 sub-chart.

Step 3. If $T^2 \leq \text{UCL}$, the sample is classified as a conforming sample and the control flow returns to Step 2. Otherwise, the sample is considered as a nonconforming sample and the control flow proceeds to Step 4.

Step 4. Check the number of T^2 samples between the present and the last nonconforming samples. This number is the CRL value of the CRL sub-chart.

Step 5. If $\text{CRL} \geq L$, conclude that the process is in-control and the control flow returns to Step 2. Otherwise, the process is considered as out-of-control and the control flow proceeds to Step 6.

Step 6. Signal an out-of-control status.

Step 7. Find and remove the assignable cause. Then the control flow returns to Step 2.

2.3. *Multivariate Charts for Skewed Populations Using Weighted Standard Deviations*

2.3.1. WSD–T^2 Chart. Chang and Bai (2004) proposed the WSD–T^2 chart based on a modification of the T^2 statistic in Eq. (1) with the WSD method. For this case, when the ith sample is obtained, it is standardized as (Chang and Bai, 2004)

$$\mathbf{Z}_i^W = \mathbf{W}^{-1}\mathbf{Z}_i, \quad \text{for } i = 1, 2, \ldots, \tag{3}$$

of which the jth element is

$$Z_{ji}^W = \frac{\overline{X}_{ji} - \mu_j}{W_j \sigma_j / \sqrt{n}} = \begin{cases} \dfrac{1}{2P_j} Z_{ji}, & \text{if } X_j > \mu_j \\ \dfrac{1}{2(1-P_j)} Z_{ji}, & \text{otherwise} \end{cases}. \tag{4}$$

Then the WSD–T^2 statistic for the ith sample is defined as

$$T_{W.i}^2 = (\mathbf{Z}_i^W)' \rho^{-1} \mathbf{Z}_i^W, \quad \text{for } i = 1, 2, \ldots, \tag{5}$$

where $\boldsymbol{Z}_i^{\mathrm{W}} = (Z_{1i}^W, Z_{2i}^W, \ldots, Z_{pi}^W)'$. Because the WSD method approximates a probability density function with segments from p-variate normal distributions, the control limit $h = \chi_\alpha^2(p)$ can be used approximately for the WSD–T^2 chart (Chang and Bai, 2004). Thus, an out-of-control signal is given when $T_{W.i}^2 > h$. Note that in Eq. (3),

$$\boldsymbol{W} = \text{diag}\{W_1, W_2, \ldots, W_p\}, \tag{6}$$

where

$$W_j = \begin{cases} 2P_j, & \text{if } \overline{X}_{ji} > \mu_j \\ 2(1-P_j), & \text{otherwise} \end{cases}. \tag{7}$$

Here, $P_j = \Pr(\overline{X}_{ji} \leq \mu_j)$. For symmetric distributions, the WSD–T^2 chart reduces to the T^2 chart.

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2.3.2. WSD–MEWMA Chart. Chang (2007) developed the WSD–MEWMA chart to improve the performance of the standard MEWMA chart for skewed populations. The WSD–MEWMA chart's statistic is defined as

$$\boldsymbol{M}_i^{\mathrm{W}} = \boldsymbol{\lambda}\boldsymbol{Z}_i^{\mathrm{W}} + (\boldsymbol{I} - \boldsymbol{\lambda})\boldsymbol{M}_{i-1}^{\mathrm{W}}, \quad \text{for } i = 1, 2, \ldots, \tag{8}$$

where $\boldsymbol{M}_0^{\mathrm{W}} = \mathbf{0}$ and $\boldsymbol{\lambda} = \mathrm{diag}\{\lambda_1, \lambda_2, \ldots, \lambda_p\}$, $0 < \lambda_j \leq 1$, for $j = 1, 2, \ldots, p$. Since $\boldsymbol{Z}_i^{W}$ follows a multivariate normal distribution with mean vector $\mathbf{0}$ and variance–covariance matrix $\boldsymbol{\rho}$ approximately, the asymptotic variance–covariance matrix of $\boldsymbol{M}_i^{\mathrm{W}}$ is $\Sigma_M = [\lambda/(2 - \lambda)]\boldsymbol{\rho}$. An out-of-control signal is detected when the chart's statistic,

$$E_i^{W} = (\boldsymbol{M}_i^{\mathrm{W}})'\Sigma_M^{-1}\boldsymbol{M}_i^{\mathrm{W}} \tag{9}$$

exceeds the control limit, h_E, where h_E is determined based on a desired in-control ARL when the underlying process is assumed to follow a multivariate normal distribution. It was noted by Chang (2007) that since the WSD method approximates the probability density function of the underlying distribution with segments from p-variate normal distributions, the control limit of the MEWMA chart can be used approximately for the WSD–MEWMA chart. The WSD–MEWMA chart reduces to the MEWMA chart when the distribution of the underlying process is symmetric.

2.3.3. WSD–MCUSUM Chart. Chang (2007) also developed the WSD–MCUSUM chart by considering the MCUSUM chart of Crosier (1988). For this case, when the ith sample is taken, it is standardized using Eqs. (3) and (4). Then $\boldsymbol{Z}_i^{\mathrm{W}}$, for $i = 1, 2, \ldots$, follows a $\mathbf{N}_p(\mathbf{0}, \rho)$ distribution approximately. Next, the cumulative statistic at i, i.e., $\boldsymbol{A}_i^{\mathrm{W}} = \boldsymbol{S}_{i-1}^{\mathrm{W}} + \boldsymbol{Z}_i^{\mathrm{W}}$, for $i = 1, 2, \ldots$, is computed, where $\boldsymbol{S}_0^{\mathrm{W}} = \mathbf{0}$. The length of $\boldsymbol{A}_i^{\mathrm{W}}$, which is $C_i^{\mathrm{W}} = [(\boldsymbol{A}_i^{\mathrm{W}})'\boldsymbol{\rho}^{-1}\boldsymbol{A}_i^{\mathrm{W}}]^{1/2}$, is then computed. The size of a mean shift modified according to the adjusted variance–covariance matrix with WSDs is given as (Chang, 2007)

$$d^{\mathrm{W}}(\boldsymbol{\mu}_1) = [(\boldsymbol{\mu}_1 - \boldsymbol{\mu}_0)'(\Sigma^{\mathrm{W}})^{-1}(\boldsymbol{\mu}_1 - \boldsymbol{\mu}_0)]^{1/2} \tag{10}$$

and the reference value, $k^{\mathrm{W}} = d^{\mathrm{W}}(\boldsymbol{\mu}_1)/2$, where $\Sigma^{\mathrm{W}} = \boldsymbol{W}(\boldsymbol{\mu}_1 - \boldsymbol{\mu}_0) \cdot \Sigma \cdot \boldsymbol{W}(\boldsymbol{\mu}_1 - \boldsymbol{\mu}_0)$. Here, Σ, $\boldsymbol{\mu}_0$, and $\boldsymbol{\mu}_1$ are the variance–covariance matrix, in-control mean vector, and out-of-control mean vector of $\boldsymbol{X}$, respectively. Also, $\boldsymbol{W}(\boldsymbol{\mu}_1 - \boldsymbol{\mu}_0)$ is defined as $\boldsymbol{W}(\boldsymbol{\mu}_1 - \boldsymbol{\mu}_0) = \mathrm{diag}\{W_1(\mu_1' - \mu_1), W_2(\mu_2' - \mu_2), \ldots, W_p(\mu_p' - \mu_p)\}$, where

$$W_j(x) = \begin{cases} 2P_j, & \text{if } x > 0 \\ 2(1 - P_j), & \text{otherwise} \end{cases}, \quad \text{for } j = 1, 2, \ldots, p. \tag{11}$$

Note that $\boldsymbol{\mu}_0 = (\mu_1, \mu_2, \ldots, \mu_p)'$ and $\boldsymbol{\mu}_1 = (\mu_1', \mu_2', \ldots, \mu_p')'$.

Now, set k to be

$$k = d(\boldsymbol{\mu}_1)/2 = \left[(\boldsymbol{\mu}_1 - \boldsymbol{\mu}_0)'\Sigma^{-1}(\boldsymbol{\mu}_1 - \boldsymbol{\mu}_0)\right]^{1/2}/2 \tag{12}$$

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for detecting a mean shift of size $d(\mu_1)$ at the quickest speed. The jth element of S_i^W is

$$S_{ji}^W = \begin{cases} A_{ji}^W \left(1 - \dfrac{k}{2P_j C_i^W}\right), & \text{if } C_i^W > k^W \text{ and } A_{ji}^W > 0 \\ A_{ji}^W \left(1 - \dfrac{k}{2(1-P_j) C_i^W}\right), & \text{if } C_i^W > k^W \text{ and } A_{ji}^W \leq 0 \\ 0, & \text{otherwise} \end{cases} \quad (13)$$

In Eq. (13), A_{ji}^W is the jth element of A_i^W. The WSD–MCUSUM chart issues an out-of-control signal if its statistic,

$$Y_i^W = [(S_i^W)' \rho^{-1} S_i^W]^{1/2}, \quad (14)$$

exceeds the control limit, h_c, where h_c is determined so that the chart gives a desired in-control ARL when the process distribution is multivariate normal (Chang, 2007). Here, the control limit of the MCUSUM chart can be used approximately for the WSD–MCUSUM chart based on the same explanation given in Sec. 2.3.2. The WSD–MCUSUM chart reduces to the MCUSUM chart for symmetric distributions.

3. A Proposed Multivariate Synthetic WSD–T^2 Chart

The WSD–T^2 chart was shown to provide considerable improvement over the standard T^2 chart for skewed distributions, in terms of false alarm rates and out-of-control average run lengths (ARLs) by Chang and Bai (2004). A synthetic WSD–T^2 chart is proposed in this paper to improve the performances of the WSD–T^2 chart when the underlying process distribution is skewed. It will be shown in Sec. 4 that the synthetic WSD–T^2 chart provides a remarkable improvement over all the existing charts for skewed populations under consideration, in terms of false alarm and out-of-control mean shift detection rates when the underlying process follows a skewed distribution. Note that when the underlying process distribution is symmetric, the synthetic WSD–T^2 chart reduces to the synthetic T^2 chart of Ghute and Shirke (2008a).

The synthetic WSD–T^2 chart is based on the idea of integrating the WSD method of Chang and Bai (2004) and the synthetic T^2 chart approach of Ghute and Shirke (2008a). The synthetic WSD–T^2 chart consists of a WSD–T^2 sub-chart and a CRL sub-chart. The operation of the synthetic WSD–T^2 chart is similar to that of the synthetic T^2 chart discussed in Sec. 2.2, except in Step 2 of Sec. 2.2. In this step, the WSD–T^2 statistic is computed for the WSD–T^2 sub-chart using Eq. (5), in the construction of the synthetic WSD–T^2 chart.

Note that the optimal limits, UCL_S and L_S, for the WSD–T^2 sub-chart and the CRL sub-chart, respectively, are computed assuming that the underlying process follows a multivariate normal distribution. They are also employed for the case of skewed populations because the WSD method can effectively give a multivariate normal approximation of the probability density function (PDF) of a skewed multivariate distribution (Chang and Bai, 2004).

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The optimal limits, UCL_S and L_S, are determined by using the method discussed by Ghute and Shirke (2008a) for the synthetic T^2 chart. The optimal design of the synthetic WSD–T^2 chart depends on the desired in-control average run length, $ARL_S(0)$, and the out-of-control one, $ARL_S(d)$. Here, $d = d(\mu_1) \neq 0$ (see Eq. (12)) is the magnitude of a shift considered large enough to seriously impair the quality of a process. Thus, the corresponding $ARL_S(d)$ should be as small as possible so that this shift can be detected at the quickest speed. When $d = 0$, the process is in-control. Note that $ARL_S(0)$ is decided by the requirement of the false alarm rate, α, since $ARL_S(0) = \frac{1}{\alpha}$. The in-control ARL of the synthetic WSD–T^2 chart, assuming an underlying multivariate normal distribution is given as (Ghute and Shirke, 2008a)

$$ARL_S(0) = \frac{1}{P(0)} \times \frac{1}{1 - [1 - P(0)]^{L_S}}, \tag{15}$$

where

$$P(0) = \Pr(T^2_{W,i} > UCL_S \mid d = 0) = \Pr(T^2_i > UCL_S \mid d = 0) = 1 - F_p(UCL_S). \tag{16}$$

Here, $F_p(x) = \Pr(X \leq x)$, where X follows a central chi-square distribution with p degrees of freedom while $T^2_{W,i}$ is the charting statistic of the WSD–T^2 sub-chart. The control charting statistic of the WSD–T^2 sub-chart discussed here is similar to that of the WSD–T^2 chart presented in Sec. 2.3.1.

The design of the synthetic WSD–T^2 chart, based on an underlying multivariate normal distribution is made by solving an optimization problem. The objective function to be minimized is (Ghute and Shirke, 2008a)

$$ARL_S(d) = \frac{1}{P(d)} \times \frac{1}{1 - [1 - P(d)]^{L_S}}, \tag{17}$$

subject to the equality constraint in Eq. (15). In Eq. (17),

$$P(d) = \Pr(T^2_{W,i} > UCL_S \mid d = d(\mu_1) \neq 0) = 1 - F_{p,\lambda^2}(UCL_S), \tag{18}$$

where $F_{p,\lambda^2}(x) = \Pr(X' \leq x)$. Here, X' follows a non central chi-square distribution with p degrees of freedom and non centrality parameter λ^2_*. The value of λ^2_* is

$$\lambda^2_* = n(\mu_1 - \mu_0)'\Sigma^{-1}(\mu_1 - \mu_0) = nd^2. \tag{19}$$

The optimal design procedure for the synthetic WSD–T^2 chart is described as follows: This procedure is similar to that of the synthetic T^2 chart (see Ghute and Shirke, 2008a).

Step 1. Set n, p, d, Σ, and $ARL_S(0)$.

Step 2. Initialize L_S as 1.

Step 3. Compute $P(0)$ using Eq. (15). From this value of $P(0)$, compute UCL_S using Eq. (16).

Step 4. Compute $P(d)$ using Eq. (18) from the current values of L_S and UCL_S. Then obtain $ARL_S(d)$ using Eq. (17).

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Table 1
Values of $ARL_S(d)$ for different (L_S, UCL_S) combinations

L_S	UCL_S	$ARL_S(d)$
1	5.9135	3.839
2	6.5879	3.138
3	6.9815	2.927
4	7.2599	2.855
5	**7.4753**	**2.840**
6	7.6509	2.853
7	7.7989	2.880
8	7.9268	2.915

Step 5. If $L_S = 1$, then increase L_S by 1 and go back to Step 3. If $L_S \geq 2$ and $ARL_S(d)$ has been reduced, then increase L_S by 1 and go back to Step 3, otherwise, go to the next step.

Step 6. Stop the iteration. Choose the last (L_S, UCL_S) combination before an increase in the value of $ARL_S(d)$ as the optimal limits because this combination gives the lowest value of $ARL_S(d)$.

To illustrate the design of the synthetic WSD–T^2 chart, consider the case when $n = 5$, $p = 2$, $d = 1$, and $ARL_S(0) = 370$. The $ARL_S(d)$ values for the different combinations of (L_S, UCL_S) limits are shown in Table 1. It is seen that the $ARL_S(d)$ values first decline and then increase. The minimum $ARL_S(d)$ is 2.840 when $L_S = 5$ and $UCL_S = 7.4753$. Thus, the optimal limits for the synthetic WSD–T^2 chart are $L_S = 5$ for the CRL sub-chart and $UCL_S = 7.4753$ for the WSD–T^2 sub-chart.

The optimal limits of the synthetic T^2 chart provided by Ghute and Shirke (2008a) for some cases of n, p, d, and $ARL_S(0)$ can also be used for the synthetic WSD–T^2 chart because the limits for the latter are obtained assuming that the process follows an underlying multivariate normal distribution. If other choices of n, p, d, and $ARL_S(0)$ are required, the six steps approach discussed above is employed to compute the optimal (L_S, UCL_S) limits for the synthetic WSD–T^2 chart.

The main idea in the construction of the synthetic WSD–T^2 chart is to use the heuristic WSD method proposed by Chang and Bai (2004) to construct the WSD–T^2 sub-chart and then merging the WSD–T^2 sub-chart with the CRL sub-chart to form the synthetic WSD–T^2 chart. The WSD method adjusts the variance–covariance matrix of the quality characteristics in the WSD–T^2 sub-chart's statistic according to the direction and degree of skewness. Chang and Bai (2004) pointed out that the WSD method effectively approximates the probability density function of a skewed distribution using several multivariate normal distributions. Hence, the (L_S, UCL_S) limits of the synthetic WSD–T^2 chart obtained based on the multivariate normality assumption are still reasonably optimal for multivariate skewed populations.

4. Performance of the Synthetic WSD–T^2 Chart

In this section, we compare the performance of the proposed synthetic WSD–T^2 chart with that of the Hotelling's T^2 chart, the WSD–T^2 chart suggested by Chang

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and Bai (2004), and the WSD–MEWMA and WSD–MCUSUM charts developed by Chang (2007).

These charts are compared based on the false alarm rates when the process is in-control and the mean shift detection rates when the process is out-of-control. The mean shift detection rates are computed under the zero state mode, where a mean shift is assumed to occur immediately after inspecting an entire sample or at the beginning of a new sample. In this paper, the bivariate normal, bivariate lognormal, Lee's bivariate Weibull (Lee, 1979), and Cheriyan and Ramabhadran's bivariate gamma distributions (Cheriyan, 1941; Ramabhadran, 1951) are considered in the computation of the false alarm rates when the process is in-control. However, only the bivariate lognormal distribution is considered in the computation of the mean shift detection rates when the process is out-of-control because of space constraint. Kotz et al. (2000) provided an excellent discussion on these distributions. Note that the lognormal distribution can represent various skewnesses and correlations but the Weibull and gamma distributions can only represent some positive correlations (Kotz et al., 2000). It is assumed that the nominal false alarm rate, $\alpha = 0.0027$, when the underlying distribution is bivariate normal. The control limits of the various charts are determined using the approach discussed in Sec. 2, based on $\alpha = 0.0027$.

The limits of the standard Hotelling's T^2 and the WSD–T^2 charts are computed as $h = \chi^2_{0.0027}(2) = 11.829$. The WSD–MEWMA, WSD–MCUSUM, and synthetic WSD–T^2 charts are designed to be optimal for a process mean shift of size $d(\mu_1) = 1$. Here, the optimal $\lambda = \text{diag}\{0.125, 0.125\}$ and the corresponding limit of the WSD–MEWMA chart for the bivariate case is determined to be $h_E = 10.4$. These optimal parameters are determined using the method discussed by Lee and Khoo (2006a). The optimal limit of the WSD–MCUSUM chart is found to be $h_C = 6.213$ using the procedure described in Lee and Khoo (2006b). For the synthetic WSD–T^2 chart, the optimal limits of the WSD–T^2 sub-chart and the CRL sub-chart are UCL_S and L_S, respectively. These limits are given in Table 2 for sample sizes, $n = 1, 3, 5$ and 7 for the bivariate case. Note that for $n = 1$ and 3, the optimal (L_S, UCL_S) limits are computed using the 6 steps procedure presented in Sec. 3, while for $n = 5$ and 7, the optimal limits given by Ghute and Shirke (2008a) are used.

The correlation coefficients, $\rho = 0.3$ and 0.8, are considered. For convenience, the location parameters of (0, 0) are chosen for the bivariate lognormal distribution since the skewness does not depend on them. Similarly, the scale parameters of (1, 1) are chosen for the Weibull and gamma distributions. The scale parameters of the quality characteristics (X_1, X_2) from the bivariate lognormal distribution are chosen to give the desired skewnesses (γ_1, γ_2). For the bivariate Weibull and gamma

Table 2
Optimal limits (L_S, UCL_S) of the synthetic WSD–T^2 chart for the bivariate case, when $n = 1, 3, 5$, and 7

n	L_S	UCL_S
1	25	9.0017
3	8	7.9268
5	5	7.4749
7	4	7.2598

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distributions, the shape parameters of (X_1, X_2) are determined so that the desired skewnesses (γ_1, γ_2) are attained. The false alarm rates are computed for $(\gamma_1, \gamma_2) \in \{(1, 1), (1, 2), (1, 3), (2, 2), (2, 3), (3, 3)\}$ and the sample sizes, $n \in \{1, 3, 5, 7\}$ when the process is in-control, while the mean shift detection rates are computed for $(\gamma_1, \gamma_2) \in \{(1, 1), (2, 2), (3, 3)\}$ and $n = 5$ when the process is out-of-control. Six different directions of the process mean shifts from $\boldsymbol{\mu}_0 = (\mu_1, \mu_2)'$ to $\boldsymbol{\mu}_1 = (\mu_1 + \delta_1\sigma_1, \mu_2 + \delta_2\sigma_2)'$ are considered and the size of the mean shift is $d(\boldsymbol{\mu}_1) = [(\boldsymbol{\mu}_1 - \boldsymbol{\mu}_0)'\Sigma^{-1}(\boldsymbol{\mu}_1 - \boldsymbol{\mu}_0)]^{1/2}$. These directions are similar to that considered by Chang (2007), in his study of the performances of the WSD–MEWMA and WSD–MCUSUM charts. The directions of the shifts are shown in Fig. 1. Here, the mean shifts if at least one $\delta_j \neq 0$, for $j = 1$ and 2. The process is in-control if $\delta_1 = \delta_2 = 0$.

The false alarm rates and mean shift detection rates are obtained from Monte Carlo simulations using the Statistical Analysis System (SAS) version 9.1 program, run on an Intel Pentium 2.8 GHz computer, where all the results are averages of 10,000 repetitions. For cases with unknown parameters, process parameters are estimated from 30 preliminary in-control subgroups, each of size, n.

The false alarm rates of the charts are given in Tables 3 and 4 when parameters are known and unknown, respectively.

Note that the false alarm rates, marked as "*" in these tables for the Cheriyan and Ramabhadran's bivariate gamma distribution cannot be computed because the corresponding shape parameters of one of the gamma distributed components, used in the transformation to compute variate X_2 have negative values. This is consistent with the false alarm rates obtained by Chang and Bai (2004) as the same bivariate gamma distribution is considered here. Table 3 shows that the synthetic WSD–T^2 chart performs significantly better than the standard T^2 and the WSD–T^2 charts for all skewnesses (γ_1, γ_2) and sample sizes, n, when parameters are known. It also shows that generally, the synthetic WSD–T^2 chart outperforms the WSD–MEWMA chart for all skewnesses, when $n = 3, 5$, and 7. On the whole, if parameters are known, it is seen that the WSD–MCUSUM chart produces the lowest false alarm rates for $n = 1$ and 3, when $\rho = 0.3$ and 0.8, while the synthetic WSD–T^2 chart gives the smallest false alarm rates for $n = 5$ and 7, when $\rho = 0.3$. When process parameters are unknown, Table 4 shows that generally, the synthetic WSD–T^2 chart has lower false alarm rates than the standard T^2, WSD–T^2, WSD–MCUSUM and WSD–MEWMA charts for all skewnesses (γ_1, γ_2) and sample sizes, $n = 3, 5$ and 7, except for the case involving the Weibull distribution and $\rho = 0.8$. For this case,

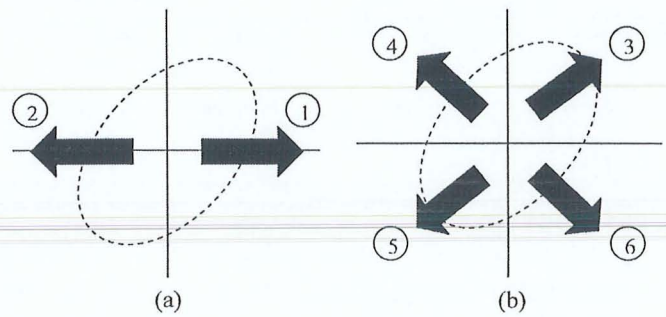

Figure 1. Directions of the mean shift when (a) only μ_1 is shifted (b) both μ_1 and μ_2 are shifted.

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Table 3
False alarm rates for bivariate distributions (parameters known)

		Sample size, n																			
		1					3					5					7				
Distribution	(γ_1, γ_2)	Synth WSD-T^2	Std-T^2	WSD-T^2	WSD-MCUSUM	WSD-MEWMA	Synth WSD-T^2	Std-T^2	WSD-T^2	WSD-MCUSUM	WSD-MEWMA	Synth WSD-T^2	Std-T^2	WSD-T^2	WSD-MCUSUM	WSD-MEWMA	Synth WSD-T^2	Std-T^2	WSD-T^2	WSD-MCUSUM	WSD-MEWMA
										$\rho = 0.3$											
Normal	(0, 0)	0.0027	0.0027	0.0027	0.0027	0.0028	0.0027	0.0027	0.0026	0.0027	0.0027	0.0027	0.0027	0.0027	0.0027	0.0027	0.0027	0.0027	0.0027	0.0027	0.0028
Lognormal	(1, 1)	0.0058	0.0148	0.0085	0.0026	0.0029	0.0024	0.0083	0.0042	0.0026	0.0036	0.0022	0.0064	0.0033	0.0028	0.0038	0.0022	0.0056	0.0031	0.0029	0.0040
	(1, 2)	0.0074	0.0201	0.0110	0.0026	0.0031	0.0025	0.0128	0.0058	0.0026	0.0046	0.0022	0.0099	0.0043	0.0029	0.0053	0.0023	0.0084	0.0037	0.0030	0.0057
	(1, 3)	0.0076	0.0219	0.0117	0.0026	0.0032	0.0026	0.0155	0.0069	0.0025	0.0056	0.0022	0.0128	0.0052	0.0029	0.0068	0.0023	0.0111	0.0045	0.0031	0.0076
	(2, 2)	0.0091	0.0252	0.0133	0.0026	0.0032	0.0027	0.0168	0.0074	0.0025	0.0054	0.0023	0.0134	0.0053	0.0028	0.0064	0.0024	0.0113	0.0043	0.0031	0.0070
	(2, 3)	0.0093	0.0270	0.0141	0.0027	0.0032	0.0028	0.0196	0.0085	0.0024	0.0064	0.0022	0.0161	0.0063	0.0028	0.0079	0.0024	0.0140	0.0052	0.0031	0.0087
	(3, 3)	0.0095	0.0289	0.0149	0.0028	0.0033	0.0030	0.0225	0.0096	0.0024	0.0073	0.0021	0.0190	0.0073	0.0027	0.0092	0.0023	0.0166	0.0060	0.0032	0.0103
Weibull	(1, 1)	0.0065	0.0152	0.0079	0.0023	0.0030	0.0020	0.0079	0.0035	0.0026	0.0039	0.0019	0.0061	0.0028	0.0027	0.0041	0.0021	0.0051	0.0027	0.0028	0.0043
	(1, 2)	0.0083	0.0225	0.0110	0.0022	0.0034	0.0022	0.0129	0.0049	0.0026	0.0061	0.0020	0.0097	0.0036	0.0030	0.0071	0.0024	0.0081	0.0031	0.0033	0.0076
	(1, 3)	0.0087	0.0252	0.0122	0.0023	0.0040	0.0025	0.0169	0.0063	0.0026	0.0092	0.0023	0.0136	0.0046	0.0032	0.0113	0.0028	0.0116	0.0039	0.0036	0.0126
	(2, 2)	0.0104	0.0293	0.0140	0.0021	0.0038	0.0025	0.0177	0.0063	0.0026	0.0079	0.0022	0.0133	0.0043	0.0032	0.0096	0.0028	0.0110	0.0035	0.0036	0.0105
	(2, 3)	0.0109	0.0231	0.0151	0.0023	0.0043	0.0030	0.0217	0.0077	0.0025	0.0108	0.0024	0.0171	0.0054	0.0032	0.0134	0.0031	0.0144	0.0044	0.0038	0.0149
	(3, 3)	0.0114	0.0348	0.0163	0.0025	0.0047	0.0034	0.0256	0.0093	0.0023	0.0133	0.0024	0.0207	0.0066	0.0032	0.0167	0.0034	0.0176	0.0053	0.0040	0.0187
Gamma	(1, 1)	0.0059	0.0150	0.0083	0.0025	0.0028	0.0023	0.0084	0.0039	0.0027	0.0036	0.0021	0.0064	0.0031	0.0029	0.0041	0.0023	0.0054	0.0029	0.0029	0.0042
	(1, 2)	0.0075	0.0217	0.0106	0.0022	0.0033	0.0022	0.0126	0.0049	0.0027	0.0061	0.0020	0.0095	0.0034	0.0030	0.0071	0.0025	0.0082	0.0032	0.0032	0.0079
	(1, 3)	0.0075	0.0245	0.0112	0.0022	0.0047	0.0023	0.0162	0.0054	0.0027	0.0118	0.0022	0.0125	0.0041	0.0035	0.0147	0.0030	0.0106	0.0037	0.0041	0.0162
	(2, 2)	0.0099	0.0284	0.0133	0.0021	0.0033	0.0025	0.0173	0.0062	0.0025	0.0078	0.0019	0.0132	0.0041	0.0031	0.0096	0.0027	0.0113	0.0034	0.0036	0.0105
	(2, 3)	0.0100	0.0318	0.0141	0.0020	0.0043	0.0028	0.0211	0.0071	0.0024	0.0131	0.0021	0.0167	0.0049	0.0034	0.0162	0.0032	0.0139	0.0039	0.0042	0.0180
	(3, 3)	0.0106	0.0353	0.0154	0.0021	0.0052	0.0029	0.0252	0.0081	0.0022	0.0174	0.0020	0.0201	0.0057	0.0036	0.0219	0.0037	0.0170	0.0045	0.0048	0.0241
										$\rho = 0.8$											
Normal	(0, 0)	0.0027	0.0027	0.0027	0.0027	0.0028	0.0027	0.0027	0.0027	0.0027	0.0027	0.0027	0.0027	0.0027	0.0027	0.0027	0.0027	0.0027	0.0027	0.0027	0.0028
Lognormal	(1, 1)	0.0065	0.0162	0.0094	0.0026	0.0027	0.0024	0.0092	0.0046	0.0026	0.0032	0.0022	0.0070	0.0035	0.0027	0.0035	0.0021	0.0060	0.0031	0.0028	0.0037
	(1, 2)	0.0075	0.0208	0.0114	0.0027	0.0030	0.0025	0.0139	0.0062	0.0026	0.0044	0.0022	0.0112	0.0047	0.0028	0.0052	0.0022	0.0095	0.0040	0.0030	0.0057
	(1, 3)	0.0067	0.0209	0.0111	0.0029	0.0033	0.0024	0.0162	0.0071	0.0026	0.0058	0.0020	0.0141	0.0058	0.0029	0.0076	0.0020	0.0126	0.0050	0.0032	0.0085
	(2, 2)	0.0095	0.0265	0.0142	0.0028	0.0026	0.0028	0.0184	0.0081	0.0023	0.0042	0.0022	0.0148	0.0058	0.0025	0.0052	0.0022	0.0127	0.0047	0.0027	0.0057
	(2, 3)	0.0092	0.0276	0.0146	0.0030	0.0027	0.0029	0.0209	0.0091	0.0023	0.0049	0.0021	0.0177	0.0096	0.0025	0.0063	0.0022	0.0155	0.0058	0.0027	0.0072
	(3, 3)	0.0094	0.0294	0.0154	0.0032	0.0025	0.0031	0.0239	0.0103	0.0022	0.0051	0.0021	0.0207	0.0080	0.0024	0.0067	0.0021	0.0183	0.0067	0.0026	0.0077
Weibull	(1, 1)	0.0193	0.0277	0.0220	0.0032	0.0038	0.0060	0.0154	0.0112	0.0024	0.0034	0.0037	0.0106	0.0074	0.0024	0.0035	0.0029	0.0085	0.0060	0.0022	0.0035
	(1, 2)	0.0202	0.0326	0.0232	0.0038	0.0052	0.0089	0.0223	0.0145	0.0032	0.0069	0.0051	0.0170	0.0107	0.0034	0.0078	0.0045	0.0142	0.0090	0.0035	0.0084
	(1, 3)	0.0199	0.0344	0.0236	0.0048	0.0076	0.0099	0.0278	0.0175	0.0047	0.0138	0.0071	0.0236	0.0144	0.0053	0.0173	0.0068	0.0207	0.0128	0.0059	0.0191
	(2, 2)	0.0197	0.0360	0.0237	0.0039	0.0046	0.0091	0.0273	0.0171	0.0030	0.0067	0.0058	0.0215	0.0136	0.0031	0.0079	0.0055	0.0181	0.0115	0.0033	0.0086
	(2, 3)	0.0190	0.0372	0.0237	0.0043	0.0052	0.0103	0.0317	0.0195	0.0033	0.0100	0.0072	0.0267	0.0167	0.0037	0.0124	0.0070	0.0232	0.0150	0.0041	0.0139
	(3, 3)	0.0179	0.0380	0.0234	0.0046	0.0050	0.0110	0.0349	0.0211	0.0034	0.0109	0.0081	0.0304	0.0193	0.0037	0.0139	0.0080	0.0272	0.0180	0.0041	0.0157
Gamma	(1, 1)	0.0113	0.0193	0.0130	0.0029	0.0033	0.0037	0.0104	0.0066	0.0027	0.0036	0.0029	0.0078	0.0049	0.0029	0.0040	0.0027	0.0066	0.0044	0.0029	0.0040
	(1, 2)	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*
	(1, 3)	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*
	(2, 2)	0.0171	0.0331	0.0202	0.0031	0.0041	0.0071	0.0104	0.0066	0.0027	0.0070	0.0048	0.0173	0.0106	0.0032	0.0084	0.0047	0.0145	0.0092	0.0034	0.0094
	(2, 3)	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*
	(3, 3)	0.0168	0.0370	0.0214	0.0035	0.0046	0.0101	0.0302	0.0179	0.0027	0.0142	0.0081	0.0253	0.0174	0.0035	0.0180	0.0082	0.0218	0.0165	0.0042	0.0201

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Table 4
False alarm rates for bivariate distributions (parameters unknown)

		Sample size, n																			
		1					3					5					7				
Distribution	(γ_1, γ_2)	Synth WSD-T^2	Std-T^2	WSD-T^2	WSD-MCUSUM	WSD-MEWMA	Synth WSD-T^2	Std-T^2	WSD-T^2	WSD-MCUSUM	WSD-MEWMA	Synth WSD-T^2	Std-T^2	WSD-T^2	WSD-MCUSUM	WSD-MEWMA	Synth WSD-T^2	Std-T^2	WSD-T^2	WSD-MCUSUM	WSD-MEWMA
										$\rho = 0.3$											
Normal	(0, 0)	0.0200	0.0095	0.0127	0.0134	0.0152	0.0082	0.0040	0.0091	0.0221	0.0271	0.0054	0.0029	0.0067	0.0240	0.0275	0.0047	0.0031	0.0063	0.0252	0.0280
Lognormal	(1, 1)	0.0244	0.0258	0.0206	0.0140	0.0166	0.0081	0.0133	0.0090	0.0108	0.0127	0.0049	0.0093	0.0061	0.0100	0.0124	0.0041	0.0077	0.0051	0.0096	0.0114
	(1, 2)	0.0280	0.0342	0.0252	0.0146	0.0182	0.0092	0.0195	0.0116	0.0112	0.0146	0.0055	0.0141	0.0077	0.0103	0.0145	0.0046	0.0117	0.0062	0.0100	0.0135
	(1, 3)	0.0302	0.0390	0.0280	0.0152	0.0199	0.0101	0.0243	0.0140	0.0116	0.0168	0.0061	0.0185	0.0096	0.0106	0.0170	0.0052	0.0156	0.0078	0.0104	0.0161
	(2, 2)	0.0321	0.0427	0.0300	0.0153	0.0197	0.0102	0.0256	0.0143	0.0115	0.0163	0.0061	0.0189	0.0094	0.0106	0.0165	0.0052	0.0156	0.0075	0.0103	0.0155
	(2, 3)	0.0345	0.0474	0.0329	0.0159	0.0214	0.0112	0.0305	0.0167	0.0119	0.0184	0.0067	0.0232	0.0113	0.0108	0.0187	0.0058	0.0195	0.0090	0.0106	0.0178
	(3, 3)	0.0370	0.0520	0.0357	0.0166	0.0230	0.0123	0.0351	0.0191	0.0122	0.0204	0.0074	0.0275	0.0132	0.0111	0.0209	0.0064	0.0234	0.0106	0.0109	0.0200
Weibull	(1, 1)	0.0290	0.0304	0.0234	0.0136	0.0163	0.0149	0.0138	0.0088	0.0106	0.0127	0.0048	0.0094	0.0057	0.0097	0.0116	0.0041	0.0077	0.0049	0.0095	0.0115
	(1, 2)	0.0325	0.0404	0.0281	0.0140	0.0181	0.0155	0.0207	0.0109	0.0109	0.0156	0.0053	0.0143	0.0068	0.0102	0.0151	0.0048	0.0116	0.0056	0.0100	0.0151
	(1, 3)	0.0342	0.0455	0.0307	0.0148	0.0210	0.0166	0.0265	0.0133	0.0113	0.0198	0.0061	0.0194	0.0084	0.0106	0.0199	0.0057	0.0161	0.0069	0.0106	0.0202
	(2, 2)	0.0374	0.0507	0.0335	0.0145	0.0200	0.0163	0.0273	0.0132	0.0112	0.0182	0.0060	0.0191	0.0080	0.0104	0.0179	0.0057	0.0155	0.0064	0.0105	0.0183
	(2, 3)	0.0397	0.0560	0.0366	0.0154	0.0229	0.0176	0.0332	0.0159	0.0115	0.0222	0.0068	0.0242	0.0099	0.0108	0.0224	0.0068	0.0200	0.0078	0.0110	0.0230
	(3, 3)	0.0424	0.0616	0.0399	0.0162	0.0257	0.0190	0.0391	0.0187	0.0119	0.0261	0.0078	0.0292	0.0118	0.0113	0.0267	0.0082	0.0243	0.0093	0.0116	0.0275
Gamma	(1, 1)	0.0237	0.0264	0.0203	0.0135	0.0163	0.0077	0.0130	0.0083	0.0107	0.0127	0.0047	0.0093	0.0056	0.0099	0.0117	0.0040	0.0076	0.0048	0.0098	0.0114
	(1, 2)	0.0275	0.0367	0.0248	0.0140	0.0182	0.0081	0.0194	0.0101	0.0112	0.0157	0.0051	0.0137	0.0066	0.0104	0.0154	0.0047	0.0111	0.0054	0.0101	0.0154
	(1, 3)	0.0298	0.0428	0.0275	0.0142	0.0220	0.0092	0.0253	0.0121	0.0116	0.0228	0.0060	0.0184	0.0078	0.0111	0.0231	0.0063	0.0150	0.0065	0.0114	0.0240
	(2, 2)	0.0340	0.0484	0.0309	0.0141	0.0193	0.0092	0.0267	0.0130	0.0112	0.0183	0.0057	0.0189	0.0079	0.0105	0.0181	0.0056	0.0153	0.0063	0.0104	0.0185
	(2, 3)	0.0368	0.0552	0.0344	0.0144	0.0231	0.0106	0.0330	0.0151	0.0113	0.0242	0.0067	0.0243	0.0095	0.0111	0.0251	0.0073	0.0197	0.0074	0.0114	0.0259
	(3, 3)	0.0408	0.0632	0.0384	0.0149	0.0264	0.0125	0.0396	0.0178	0.0114	0.0299	0.0080	0.0294	0.0113	0.0115	0.0309	0.0095	0.0240	0.0089	0.0121	0.0322
										$\rho = 0.8$											
Normal	(0, 0)	0.0219	0.0095	0.0142	0.0145	0.0174	0.0085	0.0293	0.0408	0.0355	0.0417	0.0056	0.0058	0.0106	0.0209	0.0252	0.0048	0.0038	0.0070	0.0194	0.0228
Lognormal	(1, 1)	0.0272	0.0277	0.0229	0.0151	0.0189	0.0087	0.0144	0.0098	0.0111	0.0132	0.0052	0.0101	0.0066	0.0101	0.0118	0.0042	0.0083	0.0054	0.0097	0.0114
	(1, 2)	0.0306	0.0361	0.0273	0.0158	0.0211	0.0099	0.0214	0.0128	0.0116	0.0156	0.0058	0.0158	0.0086	0.0106	0.0145	0.0048	0.0133	0.0070	0.0103	0.0141
	(1, 3)	0.0317	0.0396	0.0291	0.0168	0.0241	0.0107	0.0266	0.0152	0.0124	0.0192	0.0064	0.0210	0.0109	0.0113	0.0183	0.0055	0.0184	0.0090	0.0110	0.0182
	(2, 2)	0.0357	0.0454	0.0329	0.0165	0.0224	0.0113	0.0280	0.0159	0.0117	0.0165	0.0065	0.0209	0.0106	0.0105	0.0152	0.0054	0.0175	0.0084	0.0102	0.0148
	(2, 3)	0.0380	0.0497	0.0355	0.0173	0.0245	0.0124	0.0330	0.0185	0.0122	0.0187	0.0072	0.0256	0.0127	0.0109	0.0175	0.0060	0.0220	0.0103	0.0105	0.0172
	(3, 3)	0.0412	0.0550	0.0388	0.0181	0.0261	0.0138	0.0380	0.0212	0.0125	0.0202	0.0080	0.0302	0.0149	0.0110	0.0188	0.0067	0.0261	0.0122	0.0108	0.0186
Weibull	(1, 1)	0.0549	0.0546	0.0510	0.0231	0.0264	0.0248	0.0329	0.0290	0.0133	0.0156	0.0129	0.0217	0.0184	0.0110	0.0131	0.0089	0.0167	0.0141	0.0105	0.0125
	(1, 2)	0.0521	0.0570	0.0486	0.0211	0.0266	0.0226	0.0357	0.0279	0.0134	0.0190	0.0122	0.0252	0.0184	0.0116	0.0174	0.0093	0.0201	0.0144	0.0112	0.0172
	(1, 3)	0.0470	0.0554	0.0446	0.0197	0.0284	0.0199	0.0371	0.0257	0.0135	0.0244	0.0112	0.0279	0.0175	0.0121	0.0238	0.0092	0.0232	0.0140	0.0119	0.0242
	(2, 2)	0.0583	0.0646	0.0551	0.0242	0.0295	0.0273	0.0445	0.0347	0.0141	0.0205	0.0157	0.0328	0.0244	0.0118	0.0185	0.0121	0.0264	0.0195	0.0113	0.0183
	(2, 3)	0.0568	0.0655	0.0542	0.0235	0.0314	0.0273	0.0477	0.0355	0.0143	0.0246	0.0162	0.0367	0.0260	0.0122	0.0232	0.0132	0.0304	0.0213	0.0118	0.0234
	(3, 3)	0.0606	0.0699	0.0584	0.0263	0.0345	0.0307	0.0533	0.0402	0.0152	0.0272	0.0188	0.0426	0.0309	0.0128	0.0258	0.0158	0.0359	0.0259	0.0123	0.0261
Gamma	(1, 1)	0.0365	0.0334	0.0302	0.0154	0.0191	0.0113	0.0168	0.0129	0.0112	0.0135	0.0066	0.0114	0.0084	0.0103	0.0120	0.0051	0.0090	0.0098	0.0098	0.0115
	(1, 2)	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*
	(1, 3)	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*
	(2, 2)	0.0555	0.0575	0.0484	0.0180	0.0243	0.0204	0.0343	0.0252	0.0125	0.0191	0.0118	0.0251	0.0179	0.0112	0.0181	0.0097	0.0203	0.0145	0.0109	0.0178
	(2, 3)	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*
	(3, 3)	0.0647	0.0707	0.0596	0.0228	0.0333	0.0321	0.0491	0.0393	0.0143	0.0296	0.0202	0.0388	0.0309	0.0129	0.0298	0.0181	0.0321	0.0271	0.0128	0.0300

the WSD–MCUSUM chart has the best performance. For the case of individual measurements, i.e., $n = 1$, we observe from Table 4 that the WSD–T^2 chart has slightly lower false alarm rates than the synthetic WSD–T^2 chart for the various skewnesses. When $n = 1$, the WSD–MCUSUM chart gives the lowest false alarm rates.

Tables 5–8 present the mean shift detection rates for the bivariate lognormal cases with $n = 5$, where $\rho = 0.3$ and 0.8.

When parameters are known, Tables 5 and 6 show that the synthetic WSD–T^2 chart significantly outperforms all the other charts for cases 2, 4, 5, and 6, when $d(\mu_1) \geq 1$ and for cases 1 and 3, when $d(\mu_1) \geq 1.5$, for all skewnesses, $(\gamma_1, \gamma_2) = (1, 1), (2, 2)$ and $(3, 3)$, and correlation coefficients, $\rho = 0.3$ and 0.8. From the results in Tables 7 and 8, for unknown parameters, we observe that on the whole, the synthetic WSD–T^2 chart has higher mean shift detection rates than all the other charts for cases 1, 2, 4, 5, and 6, when $d(\mu_1) \geq 1$ and for case 3, when $d(\mu_1) \geq 1.25$, for all skewnesses and correlation coefficients. Although not shown here because of space constraint, simulations made using SAS version 9.1 to compute the mean shift detection rates for all the charts based on the Lee's bivariate Weibull and Cheriyan and Ramabhadran's bivariate gamma distributions when parameters are known and unknown, where the sample size, $n = 5$, yield similar results, i.e., the synthetic WSD–T^2 chart generally provides better mean shift detection rates than the other charts.

Overall, the synthetic WSD–T^2 chart is found to have the most favorable performance among all the charts considered for cases with known and unknown process parameters. The synthetic WSD–T^2 chart gives lower false alarm rates compared to all the other charts for skewed distributions, except the WSD–MCUSUM chart, for most skewnesses (γ_1, γ_2) when subgrouping is considered. The synthetic WSD–T^2 chart also provides the highest mean shift detection rates among all the charts for skewed populations for moderate and large shifts based on the various skewnesses, when process parameters are known and unknown. Since a moderate or a large mean shift will have a more serious effect on a process than a small mean shift, a desirable control chart is the one with the quickest response to moderate and large shifts. The synthetic WSD–T^2 chart is shown to have this desirable property among all the control charts for skewed populations.

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5. An Illustrative Example

We illustrate how the synthetic WSD–T^2 chart is constructed using simulated process data. The Hotelling's T^2, WSD–T^2, WSD–MEWMA, and WSD–MCUSUM charts are also considered for the sake of comparison. These data are 30 successive random samples, each of size, $n = 5$, generated from a 4 variate lognormal distribution using the SAS version 9.1 program. For convenience, the location parameters (0, 0, 0, 0) are chosen. The scale parameters $(\sigma_1, \sigma_2, \sigma_3, \sigma_4) = (0.1656, 0.5593, 0.4484, 0.6525)$ are selected so that the skewnesses are $(\gamma_1, \gamma_2, \gamma_3, \gamma_4) = (0.5, 2.0, 1.5, 2.5)$. The correlation matrix is

$$\rho = \begin{pmatrix} 1.0 & -0.5 & -0.3 & 0.2 \\ & 1.0 & 0.7 & 0.5 \\ & & 1.0 & 0.1 \\ & & & 1.0 \end{pmatrix}.$$

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Table 5

Mean shift detection rates when only μ_1 is shifted, $n = 5$ and optimal $d(\mu_1) = 1$ for bivariate lognormal distribution (parameters known)

			$\rho = 0.3$								$\rho = 0.8$				
$d(\mu_1)$	Case	(γ_1, γ_2)	Synth WSD–T^2	Std–T^2	WSD–T^2	WSD–MCUSUM	WSD–MEWMA	$d(\mu_1)$	Case	(γ_1, γ_2)	Synth WSD–T^2	Std–T^2	WSD–T^2	WSD–MCUSUM	WSD–MEWMA
0.5	1	(1, 1)	0.0175	0.0292	0.0154	0.0591	0.0882	0.5	1	(1, 1)	0.0223	0.0268	0.0168	0.1008	0.0974
		(2, 2)	0.0122	0.0377	0.0152	0.0904	0.0807			(2, 2)	0.0191	0.0347	0.0164	0.1003	0.0979
		(3, 3)	0.0093	0.0425	0.0158	0.0880	0.0775			(3, 3)	0.0164	0.0390	0.0164	0.1011	0.1011
	2	(1, 1)	0.0612	0.0134	0.0290	0.1175	0.1266		2	(1, 1)	0.0494	0.0178	0.0258	0.1116	0.1182
		(2, 2)	0.1094	0.0150	0.0427	0.1310	0.1490			(2, 2)	0.0769	0.0223	0.0334	0.1203	0.1350
		(3, 3)	0.1626	0.0189	0.0568	0.1424	0.1680			(3, 3)	0.1060	0.0262	0.0392	0.1283	0.1504
1	1	(1, 1)	0.2012	0.1512	0.0918	0.2259	0.2234	1	1	(1, 1)	0.2543	0.1487	0.1115	0.2351	0.2352
		(2, 2)	0.1395	0.1450	0.0652	0.2140	0.2079			(2, 2)	0.2268	0.1379	0.0946	0.2309	0.2299
		(3, 3)	0.0941	0.1373	0.0523	0.2067	0.1991			(3, 3)	0.2201	0.1268	0.0856	0.2294	0.2286
	2	(1, 1)	0.4970	0.1496	0.2690	0.2723	0.2845		2	(1, 1)	0.4556	0.1482	0.2389	0.2641	0.2745
		(2, 2)	0.6249	0.1346	0.3988	0.2988	0.3188			(2, 2)	0.5709	0.1293	0.3414	0.2845	0.3019
		(3, 3)	0.7061	0.1128	0.5062	0.2317	0.3484			(3, 3)	0.6535	0.1075	0.4366	0.3035	0.3273
1.25	1	(1, 1)	0.4289	0.2956	0.1944	0.2860	0.2850	1.25	1	(1, 1)	0.7138	0.2991	0.2346	0.2956	0.2966
		(2, 2)	0.3188	0.2740	0.1339	0.2706	0.2662			(2, 2)	0.6668	0.2749	0.2011	0.2877	0.2872
		(3, 3)	0.2485	0.2542	0.1018	0.2612	0.2553			(3, 3)	0.4279	0.2504	0.1856	0.2836	0.2829
	2	(1, 1)	0.7185	0.3303	0.4860	0.3449	0.3578		2	(1, 1)	0.6878	0.3213	0.4478	0.3359	0.3472
		(2, 2)	0.8004	0.3358	0.6274	0.3778	0.3964			(2, 2)	0.7655	0.3143	0.5745	0.3626	0.3799
		(3, 3)	0.8461	0.3331	0.7185	0.4033	0.4239			(3, 3)	0.8146	0.2972	0.6673	0.3862	0.4065

(continued)

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Table 5
Continued

			$\rho = 0.3$								$\rho = 0.8$				
$d(\mu_1)$	Case	(γ_1, γ_2)	Synth WSD–T^2	Std–T^2	WSD–T^2	WSD–MCUSUM	WSD–MEWMA	$d(\mu_1)$	Case	(γ_1, γ_2)	Synth WSD–T^2	Std–T^2	WSD–T^2	WSD–MCUSUM	WSD–MEWMA
1.5	1	(1, 1)	0.6721	0.4995	0.3605	0.3422	0.3422	1.5	1	(1, 1)	0.7138	0.5088	0.4164	0.3517	0.3534
		(2, 2)	0.5722	0.4732	0.2575	0.3238	0.3205			(2, 2)	0.6668	0.4903	0.3627	0.3400	0.3409
		(3, 3)	0.4970	0.4498	0.1976	0.3128	0.3077			(3, 3)	0.6404	0.4721	0.3364	0.3333	0.3332
	2	(1, 1)	0.8609	0.5497	0.6936	0.4132	0.4248		2	(1, 1)	0.8438	0.5405	0.6610	0.4044	0.4151
		(2, 2)	0.9003	0.5703	0.7962	0.4425	0.4558			(2, 2)	0.8809	0.5531	0.7597	0.4311	0.4440
		(3, 3)	0.9212	0.5866	0.8512	0.4608	0.4755			(3, 3)	0.9029	0.5629	0.8185	0.4499	0.4634
2	1	(1, 1)	0.9571	0.8862	0.7790	0.4534	0.4540	2	1	(1, 1)	0.9562	0.8850	0.8067	0.4608	0.4621
		(2, 2)	0.9419	0.9030	0.6744	0.4267	0.4241			(2, 2)	0.9391	0.9001	0.7467	0.4453	0.4444
		(3, 3)	0.9282	0.9238	0.5916	0.4070	0.4028			(3, 3)	0.9252	0.9173	0.7075	0.4350	0.4332
	2	(1, 1)	0.9764	0.8750	0.9306	0.5052	0.5131		2	(1, 1)	0.9740	0.8763	0.9209	0.5010	0.5079
		(2, 2)	0.9785	0.8780	0.9517	0.5451	0.5662			(2, 2)	0.9737	0.8824	0.9400	0.5305	0.5478
		(3, 3)	0.9796	0.8836	0.9611	0.6025	0.6396			(3, 3)	0.9739	0.8913	0.9495	0.5742	0.6072
3	1	(1, 1)	1.0000	1.0000	0.9999	0.5675	0.5682	3	1	(1, 1)	1.0000	0.9999	0.9995	0.5812	0.5845
		(2, 2)	1.0000	1.0000	1.0000	0.5297	0.5285			(2, 2)	1.0000	1.0000	0.5427	0.5427	0.5443
		(3, 3)	1.0000	1.0000	1.0000	0.5172	0.5161			(3, 3)	1.0000	1.0000	0.9977	0.5254	0.5270
	2	(1, 1)	0.9998	0.9978	0.9990	0.8341	0.8527		2	(1, 1)	0.9998	0.9985	0.9990	0.8157	0.8357
		(2, 2)	0.9991	0.9947	0.9979	0.9162	0.9300			(2, 2)	0.9989	0.9964	0.9973	0.8989	0.9141
		(3, 3)	0.9983	0.9921	0.9969	0.9477	0.9571			(3, 3)	0.9979	0.9947	0.9956	0.9329	0.9431

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Table 6
Mean shift detection rates when both μ_1 and μ_2 are shifted, $n = 5$ and optimal $d(\mu_1) = 1$ for bivariate lognormal distribution (parameters known)

			$\rho = 0.3$								$\rho = 0.8$				
$d(\mu_1)$	Case	(γ_1, γ_2)	Synth WSD–T^2	Std–T^2	WSD–T^2	WSD–MCUSUM	WSD–MEWMA	$d(\mu_1)$	Case	(γ_1, γ_2)	Synth WSD–T^2	Std–T^2	WSD–T^2	WSD–MCUSUM	WSD–MEWMA
0.5	3	(1, 1)	0.0144	0.0334	0.0152	0.0894	0.0804	0.5	3	(1, 1)	0.0146	0.0347	0.0160	0.0904	0.0814
		(2, 2)	0.0097	0.0443	0.0155	0.0803	0.0660			(2, 2)	0.0101	0.0455	0.0168	0.0818	0.0673
		(3, 3)	0.0079	0.0496	0.0164	0.0752	0.0575			(3, 3)	0.0091	0.0506	0.0178	0.0766	0.0587
	4	(1, 1)	0.0348	0.0218	0.0205	0.1062	0.1087		4	(1, 1)	0.0344	0.0228	0.0208	0.1062	0.1082
		(2, 2)	0.0444	0.0275	0.0226	0.1109	0.1191			(2, 2)	0.0421	0.0300	0.0230	0.1101	0.1177
		(3, 3)	0.0542	0.0315	0.0236	0.1157	0.1302			(3, 3)	0.0494	0.0330	0.0242	0.1144	0.1280
	5	(1, 1)	0.0732	0.0080	0.0311	0.1234	0.1333		5	(1, 1)	0.0739	0.0073	0.0289	0.1224	0.1321
		(2, 2)	0.1430	0.0064	0.0501	0.1412	0.1604			(2, 2)	0.1444	0.0070	0.0467	0.1392	0.1581
		(3, 3)	0.2195	0.0096	0.0721	0.1556	0.1824			(3, 3)	0.2201	0.0112	0.0653	0.1528	0.1794
	6	(1, 1)	0.0346	0.0219	0.0206	0.1065	0.1088		6	(1, 1)	0.0342	0.0228	0.0208	0.1062	0.1083
		(2, 2)	0.0441	0.0273	0.0227	0.1109	0.1191			(2, 2)	0.0421	0.0290	0.0230	0.1101	0.1178
		(3, 3)	0.0556	0.0316	0.0238	0.1157	0.1301			(3, 3)	0.0493	0.0330	0.0242	0.1143	0.1281
1	3	(1, 1)	0.1711	0.1596	0.0858	0.2205	0.2170	1	3	(1, 1)	0.1693	0.1588	0.0863	0.2207	0.2171
		(2, 2)	0.0966	0.1580	0.0606	0.2048	0.1963			(2, 2)	0.0946	0.1555	0.0614	0.2047	0.1962
		(3, 3)	0.0632	0.1521	0.0498	0.1950	0.1834			(3, 3)	0.0608	0.1487	0.0507	0.1949	0.1831
	4	(1, 1)	0.3652	0.1486	0.1731	0.2500	0.2559		4	(1, 1)	0.3640	0.1476	0.1742	0.2500	0.2558
		(2, 2)	0.4348	0.1344	0.2164	0.2587	0.2691			(2, 2)	0.4322	0.1322	0.2195	0.2595	0.2692
		(3, 3)	0.5033	0.1200	0.2667	0.2680	0.2834			(3, 3)	0.5005	0.1166	0.2732	0.2698	0.2839
	5	(1, 1)	0.5376	0.1396	0.2877	0.2782	0.2908		5	(1, 1)	0.5431	0.1387	0.2902	0.2780	0.2906
		(2, 2)	0.6879	0.1197	0.4403	0.3093	0.3302			(2, 2)	0.6942	0.1159	0.4477	0.3089	0.3297
		(3, 3)	0.7777	0.0984	0.5645	0.3357	0.3630			(3, 3)	0.7819	0.0897	0.5735	0.3353	0.3627
	6	(1, 1)	0.3645	0.1484	0.1731	0.2500	0.2560		6	(1, 1)	0.3635	0.1475	0.1738	0.2502	0.2559
		(2, 2)	0.4338	0.1348	0.2158	0.2588	0.2693			(2, 2)	0.4316	0.1321	0.2187	0.2596	0.2693
		(3, 3)	0.5029	0.1202	0.2661	0.2681	0.2836			(3, 3)	0.4992	0.1170	0.2724	0.2698	0.2839
1.25	3	(1, 1)	0.3835	0.3005	0.1801	0.2809	0.2790	1.25	3	(1, 1)	0.3793	0.2981	0.1788	0.2810	0.2790
		(2, 2)	0.2455	0.2833	0.1189	0.2619	0.2554			(2, 2)	0.2382	0.2781	0.1179	0.2617	0.2553
		(3, 3)	0.1672	0.2666	0.0905	0.2499	0.2408			(3, 3)	0.1591	0.2590	0.0897	0.2499	0.2407
	4	(1, 1)	0.6100	0.3106	0.3483	0.3160	0.3230		4	(1, 1)	0.6077	0.3095	0.3489	0.3164	0.3229
		(2, 2)	0.6639	0.2956	0.4171	0.2366	0.3380			(2, 2)	0.6572	0.2905	0.4187	0.3278	0.3380
		(3, 3)	0.7125	0.2750	0.4878	0.3384	0.3546			(3, 3)	0.7009	0.2656	0.4887	0.3405	0.3546

(continued)

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Table 6
Continued

			$\rho = 0.3$								$\rho = 0.8$				
$d(\mu_1)$	Case	(γ_1, γ_2)	Synth WSD–T^2	Std–T^2	WSD–T^2	WSD–MCUSUM	WSD–MEWMA	$d(\mu_1)$	Case	(γ_1, γ_2)	Synth WSD–T^2	Std–T^2	WSD–T^2	WSD–MCUSUM	WSD–MEWMA
	5	(1, 1)	0.7517	0.3255	0.5128	0.3508	0.3640		5	(1, 1)	0.7546	0.3284	0.5171	0.3508	0.3639
		(2, 2)	0.8489	0.3286	0.6751	0.3881	0.4071			(2, 2)	0.8503	0.3333	0.6818	0.3884	0.4074
		(3, 3)	0.8988	0.3261	0.7764	0.4162	0.4071			(3, 3)	0.8987	0.3307	0.7817	0.4165	0.4368
	6	(1, 1)	0.6091	0.3106	0.3483	0.3160	0.3230		6	(1, 1)	0.6073	0.3093	0.3491	0.3165	0.3231
		(2, 2)	0.6632	0.2956	0.4171	0.3266	0.3379			(2, 2)	0.6575	0.2904	0.4192	0.3279	0.3381
		(3, 3)	0.7118	0.2750	0.4878	0.3384	0.3545			(3, 3)	0.7012	0.2654	0.4896	0.3406	0.3546
1.5	3	(1, 1)	0.6291	0.4968	0.3343	0.3375	0.3367	1.5	3	(1, 1)	0.6259	0.4933	0.3315	0.3375	0.3367
		(2, 2)	0.4808	0.4693	0.2232	0.3155	0.3105			(2, 2)	0.4724	0.4624	0.2187	0.3155	0.3105
		(3, 3)	0.3686	0.4457	0.1655	0.3017	0.2942			(3, 3)	0.3572	0.4361	0.1611	0.3016	0.2941
	4	(1, 1)	0.7960	0.5252	0.5591	0.3796	0.3872		4	(1, 1)	0.7923	0.5253	0.5581	0.3798	0.3868
		(2, 2)	0.8220	0.5241	0.6255	0.3922	0.4031			(2, 2)	0.8116	0.5238	0.6185	0.3926	0.4018
		(3, 3)	0.8457	0.5225	0.6851	0.4052	0.4188			(3, 3)	0.8296	0.5215	0.6714	0.4053	0.4160
	5	(1, 1)	0.8850	0.5531	0.7213	0.4189	0.4305		5	(1, 1)	0.8849	0.5572	0.7240	0.4187	0.4305
		(2, 2)	0.9336	0.5769	0.8383	0.4518	0.4650			(2, 2)	0.9321	0.5846	0.8405	0.4513	0.4639
		(3, 3)	0.9564	0.5949	0.8970	0.4725	0.4880			(3, 3)	0.9541	0.6052	0.8968	0.4704	0.4847
	6	(1, 1)	0.7959	0.5248	0.5593	0.3796	0.3872		6	(1, 1)	0.7921	0.5248	0.5582	0.3799	0.3869
		(2, 2)	0.8221	0.5240	0.6256	0.3923	0.4032			(2, 2)	0.8121	0.5239	0.6194	0.3926	0.4019
		(3, 3)	0.8456	0.5225	0.6854	0.4053	0.4189			(3, 3)	0.8301	0.5216	0.6716	0.4053	0.4161
2	3	(1, 1)	0.9419	0.8743	0.7473	0.4481	0.4483	2	3	(1, 1)	0.9441	0.8761	0.7465	0.4482	0.4484
		(2, 2)	0.9028	0.8772	0.6047	0.4166	0.4127			(2, 2)	0.9079	0.8818	0.5992	0.4162	0.4122
		(3, 3)	0.8618	0.8854	0.4905	0.3949	0.3889			(3, 3)	0.8691	0.8942	0.4816	0.3937	0.3880
	4	(1, 1)	0.9684	0.8795	0.8836	0.4829	0.4867		4	(1, 1)	0.9650	0.8803	0.8773	0.4824	0.4861
		(2, 2)	0.9656	0.8871	0.8969	0.4875	0.4927			(2, 2)	0.9579	0.8897	0.8813	0.4866	0.4920
		(3, 3)	0.9648	0.8967	0.9095	0.4926	0.5010			(3, 3)	0.9545	0.9014	0.8883	0.4929	0.5024
	5	(1, 1)	0.9840	0.8846	0.9452	0.5087	0.5167		5	(1, 1)	0.9829	0.8844	0.9437	0.5078	0.5158
		(2, 2)	0.9896	0.8948	0.9710	0.5528	0.5748			(2, 2)	0.9882	0.8947	0.9685	0.5519	0.5750
		(3, 3)	0.9924	0.9046	0.9814	0.6147	0.6538			(3, 3)	0.9907	0.9044	0.9788	0.6168	0.6574
	6	(1, 1)	0.9682	0.8795	0.8836	0.4829	0.4868		6	(1, 1)	0.9647	0.8804	0.8777	0.4824	0.4861
		(2, 2)	0.9655	0.8871	0.8969	0.4874	0.4927			(2, 2)	0.9577	0.8900	0.8815	0.4867	0.4921
		(3, 3)	0.9645	0.8968	0.9097	0.4926	0.5007			(3, 3)	0.9544	0.9017	0.8883	0.4928	0.5026

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Table 7

Mean shift detection rates when only μ_1 is shifted, $n = 5$ and optimal $d(\mu_1) = 1$ for bivariate lognormal distribution (parameters unknown)

			$\rho = 0.3$								$\rho = 0.8$				
$d(\mu_1)$	Case	(γ_1, γ_2)	Synth WSD–T^2	Std–T^2	WSD–T^2	WSD–MCUSUM	WSD–MEWMA	$d(\mu_1)$	Case	(γ_1, γ_2)	Synth WSD–T^2	Std–T^2	WSD–T^2	WSD–MCUSUM	WSD–MEWMA
0.5	1	(1, 1)	0.0311	0.0382	0.0237	0.0998	0.0939	0.5	1	(1, 1)	0.0450	0.0354	0.0260	0.1054	0.1029
		(2, 2)	0.0289	0.0518	0.0258	0.0970	0.0897			(2, 2)	0.0491	0.0485	0.0300	0.1077	0.1066
		(3, 3)	0.0298	0.0628	0.0628	0.0988	0.0909			(3, 3)	0.0587	0.0591	0.0357	0.1132	0.1149
	2	(1, 1)	0.0780	0.0199	0.0394	0.1215	0.1305		2	(1, 1)	0.0791	0.0250	0.0371	0.1162	0.1228
		(2, 2)	0.1386	0.0231	0.0618	0.1368	0.1546			(2, 2)	0.1249	0.0325	0.0547	0.1278	0.1423
		(3, 3)	0.2095	0.0293	0.0926	0.1516	0.1767			(3, 3)	0.1810	0.0409	0.0783	0.1409	0.1622
1	1	(1, 1)	0.2396	0.1749	0.1184	0.2310	0.2288	1	1	(1, 1)	0.3075	0.1727	0.1398	0.2403	0.2406
		(2, 2)	0.1970	0.1821	0.1003	0.2221	0.2174			(2, 2)	0.3054	0.1792	0.1400	0.2408	0.2404
		(3, 3)	0.1872	0.1948	0.0992	0.2215	0.2145			(3, 3)	0.3321	0.1918	0.1575	0.2467	0.2469
	2	(1, 1)	0.5086	0.1697	0.2875	0.2768	0.2889		2	(1, 1)	0.4836	0.1700	0.2602	0.2691	0.2794
		(2, 2)	0.6377	0.1740	0.4220	0.3064	0.3261			(2, 2)	0.5963	0.1739	0.3729	0.2494	0.3114
		(3, 3)	0.7213	0.1879	0.5360	0.3346	0.3595			(3, 3)	0.6809	0.1888	0.4812	0.3213	0.3433
1.25	1	(1, 1)	0.4498	0.3215	0.2295	0.2916	0.2907	1.25	1	(1, 1)	0.5211	0.3254	0.2687	0.3014	0.3026
		(2, 2)	0.3789	0.3199	0.1873	0.2795	0.2764			(2, 2)	0.5029	0.3284	0.2607	0.2989	0.2999
		(3, 3)	0.3578	0.3349	0.1804	0.2776	0.2719			(3, 3)	0.5174	0.3467	0.2810	0.3032	0.3034
	2	(1, 1)	0.7223	0.3494	0.4988	0.3495	0.3621		2	(1, 1)	0.6960	0.3433	0.4630	0.3412	0.3522
		(2, 2)	0.8072	0.3695	0.6407	0.3843	0.4022			(2, 2)	0.7751	0.3592	0.5930	0.3720	0.3881
		(3, 3)	0.8533	0.3950	0.7332	0.4133	0.4339			(3, 3)	0.8272	0.3867	0.6902	0.4016	0.4208

(continued)

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Table 7
Continued

			$\rho = 0.3$								$\rho = 0.8$				
$d(\mu_1)$	Case	(γ_1, γ_2)	Synth WSD–T^2	Std–T^2	WSD–T^2	WSD–MCUSUM	WSD–MEWMA	$d(\mu_1)$	Case	(γ_1, γ_2)	Synth WSD–T^2	Std–T^2	WSD–T^2	WSD–MCUSUM	WSD–MEWMA
1.5	1	(1, 1)	0.6695	0.5157	0.3925	0.3487	0.3487	1.5	1	(1, 1)	0.7182	0.5255	0.4443	0.3588	0.3605
		(2, 2)	0.5918	0.5121	0.3256	0.3347	0.3317			(2, 2)	0.6870	0.5313	0.4208	0.3532	0.3535
		(3, 3)	0.5608	0.5258	0.3072	0.3304	0.3257			(3, 3)	0.6874	0.5554	0.4351	0.3562	0.3562
	2	(1, 1)	0.8617	0.5616	0.6989	0.4164	0.4276		2	(1, 1)	0.4886	0.5541	0.6668	0.4083	0.4186
		(2, 2)	0.9030	0.5867	0.8022	0.4493	0.4638			(2, 2)	0.8847	0.5780	0.7677	0.4395	0.4532
		(3, 3)	0.9248	0.6102	0.8574	0.4788	0.4987			(3, 3)	0.9096	0.6072	0.8295	0.4703	0.4885
2	1	(1, 1)	0.9413	0.8719	0.7667	0.4531	0.4536	2	1	(1, 1)	0.9463	0.8755	0.7994	0.4612	0.4625
		(2, 2)	0.9038	0.8711	0.6819	0.4345	0.4323			(2, 2)	0.9221	0.8810	0.7526	0.4503	0.4500
		(3, 3)	0.8719	0.8706	0.6430	0.4269	0.4232			(3, 3)	0.9069	0.8868	0.7400	0.4488	0.4479
	2	(1, 1)	0.9761	0.8745	0.9299	0.5147	0.5240		2	(1, 1)	0.9729	0.8756	0.9191	0.5107	0.5193
		(2, 2)	0.9788	0.8756	0.9523	0.5697	0.5912			(2, 2)	0.9742	0.8807	0.9409	0.5593	0.5780
		(3, 3)	0.9804	0.8767	0.9623	0.6428	0.6751			(3, 3)	0.9757	0.8864	0.9524	0.6277	0.6561
3	1	(1, 1)	0.9999	0.9999	0.9982	0.6054	0.6079	3	1	(1, 1)	0.9999	0.9997	0.9979	0.6181	0.6221
		(2, 2)	0.9990	0.9992	0.9895	0.5806	0.5800			(2, 2)	0.9990	0.9991	0.9906	0.6002	0.6017
		(3, 3)	0.9949	0.9967	0.9714	0.5787	0.5763			(3, 3)	0.9963	0.9975	0.9794	0.6070	0.6074
	2	(1, 1)	0.9997	0.9976	0.9989	0.8342	0.5824		2	(1, 1)	0.9997	0.9982	0.9988	0.8187	0.8361
		(2, 2)	0.9991	0.9941	0.9979	0.9129	0.9274			(2, 2)	0.9989	0.9959	0.9972	0.8972	0.9114
		(3, 3)	0.9983	0.9904	0.9970	0.9436	0.9543			(3, 3)	0.9980	0.9936	0.9958	0.9317	0.9422

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Table 8

Mean shift detection rates when both μ_1 and μ_2 are shifted, $n = 5$ and optimal $d(\mu_1) = 1$ for bivariate lognormal distribution (parameters unknown)

			$\rho = 0.3$								$\rho = 0.8$				
$d(\mu_1)$	Case	(γ_1, γ_2)	Synth WSD–T^2	Std–T^2	WSD–T^2	WSD–MCUSUM	WSD–MEWMA	$d(\mu_1)$	Case	(γ_1, γ_2)	Synth WSD–T^2	Std–T^2	WSD–T^2	WSD–MCUSUM	WSD–MEWMA
0.5	3	(1, 1)	0.0271	0.0436	0.0237	0.0953	0.0874	0.5	3	(1, 1)	0.0337	0.0452	0.0256	0.0967	0.0890
		(2, 2)	0.0237	0.0600	0.0265	0.0901	0.0776			(2, 2)	0.0304	0.0629	0.0293	0.0924	0.0803
		(3, 3)	0.0237	0.0725	0.0305	0.0900	0.0743			(3, 3)	0.0311	0.0755	0.0338	0.0924	0.0778
	4	(1, 1)	0.0515	0.0296	0.0300	0.1109	0.1137		4	(1, 1)	0.0607	0.0304	0.0310	0.1107	0.1133
		(2, 2)	0.0709	0.0386	0.0377	0.1179	0.1264			(2, 2)	0.0833	0.0408	0.0405	0.1177	0.1258
		(3, 3)	0.0978	0.0466	0.0467	0.1245	0.1410			(3, 3)	0.1144	0.0504	0.0535	0.1271	0.1409
	5	(1, 1)	0.1049	0.0131	0.0412	0.1262	0.1364		5	(1, 1)	0.1077	0.0124	0.0416	0.1255	0.1359
		(2, 2)	0.1912	0.0119	0.0690	0.1448	0.1644			(2, 2)	0.1984	0.0126	0.0702	0.1438	0.1635
		(3, 3)	0.2869	0.0158	0.1080	0.1622	0.1892			(3, 3)	0.2986	0.0178	0.1119	0.1715	0.1885
	6	(1, 1)	0.0594	0.0294	0.0297	0.1106	0.1133		6	(1, 1)	0.0605	0.0303	0.0309	0.1106	0.1132
		(2, 2)	0.0802	0.0384	0.0372	0.1174	0.1259			(2, 2)	0.0822	0.0408	0.0402	0.1175	0.1255
		(3, 3)	0.1094	0.0472	0.0477	0.1262	0.1405			(3, 3)	0.1133	0.0504	0.0531	0.1269	0.1406
1	3	(1, 1)	0.2130	0.1841	0.1122	0.2269	0.2237	1	3	(1, 1)	0.2332	0.1847	0.1160	0.2277	0.2248
		(2, 2)	0.1572	0.1969	0.0939	0.2163	0.2084			(2, 2)	0.1780	0.1990	0.0996	0.2180	0.2103
		(3, 3)	0.1380	0.2117	0.0914	0.2146	0.2023			(3, 3)	0.1604	0.2129	0.0976	0.2160	0.2054
	4	(1, 1)	0.3868	0.1710	0.1979	0.2550	0.2612		4	(1, 1)	0.4038	0.1706	0.1996	0.2552	0.2611
		(2, 2)	0.4641	0.1736	0.2541	0.2678	0.2785			(2, 2)	0.4785	0.1744	0.2609	0.2695	0.2794
		(3, 3)	0.5433	0.1761	0.3144	0.2799	0.2992			(3, 3)	0.5552	0.1868	0.3378	0.2881	0.3021
	5	(1, 1)	0.5573	0.1586	0.3050	0.2816	0.2944		5	(1, 1)	0.5643	0.1590	0.3110	0.2821	0.2951
		(2, 2)	0.7013	0.1562	0.4614	0.3151	0.3357			(2, 2)	0.7100	0.1599	0.4742	0.3163	0.3372
		(3, 3)	0.7908	0.1671	0.5929	0.3462	0.3721			(3, 3)	0.7979	0.1775	0.6087	0.3487	0.3746
	6	(1, 1)	0.4036	0.1704	0.1967	0.2547	0.2608		6	(1, 1)	0.4033	0.1705	0.1993	0.2551	0.2610
		(2, 2)	0.4636	0.1652	0.2417	0.2629	0.2780			(2, 2)	0.4775	0.1744	0.2596	0.2692	0.2791
		(3, 3)	0.5534	0.1827	0.3230	0.2833	0.2987			(3, 3)	0.5546	0.1870	0.3367	0.2878	0.3018
1.25	3	(1, 1)	0.4143	0.3283	0.2164	0.2878	0.2863	1.25	3	(1, 1)	0.4304	0.3272	0.2202	0.2888	0.2875
		(2, 2)	0.3183	0.3324	0.1708	0.2745	0.2686			(2, 2)	0.3394	0.3332	0.1778	0.2766	0.2709
		(3, 3)	0.2790	0.3486	0.1595	0.2718	0.2616			(3, 3)	0.3051	0.3480	0.1669	0.2735	0.2653
	4	(1, 1)	0.6188	0.3343	0.3719	0.3218	0.3289		4	(1, 1)	0.6238	0.3336	0.3728	0.3222	0.3288
		(2, 2)	0.6797	0.3408	0.4508	0.3369	0.3481			(2, 2)	0.6794	0.3414	0.4541	0.3389	0.3489
		(3, 3)	0.7356	0.3452	0.5217	0.3510	0.3703			(3, 3)	0.7312	0.3639	0.5386	0.3597	0.3725

(continued)

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Table 8
Continued

			$\rho = 0.3$								$\rho = 0.8$				
$d(\mu_1)$	Case	(γ_1, γ_2)	Synth WSD–T^2	Std–T^2	WSD–T^2	WSD–MCUSUM	WSD–MEWMA	$d(\mu_1)$	Case	(γ_1, γ_2)	Synth WSD–T^2	Std–T^2	WSD–T^2	WSD–MCUSUM	WSD–MEWMA
	5	(1, 1)	0.7568	0.3432	0.5239	0.3542	0.3674		5	(1, 1)	0.7606	0.3466	0.5311	0.3551	0.3685
		(2, 2)	0.8527	0.3628	0.6860	0.3929	0.4115			(2, 2)	0.8556	0.3720	0.6960	0.3946	0.4131
		(3, 3)	0.9036	0.3921	0.7892	0.4256	0.4453			(3, 3)	0.9046	0.4081	0.7975	0.4268	0.4471
	6	(1, 1)	0.6260	0.3335	0.3710	0.3215	0.3285		6	(1, 1)	0.6234	0.3335	0.3725	0.3220	0.3287
		(2, 2)	0.6804	0.3402	0.4492	0.3364	0.3476			(2, 2)	0.6787	0.3413	0.4531	0.3386	0.3485
		(3, 3)	0.7383	0.3579	0.5328	0.3549	0.3699			(3, 3)	0.7311	0.3638	0.5378	0.3594	0.3722
1.5	3	(1, 1)	0.6373	0.5165	0.3720	0.3453	0.3442	1.5	3	(1, 1)	0.6439	0.5141	0.3746	0.3464	0.3461
		(2, 2)	0.5286	0.5131	0.2923	0.3293	0.3248			(2, 2)	0.5422	0.5129	0.2998	0.3316	0.3273
		(3, 3)	0.4753	0.5237	0.2633	0.3236	0.3168			(3, 3)	0.4969	0.5293	0.2766	0.3275	0.3209
	4	(1, 1)	0.7980	0.5406	0.5746	0.3852	0.3925		4	(1, 1)	0.7951	0.5405	0.5724	0.3852	0.3921
		(2, 2)	0.8295	0.5548	0.6474	0.4013	0.4115			(2, 2)	0.8207	0.5562	0.6418	0.4023	0.4112
		(3, 3)	0.8585	0.5789	0.7142	0.4194	0.4318			(3, 3)	0.8462	0.5848	0.7046	0.4223	0.4332
	5	(1, 1)	0.8851	0.5632	0.7249	0.4210	0.4327		5	(1, 1)	0.8859	0.5675	0.7301	0.4219	0.4336
		(2, 2)	0.9348	0.5949	0.8421	0.4569	0.4712			(2, 2)	0.9342	0.6035	0.8458	0.4577	0.4721
		(3, 3)	0.9582	0.6280	0.9018	0.4874	0.5070			(3, 3)	0.9566	0.6395	0.9031	0.4898	0.5103
	6	(1, 1)	0.7988	0.5397	0.5738	0.3848	0.3922		6	(1, 1)	0.7948	0.5402	0.5720	0.3852	0.3920
		(2, 2)	0.8297	0.5538	0.6463	0.4008	0.4111			(2, 2)	0.8203	0.5563	0.6410	0.4020	0.4109
		(3, 3)	0.8587	0.5781	0.7133	0.4190	0.4314			(3, 3)	0.8461	0.5849	0.7043	0.4221	0.4330
2	3	(1, 1)	0.9307	0.8657	0.7482	0.4510	0.4512	2	3	(1, 1)	0.9312	0.8652	0.7479	0.4518	0.4521
		(2, 2)	0.8842	0.8657	0.6445	0.4308	0.4278			(2, 2)	0.8835	0.8651	0.6478	0.4330	0.4301
		(3, 3)	0.8476	0.8724	0.5926	0.4228	0.4176			(3, 3)	0.8479	0.8719	0.6033	0.4266	0.4216
	4	(1, 1)	0.9658	0.8765	0.8819	0.4869	0.4918		4	(1, 1)	0.9623	0.8765	0.8752	0.4868	0.4915
		(2, 2)	0.9649	0.8836	0.8995	0.4998	0.5076			(2, 2)	0.9570	0.8840	0.8850	0.5021	0.5101
		(3, 3)	0.9664	0.8925	0.9161	0.5217	0.5350			(3, 3)	0.9565	0.8926	0.8982	0.5297	0.5434
	5	(1, 1)	0.9834	0.8834	0.9438	0.5170	0.5264		5	(1, 1)	0.9824	0.8833	0.9434	0.5175	0.5274
		(2, 2)	0.9897	0.8942	0.9709	0.5739	0.5966			(2, 2)	0.9883	0.8934	0.9692	0.5785	0.6026
		(3, 3)	0.9927	0.9041	0.9819	0.5617	0.6875			(3, 3)	0.9911	0.9022	0.9797	0.6629	0.6994
	6	(1, 1)	0.9655	0.8759	0.8813	0.4866	0.4914		6	(1, 1)	0.9621	0.8763	0.8748	0.4867	0.4914
		(2, 2)	0.9645	0.8832	0.8991	0.4993	0.5070			(2, 2)	0.9567	0.8838	0.8846	0.5017	0.5097
		(3, 3)	0.9662	0.8923	0.9159	0.5212	0.5344			(3, 3)	0.9564	0.8924	0.8981	0.5294	0.5429

It follows that $\boldsymbol{\mu}_0 = (\mu_1, \mu_2, \mu_3, \mu_4)' = (1.0138, 1.1693, 1.1058, 1.2372)'$, $\boldsymbol{P} = (P_1, P_2, P_3, P_4)' = (0.5330, 0.6101, 0.5887, 0.6279)'$ and

$$\Sigma = \begin{pmatrix} 0.0286 & -0.0599 & -0.0265 & 0.0305 \\ & 0.5022 & 0.2589 & 0.3194 \\ & & 0.2723 & 0.0470 \\ & & & 0.8124 \end{pmatrix},$$

since $\mu_j = \exp(\frac{1}{2}\sigma_j^2)$, $P_j = \Pr(X_j \leq \mu_j)$, $\mathrm{Var}(X_j) = \exp(\sigma_j^2)[\exp(\sigma_j^2) - 1]$, $\mathrm{cov}(X_j, X_k) = \rho_{X_j.X_k}\sqrt{\mathrm{Var}(X_j)\mathrm{Var}(X_k)}$, for $j, k = 1, 2, 3$, and 4. Note that X_j is the jth element of the four variate lognormal vector.

The data which consist of the sample averages of the four variables for the 30 samples are given in columns 2–5 in Table 9 while the T^2, WSD–T^2,

Table 9
Four variate lognormal data and summary statistics

Sample no., i	Variables							
	$\overline{X}_{1i}$	$\overline{X}_{2i}$	$\overline{X}_{3i}$	$\overline{X}_{4i}$	T_i^2	$T_{W.i}^2$	E_i^W	Y_i^W
1	0.8949	1.3488	1.1047	1.3189	3.5618	4.5177	0.9392	1.5670
2	0.9369	1.6291	1.3241	1.2851	2.8913	1.8697	1.5341	1.9237
3	0.9824	1.0503	0.9207	0.8894	2.1495	3.6381	2.0119	2.2748
4	0.9687	1.0914	0.9696	0.7670	3.0419	5.5470	4.4182	3.6712
5	0.9298	1.6901	1.6187	1.4071	5.4613	4.7516	3.7686	3.2002
6	0.9570	0.9360	0.7526	0.9662	3.9583	5.6494	5.7470	4.0602
7	0.9707	0.9604	0.9828	0.9004	1.5158	2.1521	6.8820	4.5415
8	1.0173	0.9217	1.0988	0.7724	1.5481	2.7049	7.0983	4.6719
9	0.9341	1.0871	1.0737	0.9760	2.1713	2.4651	8.9197	5.4669
10	0.8944	1.2599	1.0740	0.9097	3.1050	3.9266	12.3409	6.7372
11	0.8698	1.1811	1.1292	1.1465	7.8843	8.6510	16.3393	8.3297
12	0.9612	1.4050	1.2768	1.1578	0.9861	0.8305	14.6020	8.2595
13	1.0415	0.5544	0.7236	0.5071	5.5116	9.0906	17.0762	9.1829
14	1.1114	0.9598	0.9633	1.5677	2.3979	2.3734	11.4214	7.9906
15	0.9919	1.1134	1.1112	1.0534	0.2859	0.4270	10.6368	7.9528
16	0.9374	1.1904	1.0307	1.2119	1.7793	2.0604	11.4780	8.5245
17	1.0056	1.3339	0.9105	1.8501	3.9645	3.7763	9.4584	8.0578
18	0.9987	1.5169	1.4615	1.3330	2.4673	1.7210	5.8160	6.8461
19	1.0506	1.3297	1.0020	1.6385	2.4937	2.5413	4.0500	5.7887
20	0.9508	1.4780	1.2732	1.2978	1.1438	0.9871	3.9928	5.7060
21	1.0125	1.4502	1.3500	1.2784	1.7744	1.1968	2.6817	4.6751
22	0.9786	1.0688	1.1289	0.9450	0.8792	1.2069	2.9397	4.7789
23	1.1395	1.3631	1.1167	3.2070	35.9350	21.4859	2.8963	4.5288
24	1.0617	0.8025	0.8638	1.0041	1.5262	2.3329	2.2417	4.0913
25	0.9732	1.1316	0.9998	1.1800	0.7307	0.9160	2.6594	4.2681
26	1.0971	1.0979	0.9497	1.7534	2.7234	2.2324	2.9682	4.0680
27	1.1018	1.0004	0.9271	1.6049	2.2289	2.0710	4.0533	4.3895
28	1.1787	1.0521	1.0511	2.0356	8.0076	6.0555	6.4866	5.2925
29	0.9504	1.4733	1.3610	1.0651	2.0926	1.8776	2.8233	3.6231
30	1.0768	0.9798	1.1414	1.0707	1.3638	1.6178	2.3781	3.1351

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WSD–MEWMA, and WSD–MCUSUM charts' statistics are shown in columns 6, 7, 8, and 9, respectively. The control charting statistic of the WSD–T^2 sub-chart, i.e., $T^2_{W,i}$ is similar to that of the WSD–T^2 chart.

Note that the statistics and limits of the T^2, WSD–T^2, WSD–MEWMA, WSD–MCUSUM, and synthetic WSD–T^2 charts are computed based on the formulae given in Secs. 2 and 3. The T^2 and WSD–T^2 charts have the same limit, i.e., $h = \chi^2_{0.0027}(4) = 16.2512$ so that $\text{ARL}_0 = 370$ for a 4 variate normal distribution. Based on similar requirements, the limits of the WSD–MEWMA and WSD–MCUSUM charts are determined as $h_E = 14.6$ and $h_c = 9.101$, respectively. Since the WSD method effectively approximates the probability density function (pdf) of the underlying distribution with multivariate normal distributions (Chang and Bai, 2004), the control limits of the standard MEWMA and standard MCUSUM charts are used approximately here for the WSD–MEWMA and WSD–MCUSUM charts, respectively. Therefore, the methods suggested by Lee and Khoo (2006a,b) are employed in computing h_E and h_c, respectively. The approximation provided by the WSD method also enables the optimal limits of the synthetic WSD–T^2 chart to be determined using the approach suggested by Ghute and Shirke (2008a) for the synthetic T^2 chart. It follows that the optimal limits for the synthetic WSD–T^2 chart are $\text{UCL}_S = 11.6383$ for the WSD–T^2 sub-chart and $L_S = 7$ for the CRL sub-chart.

The T^2, WSD–T^2, WSD–MEWMA, WSD–MCUSUM, and synthetic WSD–T^2 charts are all plotted in Fig. 2. Figure 2 shows that the T^2 and WSD–MCUSUM charts give out-of-control signals at samples 23 and 13, respectively. We also observe that the WSD–MEWMA chart gives out-of-control signals at samples 11, 12, and 13. The WSD–T^2 chart issues an out-of-control signal at sample 23. Although the WSD–T^2 statistic at the 23rd sample point plots beyond $\text{UCL}_S = 11.6383$ of the WSD–T^2 sub-chart, the CRL_1 of the corresponding CRL sub-chart is 23, i.e., greater than $L_S = 7$. This indicates that no out-of-control signal is detected by the

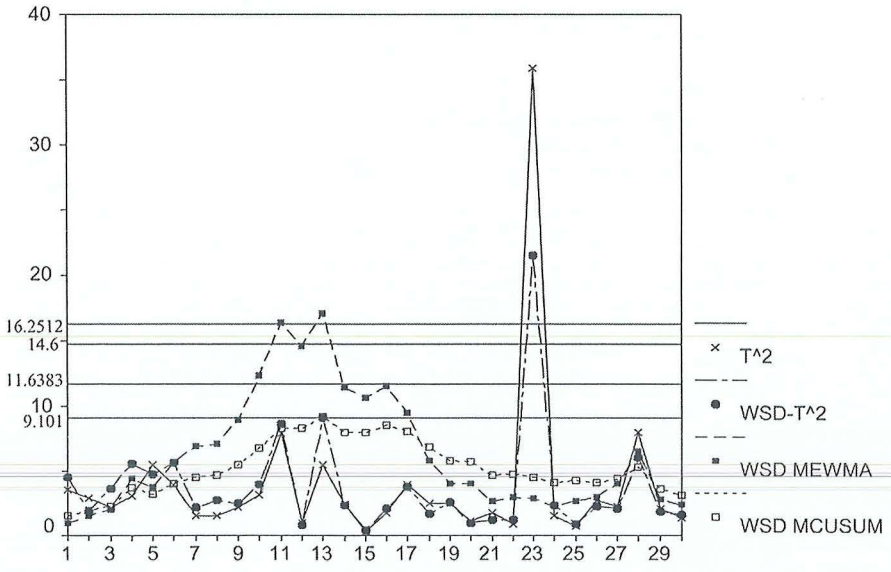

Figure 2. T^2, WSD–T^2, WSD–MEWMA, WSD–MCUSUM charts and WSD–T^2 sub-chart for the example.

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synthetic WSD–T^2 chart. Since the process is in-control for all the 30 samples, a false alarm is issued more frequently by the Hotelling's T^2, WSD–T^2, WSD–MCUSUM, and WSD–MEWMA charts compared with the synthetic WSD–T^2 chart when the underlying distribution is skewed.

6. Conclusion

In this article, a new synthetic WSD–T^2 chart based on the weighted standard deviation approach is suggested. This article shows how the standard WSD–T^2 chart's performance can be significantly enhanced by combining it with the conforming run length (CRL) chart in the construction of the synthetic WSD–T^2 chart. The simulations show that the synthetic WSD–T^2 chart gives favorable results by having the highest mean shift detection rates for moderate and large shifts when process parameters are known and unknown, based on all skewnesses (γ_1, γ_2). Besides, the synthetic WSD–T^2 chart also has lower false alarm rates than all the charts for skewed populations, except the WSD–MCUSUM chart, when subgrouping is considered. The example in Sec. 5 also shows that the synthetic WSD–T^2 chart provides a more reliable conclusion about the state of a process when the underlying distribution is skewed and the process is in-control. All the advantages of the synthetic WSD–T^2 chart mentioned above makes it a favorable alternative to all the existing multivariate charts for skewed populations based on the WSD method, discussed in Section 2.3.

Three future works on multivariate control charts for skewed populations that are worthy of pursuing are as follows.

(i) A study on the performance of the synthetic WSD–T^2 chart under a steady-state mode, where a mean shift occurs randomly at any time.
(ii) Construction of the Hotelling's T^2, MEWMA, MCUSUM, and synthetic T^2 charts incorporating the weighted variance (WV) method.
(iii) Development of new multivariate control charting methods to monitor the process dispersion of skewed populations.

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SUATU PERBANDINGAN ANTARA CARTA-CARTA KAWALAN BERSTATISTIK DENGAN CARTA KAWALAN BERDASARKAN RANGKAIAN NEURAL

(A Comparison between Statistical Control Charts and a Neural Network Based Control Chart)

ONG HONG CHOON, CHEAH SIEW CHUIN & LOW SHEAU PHIN

ABSTRAK

Carta kawalan merupakan suatu alat kualiti yang paling berpengaruh dalam kawalan proses berstatistik dan ia digunakan dengan luasnya dalam proses perkilangan. Apabila permintaan berkaitan dengan kawalan proses semakin meningkat, carta kawalan tradisional tidak lagi mencukupi untuk mengesan perubahan yang mendadak dalam sesuatu proses. Oleh yang demikian, petua larian digabungkan dalam carta kawalan Shewhart $\bar{X}$. Langkah ini, dan juga carta kawalan EWMA diperkenalkan untuk mengatasi pembatasan terhadap kepekaannya. Carta kawalan berdasarkan rangkaian neural merupakan satu langkah baru dan pencapaiannya dibandingkan dengan carta kawalan berstatistik. Data ujian yang sama dijana daripada pemboleh ubah rawak normal piawai dengan menggunakan SAS. Data ini dianggap sebagai data dalam kawalan untuk ketiga-tiga jenis carta kawalan. Kriterium yang digunakan untuk membandingkan prestasi kedua-dua jenis carta adalah purata panjang larian (ARL). Daripada keputusan yang didapati, rangkaian neural mempunyai ARL yang lebih baik berbanding dengan petua larian dalam carta kawalan tradisional dan juga carta EWMA ketika mengesan anjakan kecil dan besar dalam min proses.

Kata kunci: purata panjang larian; rangkaian neural; petua larian; EWMA; Shewhart $\bar{X}$

ABSTRACT

Control chart is one of the most powerful quality tools in statistical process control and is widely used in the manufacturing process. As the demand of the quality control increases, traditional control chart is no longer sufficient to detect the sudden change in a process. Thus, run rules are built-in into the Shewhart $\bar{X}$ control chart. This improvement, and also EWMA charts are introduced to overcome the limitation to its sensitivity. Neural network based control chart is a new approach and its performance is compared with the statistical control charts. The same test data from the standard normal random variable is generated using SAS. This data is assumed as the in-control data for the three types of control charts. The criteria to compare the performance of both types of control charts is the average run length (ARL). From the results obtained, neural network has a better ARL than the statistical control charts which includes the run rules of Shewhart $\bar{X}$ control chart and the EWMA chart when detecting small and large shifts in the process mean.

Keywords: average run length; neural network; run rules; EWMA; Shewhart $\bar{X}$

1. Pengenalan

Pada tahun 1924, Dr. Walter A. Shewhart memperkenalkan carta kawalan tradisional - carta kawalan Shewhart. Walau bagaimanapun, carta kawalan tradisional kurang sensitif untuk mengesan perubahan kecil dalam sesuatu proses. Maka, petua larian yang digabungkan dalam

carta kawalan Shewhart $\bar{X}$ diperkenalkan oleh Khoo dan Khotrun (2006) untuk menyelesaikan masalah tersebut. Dalam kajian ini, lima peraturan larian skema digunakan iaitu skema 2-dari-2, skema 2-dari-3, skema 2-dari-4, skema 3-dari-3, dan skema 3-dari-4. Pada masa yang sama, carta kawalan EWMA juga digunakan untuk mengesan perubahan kecil dalam sesuatu proses. Oleh itu dalam kajian ini, prestasi petua larian yang digabungkan dalam carta kawalan Shewhart dan prestasi carta kawalan EWMA dibandingkan dengan prestasi carta kawalan berdasarkan rangkaian neural dalam pengesanan anjakan dalam min proses.

Rangkaian neural buatan merupakan suatu kepintaran buatan yang menyerupai sistem saraf biologi dalam menyediakan kecekapan pengiraan dan berupaya untuk menyelesaikan pelbagai masalah (Dayhoff 1990). Reka bentuk rangkaian neural adalah tidak rumit. Apabila jumlah data meningkat, prestasi rangkaian neural boleh diperbaiki dengan mempersembahkan latihan tambahan. Hal ini membolehkan rangkaian neural menjadi lebih berkesan dalam mengesan corak-corak luar biasa dalam carta-carta kawalan jika dibandingkan dengan carta-carta kawalan lain yang berdasarkan statistik.

Selepas pengenalan ini, suatu penerangan ringkas berkenaan dengan ketiga-tiga jenis carta kawalan diberi dalam Bahagian 2. Penerangan lanjut disusuli dengan seni bina model perseptron multilapisan, al-Khwarizmi perambatan balik ralat dan fungsi pengaktifan yang digunakan. Kemudian, keputusan simulasi serta perbincangan tentangnya dikemukakan dan kertas kerja ini diakhiri dengan kesimpulan kajian.

2. Kaedah

2.1. *Data ujian yang dijanakan*

Data ujian dijanakan dengan menggunakan *Statistical Analysis System* (SAS) versi 9.1. Cara yang sama digunakan untuk menghasilkan proses luar kawalan untuk anjakan yang kekal dalam min proses. Data ujian dijanakan daripada pemboleh ubah rawak normal, $N(\delta, 1)$ dengan δ mewakili anjakan min proses. Set data yang sama akan digunakan dalam petua larian carta Shewhart $\bar{X}$, carta EWMA dan juga carta kawalan rangkaian neural.

2.2. *Petua larian dalam carta kawalan Shewhart* $\bar{X}$

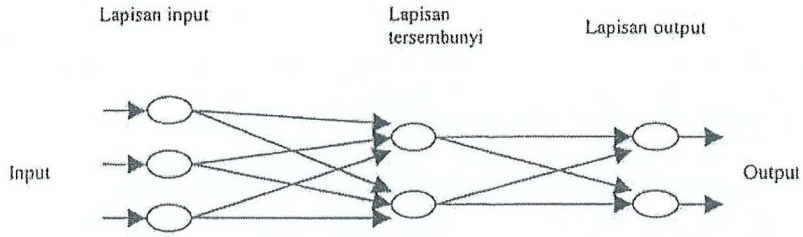
Carta kawalan Shewhart $\bar{X}$ adalah suatu carta kawalan yang popular kerana ia sangat mudah digunakan dan cepat untuk mengesan anjakan besar. Untuk meningkatkan kepekaan Shewhart $\bar{X}$ dalam pengesanan anjakan kecil, skema-skema petua larian telah dicadangkan. Dalam kajian ini, skema-skema yang dipertimbangkan termasuk carta kawalan Shewhart piawai (juga dikenali sebagai skema 1-daripada-1), skema 2-daripada-2, skema 2-daripada-3, skema 2-daripada-4, skema 3-daripada-3, dan skema 3-daripada-4.

2.3. *Carta kawalan EWMA*

Carta kawalan EWMA telah diperkenalkan oleh Roberts (1959) dan merupakan pilihan yang lain bagi mengesan anjakan yang kecil. Tambahan pula, EWMA adalah carta kawalan yang unggul apabila digunakan dengan tinjauan individu kerana carta kawalan EWMA boleh dipertimbangkan sebagai purata pemberat semua tinjauan lalu dan semasa. Dengan merujuk kepada Crowder (1989), satu carta kawalan EWMA optimum direka dalam kajian ini. Carta kawalan EWMA optimum adalah carta yang paling peka terhadap suatu magnitud anjakan tertentu berdasarkan nilai purata panjang larian (ARL) dalam kawalan yang ditetapkan.

2.4. *Rangkaian neural buatan*

Rangkaian neural buatan mengandungi unit-unit yang terdiri daripada ciri-ciri input / output (Schalkoff 1997). Ianya berupaya memproses maklumat daripada sambutan neuron dari input dan berhubung ke output. Nilai-nilai input bersama dengan sambutan yang dikenali, digunakan untuk melatih model rangkaian neural. Sepanjang latihan, pemberat-pemberat dalam rangkaian akan diubah suai sehingga output yang sesuai diperoleh. Hal ini dapat dilihat dalam Rajah 1

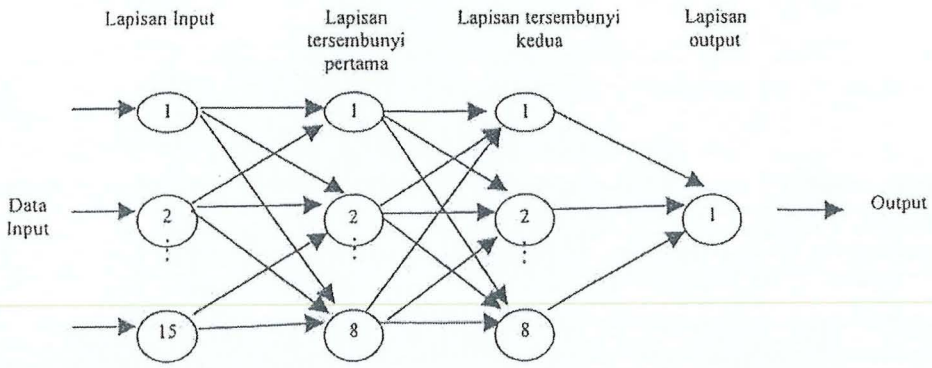


Rajah 1: Model rangkaian neural

Terdapat dua jenis seni bina rangkaian neural yang menggunakan pembelajaran yang diselia, iaitu model perseptron multilapisan (Multi-Layered Perceptron - MLP) dan rangkaian fungsi asas jejari (Radial Basis Function - RBF). Dalam kajian ini, model perseptron multilapisan (MLP) telah digunakan untuk melaksanakan proses pembelajaran rangkaian neural.

2.5. *Seni bina MLP dengan dua lapisan tersembunyi*

MLP merupakan suatu rangkaian neural dengan pembelajaran yang diselia dan nilai outputnya adalah selanjar. Menurut Cheng dan Cheng (2001), rangkaian ini menyediakan satu pemetaan daripada satu vektor input kepada satu vektor output. Dalam kajian ini, seni bina MLP dengan dua lapisan tersembunyi digunakan seperti yang ditunjukkan dalam



Rajah 2: MLP dengan struktur $15 \times 8 \times 8 \times 1$

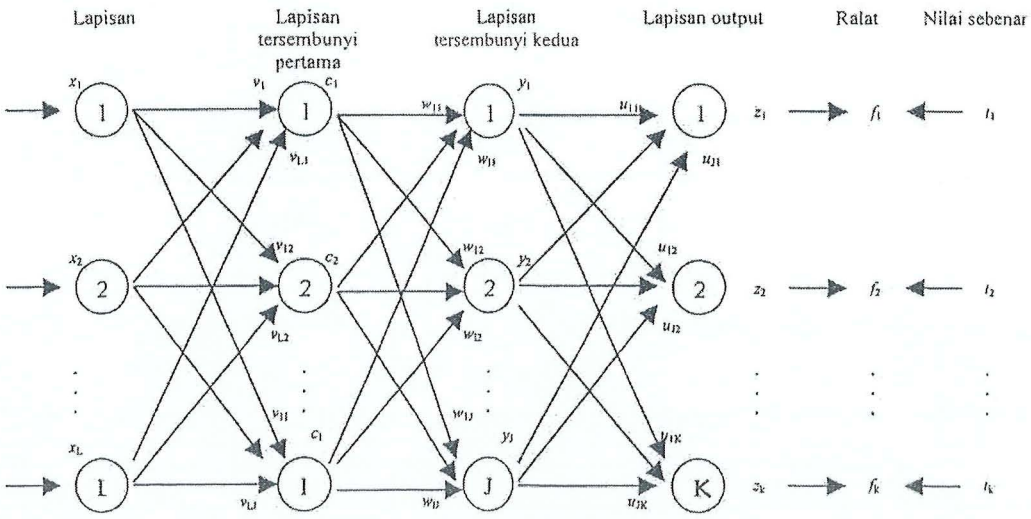
Nod dalam setiap lapisan adalah neuron manakala lapisan yang menghubungkan antara neuron mempunyai pemberat. Lapisan input mengandungi 15 neuron yang merupakan 15 titik data input yang berturutan dalam satu carta kawalan, manakala lapisan output yang mengandungi hanya satu neuron yang menentukan jenis corak yang dikemukakan di dalam kajian ini. Oleh itu, model rangkaian neural yang mempunyai empat lapisan, iaitu struktur $15 \times 8 \times 8 \times 1$ telah digunakan dalam kajian ini.

2.6. *Al-Khwarizmi latihan*

Al-Khwarizmi merujuk kepada satu kaedah atau proses untuk mencapai satu penyelesaian. Maka, al-Khwarizmi latihan dalam rangkaian neural menghasilkan output yang diinginkan melalui rangkaian neural dengan prosedur latihan. Rangkaian neural kadang kala juga dikenali sebagai al-Khwarizmi mesin pembelajaran kerana apabila rangkaian adalah terlatih, pemberat hubungan akan bertukar dan rangkaian akan mempelajari penyelesaian masalah tersebut.

2.7. *Kaedah latihan perambatan balik ralat*

Kaedah latihan perambatan balik ralat (Error backpropagation training method - EBP) merupakan al-Khwarizmi yang diguna untuk latihan rangkaian neural multilapisan. Struktur MLP yang menggunakan EBP ditunjukkan dalam Rajah 3.



Rajah 3: MLP dengan struktur dua lapisan tersembunyi (Looney 1997)

Terdapat L neuron dalam lapisan input, I neuron dalam lapisan tersembunyi pertama, J neuron dalam lapisan tersembunyi kedua dan K neuron dalam lapisan output. Katakanlah pemberat di lapisan tersembunyi pertama adalah $\{v_{li}\}$, di lapisan tersembunyi kedua adalah $\{w_{ij}\}$ dan di lapisan output adalah $\{u_{jk}\}$, yang mana $i = 1, 2, \ldots, I$, $j = 1, 2, \ldots, J$ dan $k = 1, 2, \ldots, K$.

Vektor input $x = (x_1, x_2 \ldots x_L)$ digabungkan dengan vektor pemberat dalam lapisan tersembunyi pertama, $\{v_{li}\}$. Isyarat input adalah

$$n_i = \sum_{l=1}^{L} v_{li} x_l \tag{1}$$

Kemudian, setiap satu neuron yang tersembunyi dikira dengan fungsi pengaktifan p untuk mendapatkan isyarat c_i.

$$c_i = p(n_i) = p\left(\sum_{l=1}^{L} v_{li} x_l\right) \tag{2}$$

Untuk lapisan tersembunyi kedua, setiap satu neuron tersembunyi daripada siaran-siaran lapisan tersembunyi pertama mengisyaratkan c_i kepada neuron lapisan tersembunyi kedua.

$$r_j = \sum_{i=1}^{I} w_{ij} c_i \tag{3}$$

Kemudian, setiap satu neuron tersembunyi dikira dengan fungsi pengaktifan h untuk mendapatkan isyarat y_j.

$$y_j = h(r_j) = h\left(\sum_{i=1}^{I} w_{ij} c_i\right) \tag{4}$$

Isyarat ini juga digunakan ke atas lapisan output.

$$s_k = \sum_{j=1}^{J} u_{jk} y_j \tag{5}$$

Dan isyarat s_k dikira dengan fungsi pengaktifan g untuk menghasilkan nilai output z_k

$$z_k = g(s_k) = g\left(\sum_{j=1}^{J} u_{jk} y_j\right) \tag{6}$$

Jumlah kuasa dua ralat untuk dua lapisan tersembunyi adalah

$$E = E(v, w, u)$$
$$E = \sum_{k=1}^{K} (t_k - z_k)^2 \tag{7}$$

Ralat wujud kerana output latihan yang dihasilkan z_k adalah berbeza daripada nilai sebenar t_k. Hal ini kerana lapisan neuron tersembunyi menjana output salah yang perlu dibetulkan

dengan mengubahsuaikan pemberat-pemberat. Oleh itu, ralat tersebut menjadi kriterium untuk menilai keberkesanan rangkaian neural (Looney 1997).

Terbitan separa digunakan untuk mengemaskinikan pemberat-pemberat seperti yang terdapat dalam persamaan-persamaan (8), (9) dan (10), yang η ditakrifkan sebagai kadar pembelajaran dan ia merupakan suatu nilai yang selanjar.

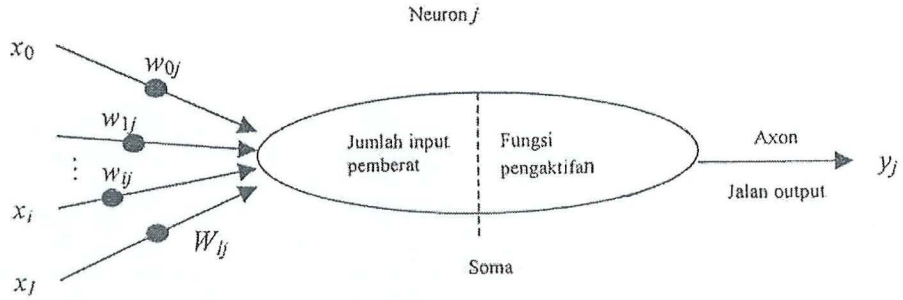
$$v_{li}^{(r+1)} = v_{li}^{(r)} - \eta\left\{\frac{\partial E(v^{(r)}, w^{(r)}, u^{(r)})}{\partial v_{li}}\right\}, \tag{8}$$

$$w_{ij}^{(r+1)} = w_{ij}^{(r)} - \eta\left\{\frac{\partial E(v^{(r)}, w^{(r)}, u^{(r)})}{\partial w_{ij}}\right\}, \tag{9}$$

$$u_{jk}^{(r+1)} = u_{jk}^{(r)} - \eta\left\{\frac{\partial E(v^{(r)}, w^{(r)}, u^{(r)})}{\partial u_{jk}}\right\}. \tag{10}$$

2.8. *Fungsi pengaktifan*

Fungsi pengaktifan, $y = f(s)$ adalah suatu fungsi terbezakan yang memetakan jumlah input-input berpemberat, $s_j = \sum w_{ij}x_i$ ke dalam satu julat yang sesuai dengan nilai output. Sementara itu, $x = \{x_1, x_2, \ldots, x_I\}$ dan $t = \{t_1, t_2, \ldots, t_K\}$ masing-masing mewakili vektor input dan vektor output terhasrat. Simbol I pula merupakan bilangan neuron yang terdapat dalam lapisan input dan J merupakan nombor neuron dalam lapisan tersembunyi. Rajah 4 menunjukkan skema proses pemetaan.



Rajah 4: Skema untuk neuron buatan (Tsoukalas & Uhrig 1997)

Dalam kajian ini, fungsi pengaktifan dwikutub yang juga dikenali sebagai fungsi sigmoid digunakan untuk mendapatkan output yang sesuai.

2.9. *Rangkaian terlatih*

Rangkaian terlatih adalah penting untuk mendapatkan output yang dikehendaki dalam pembentukan atau latihan rangkaian tersebut. Dengan menggunakan satu transformasi linear, isyarat input yang diwakili oleh unit-unit input telah dipetakan ke suatu skala yang ditentukan (Cheng 1995). Pemberat hubungan awal dijanakan untuk menghasilkan nilai-nilai dalam selang [–0.5, 0.5] dan kemudian diubah suai dengan menggunakan al-Khwarizmi perambatan balik ralat. Apabila latihan dijalankan, nilai pemberat-pemberat baru akan diperoleh dan ianya digunakan untuk menentukan taraf satu proses.

2.10. *Ukuran prestasi*

Prestasi carta kawalan statistik dan carta kawalan rangkaian neural dinilai dengan menggunakan purata panjang larian (ARL). Ianya merupakan jumlah min sampel yang dikehendaki bagi sesuatu proses untuk mensyaratkan keadaan luar kawalan. Matlamat ARL adalah untuk mengesan sebarang keadaan luar kawalan secepat mungkin (iaitu ARL yang lebih pendek) jika proses tersebut adalah di luar kawalan (ada anjakan dalam min proses) dan selewat mungkin jika proses adalah terkawal (tiada anjakan dalam min proses).

Simulasi min proses bagi menghasilkan anjakan yang berlainan, δ dilakukan dengan menggunakan SAS versi 9.1. Data ujian dijanakan daripada pemboleh ubah rawak normal $N(\delta, 1)$ dengan δ mewakili anjakan min proses. Perbandingan antara carta kawalan statistik dan carta kawalan rangkaian neural dilakukan untuk mengesan carta yang paling berkesan. Carta kawalan yang mempunyai ARL yang lebih pendek dalam proses di keadaan luar kawalan mempunyai prestasi yang lebih baik.

3. Keputusan dan Perbincangan

Profil ARL untuk semua skema adalah berdasarkan purata anjakan proses dari sigma yang bernilai dari sifar hingga empat. Purata anjakan proses dengan sigma sifar menunjukkan proses tersebut adalah dalam kawalan dan proses luar kawalan telah ditunjukkan oleh nilai-nilai yang lain sampai ke empat sigma. Berdasarkan Jadual 1, kes-kes untuk anjakan 0.00, 0.50, 0.75, 1.00, 1.25, 1.50, 1.75, 2.00, 2.25, 2.50, 2.75, 3.00, 3.50 dan 4.00 telah dilaksanakan. Had-had kawalan untuk carta kawalan berstatistik ditentukan dengan merujuk kepada kajian Crowder (1989) dan Khoo (2004). Untuk carta kawalan berstatistik dan carta kawalan berdasarkan rangkaian neural, pilihan dibuat dengan mengubah had-had kawalan untuk mendapatkan ARL dalam kawalan sebanyak lebih kurang 370.

Merujuk kepada Jadual 1 dan memberi perhatian kepada anjakan yang kecil daripada 1.0, EWMA optimum dan carta kawalan berdasarkan rangkaian neural telah menunjukkan prestasi yang lebih baik daripada carta kawalan Shewhart dan petua larian. Dengan hanya merujuk kepada EWMA optimum dan carta kawalan berdasarkan rangkaian neural, carta kawalan berdasarkan rangkaian neural dapat mengesan anjakan 0.5 lebih cepat daripada EWMA optimum. Akan tetapi, EWMA optimum pada 1.0 menunjukkan prestasi yang lebih baik daripada carta kawalan berdasarkan rangkaian neural kerana ia mencatat ARL luar kawalan yang lebih pendek.

Walaupun carta kawalan Shewhart piawai terkenal dengan kebolehannya dalam pengesanan anjakan besar, tetapi berdasarkan keputusan simulasi, carta kawalan berdasarkan rangkaian neural menunjukkan prestasi yang lebih baik dengan mencatat ARL luar kawalan yang pendek. Kesimpulannya, carta kawalan berdasarkan rangkaian neural lebih cekap dalam pengesanan anjakan kecil dan menunjukkan prestasi yang setanding dengan EWMA dalam pengesanan anjakan sederhana.

4. Kesimpulan

ARL dalam kawalan telah ditetapkan pada 370 dan merupakan sesuatu ukuran untuk membandingkan prestasi carta-carta kawalan dalam kajian ini. ARL luar kawalan yang lebih pendek menunjukkan carta kawalan tersebut mempunyai prestasi yang lebih baik dalam pengesanan anjakan kecil. Oleh kerana itu, prestasi carta-carta kawalan berdasarkan pendekatan yang berbeza telah ditentukan dengan mengambil kira ARL luar kawalan dan pada masa yang sama mempunyai ARL dalam kawalan yang sama.

Lima petua larian telah dibina dalam carta kawalan Shewhart piawai untuk meningkatkan kepekaannya dalam mengesan anjakan kecil. Tambahan pula, EWMA optimum pada 0.5 dan 1.0 telah dibina dan dipertimbangkan dalam perbandingan antara carta kawalan berstatistik dengan carta kawalan berdasarkan rangkaian neural.

Keputusan simulasi telah menunjukkan bahawa carta kawalan berdasarkan rangkaian neural lebih cekap dalam pengesanan anjakan kecil dan setanding dengan EWMA optimum dalam anjakan sederhana. Tambahan pula, carta kawalan berdasarkan rangkaian neural khasnya model perseptron multilapisan (MLP) adalah paling cekap dalam pengesanan anjakan besar.

Jadual 1: Profil ARL berdasarkan ARL dalam kawalan 370

	Shewhart	2-of-2	2-of-3	2-of-4	3-of-3	3-of-4	EWMA Optimum pada 0.5	EWMA Optimum pada 1.0	Rangkaian Neural
	Had-had kawalan								
Anjakan	± 3	± 1.78	± 1.93	± 2.01	± 1.2	± 1.39	± 0.40	± 0.76	± 2.0
0	374.88	370.00	373.31	370.92	361.63	370.61	377.04	367.74	372.17
0.5	154.97	108.69	100.03	97.49	91.76	81.12	26.55	30.79	24.55
0.75	81.63	50.72	45.68	44.58	41.66	36.28	15.44	15.11	23.79
1	43.96	25.70	23.00	22.51	21.42	18.30	10.80	9.54	13.16
1.25	24.52	14.69	13.28	12.81	12.66	11.09	8.32	6.94	9.13
1.5	15.07	9.15	8.31	8.24	8.41	7.53	6.79	5.47	5.89
1.75	9.37	6.31	5.84	5.76	6.13	5.59	5.75	4.52	4.06
2	6.34	4.60	4.35	4.33	4.93	4.56	5.01	3.86	3.03
2.25	4.45	3.61	3.46	3.48	4.16	3.93	4.45	3.39	2.11
2.5	3.22	3.02	2.92	2.95	3.71	3.54	3.99	3.03	1.95
2.75	2.49	2.62	2.57	2.61	3.43	3.33	3.65	2.74	1.51
3	2.00	2.39	2.36	2.39	3.25	3.20	3.37	2.51	1.32
3.5	1.46	2.14	2.13	2.15	3.06	3.05	2.93	2.19	1.13
4	1.19	2.04	2.04	2.05	3.02	3.01	2.57	2.00	1.04

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A synthetic double sampling control chart for the process mean

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A synthetic double sampling control chart for the process mean

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This article proposes a synthetic double sampling chart that integrates the Double Sampling (DS) $\bar{X}$ chart and the conforming run length chart. The proposed procedure offers performance improvements in terms of the zero-state Average Run Length (ARL) and Average Number of Observations to Sample (ANOS). When the size of a mean shift δ (given in terms of the number of standard deviation units) is small (i.e., between 0.4 and 0.6) and the mean sample size $n = 5$, the proposed procedure reduces the out-of-control ARL and ANOS values by nearly half, compared with both the synthetic and DS $\bar{X}$ charts. In terms of detection ability versus the Exponentially Weighted Moving Average (EWMA) chart, the synthetic DS chart is superior to the synthetic or even the DS $\bar{X}$ chart, as the former outperforms the EWMA chart for a larger range of δ values compared to the latter. The proposed procedure generally outperforms the EWMA chart in the detection of a mean shift when δ is larger than 0.5 and $n = 5$ or 10. Although the proposed procedure is less sensitive than the EWMA chart when δ is smaller than 0.5, this may not be a setback as it is usually not desirable, from a practical viewpoint, to signal very small shifts in the process to avoid too frequent process interruptions. Instead, under such circumstances, it is better to leave the process undisturbed.

Keywords: Synthetic chart, double sampling chart, exponentially weighted moving average chart, conforming run length, average run length, average number of observations to sample

1. Introduction

The performance of a control chart is mainly determined by how fast the chart reacts to a shift. Usually, the Average Run Length (ARL) is used as a performance measure of a chart when the sample size in each sampling stage is fixed. When the sample size in each sampling stage is not fixed, such as for the Double Sampling (DS) chart, the Average Number of Samples (ANOS) could be used to evaluate the chart's performance.

The synthetic DS chart combines the usefulness of the synthetic and DS $\bar{X}$ charts. Davis and Woodall (2002) pointed out that the synthetic chart can be represented as a runs rule chart with a head start feature. By having a head start feature, the synthetic chart is more sensitive in detecting an out-of-control situation at first time points, as in the case of an Exponentially Weighted Moving Average (EWMA) or cumulative sum charts with a head start (Lucas and Crosier, 1982; Rhoads *et al.*, 1996). Giving a head start to a control chart is extremely important in situations where after restarting the control chart following an alarm and checking the monitored system, one is interested to know instantaneously whether the process returns under statistical control; i.e., whether the process adjustment has successfully return the mean to its target value (Knoth, 2005).

On a different note, the DS $\bar{X}$ chart, like the Variable Sample Size (VSS) and Variable Sample Interval (VSI) charts, has warning limits that enable a quicker detection of an out-of-control signal without increasing the false alarm rate as compared with control charts with fixed sample size. However, contrary to the VSS and VSI methods, the DS approach allows the observation of a second sample without any interruption, where the second sample comes from the same population as the first (Irianto and Shinozaki, 1998).

The synthetic DS chart, which possesses the desirable properties of two basic charts, is possibly the simplest form of a VSS chart and can be successfully used when varying the sample size during production is runs. This is usually

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the case when the production level is large and it is not a problem to use a different sample size at each different sampling epoch. The synthetic DS chart samples at a higher rate when there is evidence of a change in the process and thus permits a more rapid response to an initial out-of-control situation than either the DS $\bar{X}$ or synthetic charts.

This article proposes a synthetic DS chart. This new chart is superior to any of its component charts; i.e., the synthetic chart of Wu and Spedding (2000) or the DS $\bar{X}$ chart of Daudin (1992). Section 2 gives an overview of some recent works on synthetic and DS $\bar{X}$ charts. Section 3 provides an in-depth review on the synthetic and DS $\bar{X}$ charts. Section 4 presents the proposed synthetic DS chart. Optimization programs to compute the synthetic DS chart's optimal parameters are discussed in Section 5. Section 6 explains the steps required to construct a synthetic DS chart and Section 7 illustrates the construction with an example. In Section 8, the zero-state ARL and ANOS performances of the synthetic DS, synthetic, DS $\bar{X}$, and EWMA charts are studied. The results are compared in Section 9. Finally, conclusions are drawn in Section 10.

2. The recent literature on synthetic and DS $\bar{X}$ charts

The synthetic chart, which integrates the $\bar{X}$ and Conforming Run Length (CRL) charts, was introduced by Wu and Spedding (2000). The synthetic chart outperforms the $\bar{X}$ chart for all levels of mean shift and the EWMA and joint $\bar{X}$-EWMA charts for a mean shift of δ greater than 0.8. Calzada and Scariano (2001) studied the robustness of the synthetic chart to non-normality. Davis and Woodall (2002) evaluated and improved the synthetic chart's performance. Scariano and Calzada (2003) suggested a synthetic chart for exponential data. Synthetic charts for process dispersion were proposed by Chen and Huang (2005) by merging the sample range and R and CRL charts, and Huang and Chen (2005) by combining the sample standard deviation and S and CRL charts. Chen and Huang (2006) suggested a synthetic chart and its variable sampling interval schemes by integrating the Max and CRL charts. Costa and Rahim (2006) proposed a synthetic chart based on non-central chi-square statistics. Khoo *et al.* (2008) and Castagliola and Khoo (2009) proposed synthetic charts for skewed populations. Aparisi and De Luna (2009) developed a synthetic chart that does not detect shifts in a region of admissible shifts but detects only important shifts. Scariano and Calzada (2009) suggested generalized synthetic charts.

The DS $\bar{X}$ chart was first studied by Croasdale (1974). Daudin *et al.* (1990) and Daudin (1992) proposed a DS $\bar{X}$ procedure that uses information from both samples in making a decision at the second stage. Irianto and Shinozaki (1998) suggested an optimal DS $\bar{X}$ chart. He and Grigoryan (2002) developed a DS S chart for the process standard deviation, used in agile manufacturing. He *et al.* (2002) solved the DS $\bar{X}$ chart design problem using a genetic algorithm and developed triple sampling $\bar{X}$ charts. Carot *et al.* (2002) combined the DS and variable sampling interval methods. He and Grigoryan (2003) developed an improved DS S chart that does not require the normality assumption of the sample standard deviations. He and Grigoryan (2006) presented a joint statistical design of DS $\bar{X}$ and s charts. Costa and Claro (2008) considered a DS $\bar{X}$ chart for processes from a first-order autoregressive moving average (ARMA (1, 1)) model. By minimizing the sample sizes for both in-control and out-of-control processes, Torng and Lee (2009a) proposed a modified statistical design of a DS $\bar{X}$ chart based on the multi-objective programming method and a genetic algorithm. Torng and Lee (2009b) studied the performance of a DS $\bar{X}$ chart under non-normality.

3. A review of the synthetic and DS charts

An in-depth review on the synthetic chart of Wu and Spedding (2000) and DS $\bar{X}$ chart of Daudin (1992) is provided.

3.1. *Synthetic control chart*

The synthetic chart combines the Shewhart $\bar{X}$ chart and CRL chart of Bourke (1991). It comprises a $\bar{X}/S$ sub-chart and a CRL/S sub-chart. In a synthetic chart, CRL refers to the number of inspected samples between two consecutive non-conforming samples (including the ending non-conforming sample). Assume that the state of a sequence of samples taken from a process that starts at $t = 0$ is as illustrated in Fig. 1

Here, $CRL_1 = 3$, $CRL_2 = 4$, and $CRL_3 = 2$. The steps in implementing a synthetic chart are as follows (Wu and Spedding, 2000).

Step 1. Determine the upper $UCL_{\bar{X}/S}$ and lower $LCL_{\bar{X}/S}$ control limits of the $\bar{X}/S$ sub-chart and the lower limit L' of the CRL/S sub-chart. Here,

$$UCL_{\bar{X}/S} = \mu_0 + k\sigma_{\bar{X}}, \tag{1a}$$

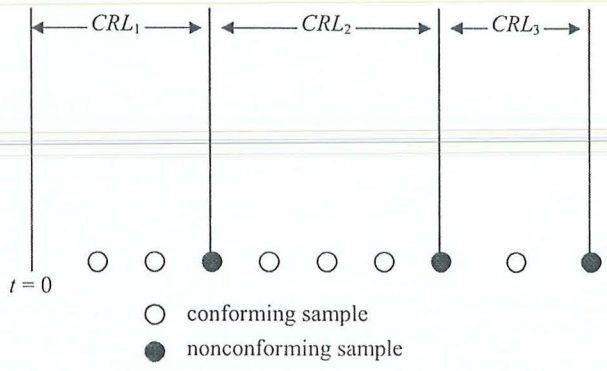

Fig. 1. Conforming run length.

and

$$LCL_{\bar{X}/S} = \mu_0 - k\sigma_{\bar{X}}, \quad (1b)$$

where μ_0 and $\sigma_{\bar{X}}$ are the in-control mean and standard deviation of the sample mean, respectively. Wu and Spedding (2000) give formulae to compute the optimal values of k and L' to obtain the minimum out-of-control ARL for a desired size of shift, δ_{opt}, based on a desired in-control ARL (ARL_0).

Step 2. At each inspection point, a random sample of size n is taken and the sample mean, $\bar{X}$, is computed.

Step 3. If $LCL_{\bar{X}/S} < \bar{X} < UCL_{\bar{X}/S}$, the sample is conforming and the control flow returns to Step 2. Otherwise, the sample is non-conforming and the control flow advances to Step 4.

Step 4. Count the number of $\bar{X}$ samples between the current (included in the count) and the last (excluded in the count) non-conforming samples. This number is the CRL value.

Step 5. If $CRL > L'$, an in-control status is concluded and the control flow returns to Step 2. Otherwise, an out-of-control status is signalled and the control flow advances to Step 6.

Step 6. Take actions to investigate and remove the assignable cause(s). Then return to Step 2.

3.2. *DS $\bar{X}$ control chart*

The DS procedure proposed by Daudin (1992) offers better ARL performance than the Shewhart $\bar{X}$ chart, without increased sampling. It can also be made to reduce the sampling without lowering the ARL performance. The DS $\bar{X}$ chart allows a second sample to be observed if the first sample signals a warning about a deviation in the mean. Using Fig. 2, the DS $\bar{X}$ charting procedure is explained as follows.

Step 1. Determine the limits L, L_1, and L_2.

Step 2. Take the first sample of size n_1 from a population with a target mean value μ_0 and an in-control standard deviation σ. Then compute the sample mean $\bar{X}_1 = \sum_{j=1}^{n_1}(X_{1j}/n_1)$ where X_{1j}, for $j = 1, 2, \ldots, n_1$, are the observations in the first sample.

Step 3. If $Z_1 = (\bar{X}_1 - \mu_0)/(\sigma/\sqrt{n_1})$ is in I_1, the process is in-control. Then the control flow returns to Step 2.

Step 4. If Z_1 is in I_3, the process is out-of-control. Then proceed to Step 8.

Step 5. If Z_1 is in I_2, a second sample of size n_2 is taken from the same population, from which the first sample was taken. Then compute $\bar{X}_2 = \sum_{j=1}^{n_2} X_{2j}/n_2$ where X_{2j}, for $j = 1, 2, \ldots, n_2$, are the observations in the second sample.

Step 6. Compute the overall sample mean $\bar{X} = (n_1\bar{X}_1 + n_2\bar{X}_2)/(n_1 + n_2)$.

Step 7. If $Z = (\bar{X} - \mu_0)/(\sigma/\sqrt{n_1 + n_2})$ is in I_4, the process is in an in-control state and the control flow returns to Step 2. Otherwise, the process is in an out-of-control state and the control flow advances to Step 8.

Step 8. An out-of-control signal is issued and corrective actions are taken to identify and remove assignable cause(s). Then return to Step 2.

Let $\delta = (|\mu_1 - \mu_0|)/\sigma$ be the size of a mean shift, where μ_0 is the in-control mean, μ_1 is the out-of-control mean, and σ is the in-control standard deviation. The probability $P(\delta)$ for the process to be in an out-of-control state is (Daudin *et al.*, 1990):

$$P(\delta) = 1 - \{\Phi(L_1 + \delta\sqrt{n_1}) - \Phi(-L_1 + \delta\sqrt{n_1})\} - \int_{z\in I_2^*} \left\{\Phi\left(cL_2 + rc\delta - z\sqrt{n_1/n_2}\right) - \Phi\left(-cL_2 + rc\delta - z\sqrt{n_1/n_2}\right)\right\}\phi(z)\,dz, \quad (2)$$

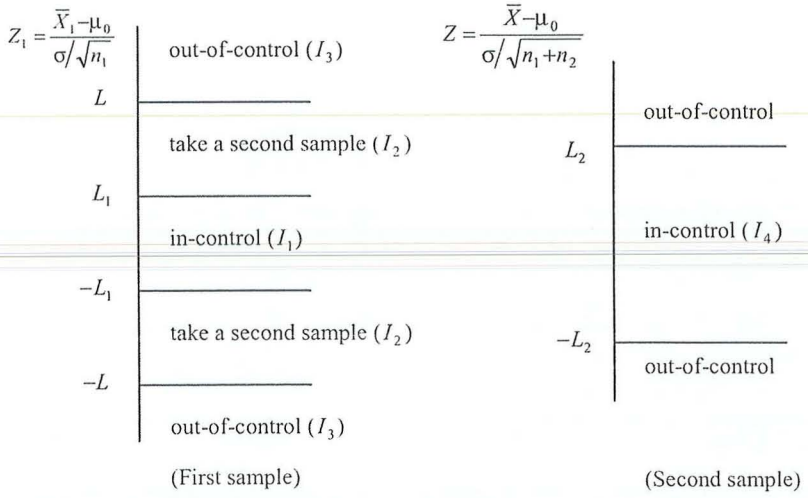

Fig. 2. Daudin's DS $\bar{X}$ control charting procedure.

where $r = \sqrt{n_1 + n_2}$, $c = r/\sqrt{n_2}$ and $I_2^* = (-L + \delta\sqrt{n_1}, -L_1 + \delta\sqrt{n_1}) \cup (L_1 + \delta\sqrt{n_1}, L + \delta\sqrt{n_1})$. The optimization of an optimal DS $\bar{X}$ chart involves determining five optimal parameters, namely, L_1, L_2, L, n_1, and n_2. The mean sample size in each sampling stage corresponding to a mean shift of size δ is $n = n_1 + n_2 \times \Pr(Z_1 \in I_2 \mid \delta)$.

Daudin *et al.* (1990) suggested an optimization procedure by minimizing the mean sample size in each sampling stage for the in-control process; that is,

$$\min_{n_1, n_2, L, L_1, L_2} n_1 + n_2 \times \Pr(Z_1 \in I_2 \mid 0), \tag{3}$$

subject to

(i) $P(0) = \alpha$;
(ii) $P(\delta) = 1 - \beta$,

where α and β are the Type-I and Type-II error sizes, respectively. An algorithm to determine the optimal parameters using this optimization procedure is provided by Daudin *et al.* (1990).

Contrary to the procedure suggested by Daudin *et al.* (1990), Irianto and Shinozaki (1998) proposed an optimization procedure by minimizing the out-of-control ARL (or similarly, maximizing $P(\delta)$) of the DS $\bar{X}$ chart in detecting a shift of size δ in the mean. Using the same notations and by setting the sample sizes, n_1 and n_2, so that the desired mean sample size, n, for the in-control process is attained, the optimization is formulated as follows:

$$\max_{L, L_1, L_2} P(\delta), \tag{4}$$

subject to

(i) $n_1 + n_2 \times \Pr(Z_1 \in I_2 \mid 0) = n$, which gives $L = \Phi^{-1}[\frac{n-n_1}{2n_2} + \Phi(L_1)]$;
(ii) $P(0) = \alpha$.

Irianto and Shinozaki (1998) showed that:

$$\Phi^{-1}\left(1 - \frac{n - n_1}{2n_2} - \frac{\alpha}{2}\right) \le L_1 \le \Phi^{-1}\left(1 - \frac{n - n_1}{2n_2}\right),$$

where this range of L_1 is quite small if α is small. For simplicity, the median of L_1 in this inequality can be used. After obtaining the value of L_1, L given in constraint (ii) can be computed. Then using constraint (iii), i.e., by obtaining $P(0)$ from Equation (2) and equating it with α, L_2 can be uniquely determined for given L_1 and L.

4. The synthetic DS chart

This section discusses the proposed synthetic DS chart. The synthetic DS chart comprises a DS/S sub-chart and a CRL/S sub-chart. The DS $\bar{X}$ chart of Daudin (1992) and the CRL chart of Bourke (1991) are considered in constructing the DS/S sub-chart and the CRL/S sub-chart, respectively. CRL denotes the number of inspected sampling stages between two consecutive non-conforming sampling stages (including the ending non-conforming sampling stage). The classification of a sampling stage as either conforming or non-conforming depends on the information about the quality given by the first sample alone or the joint information contained in the first and second samples. Thus, a sampling stage may either contain the observations of only the first sample or the observations of both the first and second samples. The first sample is of size n_1 and the second is of size n_2. Figure 2 can also be used to illustrate the DS/S sub-chart but the descriptions for in-control and out-of-control processes must be replaced with conforming and non-conforming sampling stages, respectively. The notations for the limits of the standard DS $\bar{X}$ chart, i.e., L, L_1, and L_2, are still applicable for the DS/S sub-chart. In addition, L_3 is used to denote the lower limit of the CRL/S sub-chart.

Let $ARL_S(\delta)$ represents the average number of sampling stages required to detect a mean shift of size δ, under a zero-state mode. $ARL_S(\delta)$ is computed using Equation (5):

$$ARL_S(\delta) = \frac{1}{P(\delta)} \times \frac{1}{1 - [1 - P(\delta)]^{L_3}}, \tag{5}$$

where $P(\delta)$ is given in Equation (2). When $\delta = 0$, the in-control ARL is obtained from Equation (5) as $ARL_S(0)$. The $ARL_S(\delta)$ formula in Equation (5) is similar to that in Wu and Spedding (2000) for the synthetic chart. This formula is also employed by Calzada and Scariano (2001), Davis and Woodall (2002) and many other researchers. A proof for $ARL_S(\delta)$ is given by Wu and Spedding (2000) and Calzada and Scariano (2001), to name a few. An Appendix at the end of this article shows the derivation of $ARL_S(\delta)$.

Similar to the DS $\bar{X}$ chart of Daudin (1992), the mean sample size in each sampling stage of the DS/S sub-chart, corresponding to a mean shift of size δ, is $n = n_1 + n_2 \times \Pr(Z_1 \in I_2 \mid \delta)$. Then the average number of observations required to detect a mean shift of size δ before an out-of-control signal is issued by the synthetic DS chart is computed as

$$\begin{aligned} ANOS_S(\delta) &= ARL_S(\delta) \times n \\ &= ARL_S(\delta) \times [n_1 + n_2 \times \Pr(Z_1 \in I_2 \mid \delta)] \end{aligned} \tag{6}$$

where

$$\begin{aligned} \Pr(Z_1 \in I_2 \mid \delta) = {} & [\Phi(L + \delta\sqrt{n_1}) - \Phi(L_1 + \delta\sqrt{n_1})] \\ & + [\Phi(-L_1 + \delta\sqrt{n_1}) - \Phi(-L + \delta\sqrt{n_1})] \end{aligned} \tag{7}$$

Equations (5) and (6) have been verified by a Monte Carlo simulation.

The optimal design of the synthetic DS chart requires determining a set of six parameters, i.e., n_1, n_2, L, L_1, L_2, and L_3, so that $ARL_S(\delta_{opt})$ (or similarly $ANOS_S(\delta_{opt})$) is minimized while the desired in-control ARL, i.e., $ARL_S(0)$ (or similarly, $ANOS_S(0)$), value is attained. Note that δ_{opt} represents the size of a mean shift, for which a quick detection is needed. In other words, the optimization procedure involves minimizing the out-of-control ARL of the chart for a desired size of mean shift δ_{opt}. This is similar to the optimization procedure used by Irianto and Shinozaki (1998) for the DS $\bar{X}$ chart (see Section 3.2).

The optimal parameters of a synthetic DS chart are determined as follows.

Step 1. Specify μ_0, σ, δ_{opt}, $ARL_S(0)$, and n. Here, n is the mean sample size in each sampling stage when the process is in an in-control state.

Step 2. Select the first trial values of $n_1 (< n)$ and $n_2 (\leq 2n)$, based on the constraint $n_1 + n_2 > n$, and a value of L_1 satisfying the following inequality (see Irianto and Shinozaki (1998):

$$\Phi^{-1}\left(1-\frac{n-n_1}{2n_2}-\frac{\alpha}{2}\right) \leq L_1 \leq \Phi^{-1}\left(1-\frac{n-n_1}{2n_2}\right). \tag{8}$$

For simplicity, the median of L_1 in the above inequality is used as the range of L_1 since this inequality is quite small if the size of the desired Type-I error, α, is small. Note that $\alpha = 1/ARL_S(0)$.

Step 3. Determine L using Equation (9) (see Irianto and Shinozaki (1998). Here, L is chosen so that the mean sample size in each sampling stage when the process is in an in-control state is n.

$$L = \Phi^{-1}\left[\frac{n-n_1}{2n_2} + \Phi(L_1)\right]. \tag{9}$$

Step 4. Initialize L_3 to have a value of one.

Step 5. Compute L_2 by solving Equation (10):

$$ARL_S(0) = \frac{1}{P(0)} \times \frac{1}{1-[1-P(0)]^{L_3}}, \tag{10}$$

where $P(0)$ is given in Equation (2) when $\delta = 0$.

Out[12]//MatrixForm=

ARLdelta	n1	n2	L	L1	L2	L3
1.57221	4	10	3.20638	1.63837	1.851	7
1.51362	3	10	3.20555	1.27772	1.978	6
1.59803	2	10	3.20528	1.03354	2.029	6
1.82444	1	10	3.20514	0.839213	2.044	6
1.53497	4	9	3.20621	1.58725	1.874	6
1.48399	3	9	3.20547	1.21709	1.983	5
1.56775	2	9	3.20522	0.964724	2.03	5
1.78244	1	9	3.20509	0.762445	2.085	6
1.5001	4	8	3.20605	1.52867	1.894	5
1.4616	3	8	3.20539	1.14708	1.98	4
1.54954	2	8	3.20516	0.884642	2.024	4
1.75388	1	8	3.20505	0.672367	2.089	5
1.46927	4	7	3.20588	1.46031	1.971	5
1.44733	3	7	3.2053	1.06459	2.036	4
1.54545	2	7	3.20511	0.789326	2.073	4
1.75165	1	7	3.20501	0.563963	2.087	4
1.44674	4	6	3.20572	1.37861	1.988	4
1.45264	3	6	3.20522	0.964724	2.091	4
1.57034	2	6	3.20505	0.672367	2.12	4
1.78564	1	6	3.20496	0.42887	2.136	4
1.44124	4	5	3.20555	1.27772	2.065	4
1.4867	3	5	3.20514	0.839213	2.078	3
1.64351	2	5	3.20499	0.522459	2.165	4
1.89039	1	5	3.20491	0.251599	2.186	4
1.46566	4	4	3.20539	1.14708	2.068	3
1.57169	3	4	3.20505	0.672367	2.124	3
1.79925	2	4	3.20493	0.316859	2.203	4
1.54691	4	3	3.20522	0.964724	2.128	3
1.75449	3	3	3.20496	0.42887	2.156	3
1.73936	4	2	3.20505	0.672367	2.161	3

Out[13]= {1.44124, 4, 5, 3.20555, 1.27772, 2.065, 4}

Fig. 3. $ARL_S(\delta_{opt})$ for all the (n_1, n_2) pairs and their corresponding optimal parameters when $n = 5$, $ARL_S(0) = 370$, and $\delta_{opt} = 1$.

Step 6. Calculate $ARL_S(\delta_{opt})$ using Equation (5) from the present (L_2, L_3) combination, based on the values of L, L_1, n_1, and n_2 obtained prior to Step 4.

Step 7. If $ARL_S(\delta_{opt})$ has been reduced, increase L_3 by one and return to Step 5. Otherwise, go to the next step.

Step 8. Take the current (L_2, L_3) as the optimal combination corresponding to the L, L_1, n_1, and n_2 values obtained in Steps 2 and 3.

Step 9. Repeat Steps 2 to 8 for other trial values of $n_1 (< n)$ and $n_2 (\leq 2n)$. Finally, select the optimal $(n_1, n_2, L, L_1, L_2, L_3)$ combination that gives the smallest $ARL_S(\delta_{opt})$.

Section 5 describes the optimization programs to compute the optimal parameters n_1, n_2, L, L_1, L_2, and L_3, incorporating the above nine-step procedure.

5. Optimization programs for the synthetic DS chart

Optimization programs were written in Mathematica 7 to help the end user to determine the optimal parameters of the synthetic DS chart. Mathematica 7 was used since it is widely available and easily accessible.

The first program computes the optimal L, L_1, L_2, L_3, n_1, and n_2 values when the user specifies the mean sample size, n, desired in-control ARL value, $ARL_S(0)$, and magnitude of a mean shift, in multiples of the standard deviation, where a quick detection is desired, δ_{opt}. The user does not have to input the values of μ_0 and σ into the program as the optimization procedure is based on the standardized statistics, Z_1 and Z. However, if the user wishes to compute the optimal L, L_1, L_2, and L_3 values based on predetermined n, n_1, and n_2 values, then the second program can be used. The two programs involve a global optimization procedure, where in the first program, the optimal L, L_1, L_2, L_3, n_1, and n_2 parameters computed produce the minimum $ARL_S(\delta_{opt})$ value among all pairs of (n_1, n_2) that have the same mean sample size, n, for $L_3 \leq 100$, as this upper bound of L_3 is large enough to give optimal parameters for most small shifts and $ARL_S(0)$ of practical interest. Similarly, the second program computes L, L_1, L_2, and L_3 values that give the minimum $ARL_S(\delta_{opt})$ for $L_3 \leq 100$ when n and n_2 are specified. These optimization programs can be requested from the first author.

A typical output from the first program for $n = 5$, $ARL_S(0) = 370$, and $\delta_{opt} = 1$ is shown in Fig. 3. Here, there are 30 pairs of (n_1, n_2) that produce a mean sample size of $n = 5$, where their corresponding optimal parameters are shown. The last line in the output presents the pair $(n_1, n_2) = (4, 5)$ and its corresponding optimal parameters that give the smallest $ARL_S(\delta_{opt})$ among all 30 pairs of (n_1, n_2). For ease of selection of optimal parameters for the synthetic DS chart, Table 1 provides the optimal combinations of L, L_1, L_2, L_3, n_1, and n_2 for some combinations of $ARL_S(0) \in \{250, 370\}$, $\delta_{opt} \in \{0.25, 0.5, 0.75, 1, 1.5\}$, and $n \in \{2, 3, \ldots, 10\}$, computed using the first program. For other combinations of $ARL_S(0)$, δ_{opt}, and n, the first program can be used to compute the six optimal parameters.

```
ARLdeltaopt=1.44124
n1=4
n2=5
L=3.20555
L1=1.27772
L2=2.065
L3=4
```

Fig. 4. $ARL_S(\delta_{opt})$ and the optimal parameters L, L_1, L_2, and L_3 based on $n = 5$, $n_1 = 4$, $n_2 = 5$, $ARL_S(0) = 370$, and $\delta_{opt} = 1$.

An example of the output given by the second program, say, for $n = 5$, $n_1 = 4$, $n_2 = 5$, $ARL_S(0) = 370$, and $\delta_{opt} = 1$, is shown in Fig. 4. Table 2 shows how this program identifies the (L_2, L_3) pair that gives the minimum $ARL_S(\delta_{opt})$. Here, $(L_2, L_3) = (2.065, 4)$ is found to have the lowest $ARL_S(\delta_{opt})$ for $L_3 \in \{1, 2, \ldots, 100\}$. Thus, the optimal parameters corresponding to $n = 5$, $n_1 = 4$, and $n_2 = 5$ are $(L, L_1, L_2, L_3) = (3.205\,55, 1.277\,72, 2.065, 4)$.

After computing the optimal parameters L, L_1, L_2, L_3, n_1, and n_2 for predefined values of n, $ARL_S(0)$, and δ_{opt}, the steps given in Section 6 are used to construct the optimal synthetic DS chart.

6. Constructing a synthetic DS chart

Assume that the process to be monitored follows a normal distribution with in-control mean μ_0 and standard deviation σ. A step-by-step procedure to construct the synthetic DS chart is as follows.

Step 1. Find the optimal parameters L, L_1, L_2, L_3, n_1, and n_2 using the optimization program mentioned in Section 5.

Step 2. Take the first sample with n_1 observations from a population. Then compute $\bar{X}_{1,i} = \sum_{j=1}^{n_1} X_{1j}/n_1$, where X_{1j}, for $j = 1, 2, \ldots, n_1$, represent the observations in the first sample of sampling stage i.

Step 3. Compute $Z_{1,i} = (\bar{X}_{1,i} - \mu_0)/(\sigma/\sqrt{n_1})$. If $Z_{1,i}$ is in I_1, sampling stage i is conforming, hence return to Step 2.

Step 4. If $Z_{1,i}$ is in I_3, sampling stage i is non-conforming. Then go to Step 8.

Step 5. If $Z_{1,i}$ is in I_2, take a second sample with n_2 observations from the same population from which the first sample was obtained. Then compute $\bar{X}_{2,i} = \sum_{j=1}^{n_2} X_{2j}/n_2$. Here, X_{2j}, for $j = 1, 2, \ldots, n_2$, denote the observations in the second sample of sampling stage i.

Table 1. Optimal combinations of L, L_1, L_2, L_3, n_1, n_2 for the synthetic DS chart when $ARL_S(0) = 250$ and 370

	$ARL_S(0) = 250$					$ARL_S(0) = 370$				
	δ_{opt}					δ_{opt}				
n	*0.25*	*0.50*	*0.75*	*1.00*	*1.50*	*0.25*	*0.50*	*0.75*	*1.00*	*1.50*
2	1, 4, 3.091 06, 1.145 52, 2.254, 24	1, 4, 3.091 06, 1.145 52, 2.187, 18	1, 4, 3.091 06, 1.145 52, 2.068, 11	1, 4, 3.091 06, 1.145 52, 1.988, 8	1, 3, 3.09 08, 0.963 435, 1.94, 4	1, 4, 3.205 39, 1.147 08, 2.354, 26	1, 4, 3.205 39, 1.147 08, 2.317, 22	1, 4, 3.205 39, 1.147 08, 2.178, 12	1, 4, 3.205 39, 1.147 08, 2.082, 8	1, 3, 3.205 22, 0.964 724, 2.034, 4
3	2, 6, 3.091 56, 1.376 53, 2.204, 24	2, 6, 3.091 56, 1.376 53, 2.102, 16	2, 6, 3.091 56, 1.376 53, 1.95, 9	2, 5, 3.091 31, 1.275 89, 1.935, 6	2, 3, 3.09 08, 0.963 435, 1.968, 3	2, 6, 3.205 72, 1.378 61, 2.38, 35	2, 6, 3.205 72, 1.378 61, 2.224, 18	2, 6, 3.205 72, 1.378 61, 2.078, 10	2, 5, 3.205 55, 1.277 72, 2.036, 6	2, 3, 3.205 22, 0.964 724, 2.061, 3
4	3, 8, 3.092 06, 1.526 09, 2.2, 29	3, 8, 3.092 06, 1.526 09, 2.006, 14	3, 6, 3.091 56, 1.376 53, 1.972, 7	3, 5, 3.091 31, 1.275 89, 1.969, 5	3, 3, 3.09 08, 0.9634 35, 1.918, 2	3, 8, 3.206 05, 1.528 67, 2.337, 35	3, 8, 3.206 05, 1.528 67, 2.126, 15	3, 7, 3.205 88, 1.460 31, 2.031, 8	3, 5, 3.205 55, 1.277 72, 2.069, 5	3, 3, 3.205 22, 0.964 724, 2.013, 2
5	4, 10, 3.092 56, 1.635 31, 2.141, 28	3, 10, 3.091 31, 1.275 89, 2.037, 11	3, 8, 3.091 06, 1.145 52, 1.986, 6	4, 5, 3.091 31, 1.275 89, 1.965, 4	4, 3,3.09 08, 0.9634 35, 1.947, 2	4, 10, 3.206 38, 1.638 37, 2.272, 32	3, 10, 3.205 55, 1.277 72, 2.131, 11	3, 9, 3.205 47, 1.217 09, 2.029, 6	4, 5, 3.205 55, 1.277 72, 2.065, 4	4, 4, 3.205 39, 1.147 08, 1.97, 2
6	4, 12, 3.091 56, 1.376 53, 2.204, 24	4, 12, 3.091 56, 1.376 53, 1.978, 10	4, 9, 3.091 18, 1.215 39, 1.938, 5	4, 6, 3.0908, 0.963 435, 1.968, 3	5, 4, 3.091 06, 1.145 52, 1.9, 2	4, 12, 3.205 72, 1.378 61, 2.312, 26	4, 12, 3.205 72, 1.378 61, 2.078, 10	4, 9, 3.205 47, 1.217 09, 2.038, 5	4, 7, 3.2053, 1.064 59, 2.011, 3	5, 4, 3.205 39, 1.147 08, 1.999, 2
7	5, 14, 3.091 81, 1.457 98, 2.167, 23	4, 14, 3.091 22, 1.236 48, 1.996, 9	5, 9, 3.091 18, 1.215 39, 1.981,5	5, 7, 3.090 93, 1.063 16, 1.948, 3	6, 4, 3.091 06, 1.145 52, 1.923, 2	5, 14, 3.205 88, 1.460 31, 2.288, 26	4, 14, 3.2055, 1.238 22, 2.091, 9	5, 10, 3.205 55, 1.277 72, 2.034, 5	5, 7, 3.2053, 1.064 59, 2.044, 3	6, 4, 3.205 39, 1.147 08, 2.019, 2
8	6, 16, 3.092 06, 1.526 09, 2.128, 22	5, 16, 3.091 39, 1.312 08, 1.941, 8	6, 9, 3.091 18, 1.215 39, 1.956, 4	6, 7, 3.090 93, 1.063 16, 1.974, 3	7,4, 3.091 06, 1.145 52, 1.939, 2	6, 16, 3.206 05, 1.528 67, 2.255, 25	5, 16, 3.205 61, 1.314, 2.041, 8	6, 10, 3.205 55, 1.277 72, 2.012, 4	6, 7, 3.2053, 1.064 59, 2.069, 3	7, 4, 3.205 39, 1.147 08, 2.034, 2
9	6, 18, 3.091 56, 1.376 53, 2.132, 18	6, 16, 3.091 39, 1.312 08, 1.944, 7	6, 11, 3.090 97, 1.092 25, 1.968, 4	7, 7, 3.090 93, 1.063 16, 1.893, 2	7, 5, 3.090 68, 0.838 06, 1.969, 2	6, 18, 3.205 72, 1.378 61, 2.237, 19	6, 18, 3.205 72, 1.378 61, 2.02, 8	6, 11, 3.205 33, 1.093 72, 2.065, 4	7, 7, 3.2053, 1.064 59, 1.991, 2	7, 5, 3.205 14, 0.839 213, 2.057, 2
10	7, 20, 3.091 72, 1.432 54, 2.113, 18	6, 18, 3.091 18, 1.215 39, 1.971, 7	7, 10, 3.090 89, 1.032 16, 1.955, 3	8, 7, 3.090 93, 1.063 16, 1.91, 2	8, 5, 3.090 68, 0.838 06, 1.976, 2	7, 20, 3.205 83, 1.434 79, 2.207, 18	6, 19, 3.205 51, 1.248 43, 2.043, 7	7, 11, 3.205 33, 1.093 72, 2.019, 3	8, 7, 3.2053, 1.064 59, 2.007, 2	8, 6, 3.205 22, 0.964 724, 2.039, 2

Table 2. Optimal parameters for the synthetic DS chart when $n = 5$, $\delta_{opt} = 1$, $ARL_S(0) = 370$ ($n_1 = 41$, $n_2 = 5$, $L = 3.205\ 55$, and $L_1 = 1.277\ 72$)

L_3	L_2	$ARL_S(\delta_{opt})$
1	1.682	1.831 36
2	1.886	1.502 76
3	1.994	1.446 02
4	2.065	**1.441 24**
5	2.122	1.450 79
6	2.165	1.462 74
7	2.202	1.474 86
8	2.232	1.486 34
9	2.259	1.497 04
10	2.283	1.507 02
20	2.433	1.581 31
30	2.517	1.632 01
40	2.575	1.671 54
50	2.619	1.704 25
60	2.655	1.732 45
70	2.685	1.757 32
80	2.711	1.779 65
90	2.734	1.799 98
100	2.754	1.818 63

Step 6. Compute $\bar{X}_i = (n_1\bar{X}_{1,i} + n_2\bar{X}_{2,i})/(/n_1 + n_2)$, where i denotes the sampling stage number.

Step 7. Compute $Z_i = (\bar{X}_i - \mu_0)/(\sigma/\sqrt{n_1 + n_2})$. If Z_i is in I_4, sampling stage i is conforming, hence return to Step 2. Otherwise, sampling stage i is non-conforming and go to Step 8.

Step 8. Count the number of sampling stages between the present (included in the count) and the last (excluded in the count) non-conforming stages a take it as the CRL value.

Step 9. If CRL > L_3, the process is in an in-control state, hence return to Step 2. Otherwise, an out-of-control status is signaled and go to Step 10.

Step 10. Take appropriate actions to find and remove the assignable cause(s). Then return to Step 2.

It is worth noting that unlike in the standard DS $\bar{X}$ chart, if $Z_{1,i} \in I_3$, or ($Z_{1,i} \in I_2$ and $Z_i \notin I_4$), the synthetic DS chart does not signal an out-of-control status immediately at sampling stage i. Instead, the latter only signals an out-of-control status when $CRL \leq L_3$.

7. An illustrative example

An example is given to explain how the synthetic DS chart is constructed using the steps shown in Section 6. For the sake of comparison, the construction of the DS $\bar{X}$ chart is also discussed as the synthetic DS and DS $\bar{X}$ charts are based on the same DS procedure. The synthetic and EWMA charts that do not use the DS procedure are not considered here. For a fair comparison, the same random seed is used to simulate the data for the two charts using Statistical Analysis System (SAS) 9.2. When the process is in-control, the standard normal distribution is considered, so that $\mu_0 = 0$ and $\sigma = 1$ are both known. Initially, observations for ten sampling stages are simulated with the process being in an in-control state, where the mean sample size is $n = 5$. Then observations for the next ten sampling stages are simulated from a $N(0.5, 1)$ distribution; i.e., by considering that the process mean is out of control, where $\delta = 0.5$.

7.1. *Synthetic DS chart*

Using the procedure in Section 6, the synthetic DS chart can be constructed as follows. First, using Table 1, the optimal parameters are determined as (n_1, n_2, L, L_1, L_2, L_3) = (3, 10, 3.205 55, 1.277 72, 2.131, 11) so that $ARL_S(0) = 370$, $n = 5$, and $\delta_{opt} = 0.5$ (see Step 1). Then in Step 2, say, for sampling stage 1, the first sample of size $n_1 = 3$, having observations $X_{11} = -0.6378$, $X_{12} = -0.2916$, and $X_{13} = -0.1103$, is taken (see Table 3). The first sample mean for sampling stage 1 is then computed as $\bar{X}_{1,1} = -0.3465$. Next, in Step 3, $Z_{1,1}$ is computed as $Z_{1,1} = -0.3465/(1/\sqrt{3}) = -0.6002$, where $Z_{1,1} \in I_1 = (-1.277\ 72, 1.277\ 72)$. Thus, the first sampling stage is conforming. Then return to Step 2, where the next first sample is taken for sampling stage 2. After some computation, we obtain $\bar{X}_{1,2} = -0.1447$ and $Z_{1,2} = -0.2507$ (see Table 3). Since $Z_{1,2} \in I_1$, the control flow returns to Step 2, where the following first sample in sampling stage 3 is taken. This process continues until sampling stage 5, where a second sample is taken.

In sampling stage 5, $\bar{X}_{1,5} = 1.0547$ and $= 1.8268 \in I_2 = (1.277\ 72, 3.205\ 55)$ are computed (see Table 3). Therefore, a second sample with $n_2 = 10$ observations is taken (see Step 5), where its mean is computed to be $\bar{X}_{2,5} = 0.1623$. Then in Step 6, $\bar{X}_5 = [3(1.0547) + 10(0.1623)]/(3 + 10) = 0.3682$ is obtained (see Table 3). From Step 7, $Z_5 = 0.3682/(1/\sqrt{3 + 10}) = 1.3276$ is found. Since $Z_5 \in I_4 = (-2.131, 2.131)$, the next first sample is taken for sampling stage 6 as the control flow returns to Step 2.

The process of taking the first and possibly a second sample for each sampling stage goes on until sampling stage 11, where $Z_{11} = 2.4505 \notin I_4$ (see Table 3). Then Step 8 is invoked, where the number of conforming sampling stages between two non-conforming ones (including the present non-conforming stage) is computed as $CRL_1 = 11$. Because CRL_1 is not larger than $L_3 = 11$, the first out-of-control signal occurs at sampling stage 11 (see Step 9). After appropriate actions are taken to find and remove the assignable cause(s), the control flow returns to Step 2, where the same procedure is repeated until the next out-of-control signal is detected. Other out-of-control sampling stages are detected at $i = 15$ and 17 as $CRL_2 = 4 \leq L_3$ and $CRL_3 = 2 \leq L_3$.

Table 3. Data for an illustrative example for the synthetic DS chart

Sampling stage, i	Sample 1 ($n_1 = 3$)			Sample 2 ($n_2 = 10$)										First stage		Second stage	
	X_{11}	X_{12}	X_{13}	X_{21}	X_{22}	X_{23}	X_{24}	X_{25}	X_{26}	X_{27}	X_{28}	X_{29}	$X_{2(10)}$	$\bar{X}_{1,i}$	$Z_{1,i}$	$\bar{X}_i$	Z_i
1	−0.6378	−0.2916	−0.1103											−0.3465	−0.6002		
2	−0.0485	0.0579	−0.4435											−0.1447	−0.2507		
3	2.0026	0.0064	−0.1528											0.6187	1.0717		
4	−0.8532	−0.1128	−0.2568											−0.4076	−0.7060		
5	−0.0445	1.1790	2.0296	−0.0371	1.0802	0.9777	−0.3317	−0.0771	−0.3887	−0.6519	−1.6336	3.2359	−0.5507	1.0547	1.8268	0.3682	1.3276
6	−1.1868	−0.0861	−1.3614	−1.7788	1.1385	−0.1222	0.2263	0.7939	0.5787	−0.1806	−0.5265	0.1689	−1.2629	−0.8781	−1.5209	−0.2769	−0.9982
7	0.4520	−1.2953	−0.2075											−0.3503	−0.6067		
8	0.7265	−0.0796	0.9564											0.5344	0.9257		
9	−0.2969	−0.5581	0.0519											−0.2677	−0.4637		
10	−0.8843	0.7218	0.4424											0.0933	0.1616		
11	0.8220	0.9436	0.8159	−0.2666	−0.4674	0.1327	−0.6992	0.8178	2.8078	0.4081	2.2298	0.7915	0.4993	0.8605	1.4905	0.6796	2.4505
12	0.4792	0.6192	1.9690	−0.2739	−0.9705	−0.6283	−0.5023	0.2724	1.1036	1.0335	1.8870	1.7521	−0.5838	1.0225	1.7709	0.4736	1.7077
13	0.9380	0.5357	1.3754	0.8233	−0.6218	−1.5725	0.7629	2.0284	0.4223	−0.7145	−0.0240	1.8929	0.9613	0.9497	1.6449	0.5236	1.8880
14	−1.2246	0.0019	−1.1536	1.1920	0.0993	0.1663	1.4243	1.4709	1.3218	0.7223	0.5801	0.8475	−0.1065	−0.7921	−1.3720	0.4109	1.4815
15	1.3847	0.9974	0.8157	0.1980	0.1955	−1.0122	0.6331	−0.8213	−0.1390	0.9897	1.6450	2.3262	2.2233	1.0659	1.8462	0.7259	2.6171
16	−0.4327	−0.2284	2.1275											0.4888	0.8466		
17	0.9834	1.5051	0.3958	1.0865	2.3982	2.1720	0.4764	−0.1938	0.6774	0.4277	0.3553	0.5307	1.1173	0.9614	1.6652	0.9178	3.3093
18	1.4988	0.1211	0.5504											0.7234	1.2530		
19	−0.7767	0.5087	0.8652											0.1991	0.3448		
20	−0.0669	−0.0690	0.6064											0.1568	0.2717		

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Table 4. Data for an illustrative example for the DS $\bar{X}$ chart

Sampling stage, i	*Sample 1 ($n_1 = 4$)*				*Sample 2 ($n_2 = 10$)*										*First stage*		*Second stage*	
	X_{11}	X_{12}	X_{13}	X_{14}	X_{21}	X_{22}	X_{23}	X_{24}	X_{25}	X_{26}	X_{27}	X_{28}	X_{29}	$X_{2(10)}$	$\bar{X}_{1,i}$	$Z_{1,i}$	$\bar{X}_i$	Z_i
1	−0.6378	−0.2916	−0.1103	−0.0485											−0.2720	−0.5441		
2	0.0579	−0.4435	2.0026	0.0064											0.4059	0.8117		
3	−0.1528	−0.8532	−0.1128	−0.2568											−0.3439	−0.6878		
4	−0.0445	1.1790	2.0296	−0.0371											0.7818	1.5635		
5	1.0802	0.9777	−0.3317	−0.0771											0.4123	0.8246		
6	−0.3887	−0.6519	−1.6336	3.2359											0.1404	0.2809		
7	−0.5507	−1.1868	−0.0861	−1.3614											−0.7963	−1.5925		
8	−1.7788	1.1385	−0.1222	0.2263											−0.1340	−0.2681		
9	0.7939	0.5787	−0.1806	−0.5265											0.1664	0.3327		
10	0.1689	−1.2629	0.4520	−1.2953											−0.4843	−0.9687		
11	0.2925	1.2265	0.4204	1.4564	0.2031	−0.0581	0.5519	−0.3843	1.2218	0.9424	0.8220	0.9436	0.8159	−0.2666	0.8490	1.6979	0.5848	2.1882
12	−0.4674	0.1327	−0.6992	0.8178											−0.0540	−0.1081		
13	2.8078	0.4081	2.2298	0.7915	0.4993	0.4792	0.6192	1.9690	−0.2739	−0.9705	−0.6283	−0.5023	0.2724	1.1036	1.5593	3.1186	0.6289	2.3532
14	1.0335	1.8870	1.7521	−0.5838	0.9380	0.5357	1.3754	0.8233	−0.6218	−1.5725	0.7629	2.0284	0.4223	−0.7145	1.0222	2.0444	0.5761	2.1557
15	−0.0240	1.8929	0.9613	−1.2246											0.4014	0.8028		
16	0.0019	−1.1536	1.1920	0.0993											0.0349	0.0698		
17	0.1663	1.4243	1.4709	1.3218	0.7223	0.5801	0.8475	−0.1065	1.3847	0.9974	0.8157	0.1980	0.1955	−1.0122	1.0958	2.1916	0.6433	2.4069
18	0.6331	−0.8213	−0.1390	0.9897											0.1656	0.3312		
19	1.6450	2.3262	2.2233	−0.4327	−0.2284	2.1275	0.9834	1.5051	0.3958	1.0865	2.3982	2.1720	0.4764	−0.1938	1.4405	2.8809	1.1775	4.4057
20	0.6774	0.4277	0.3553	0.5307											0.4978	0.9955		

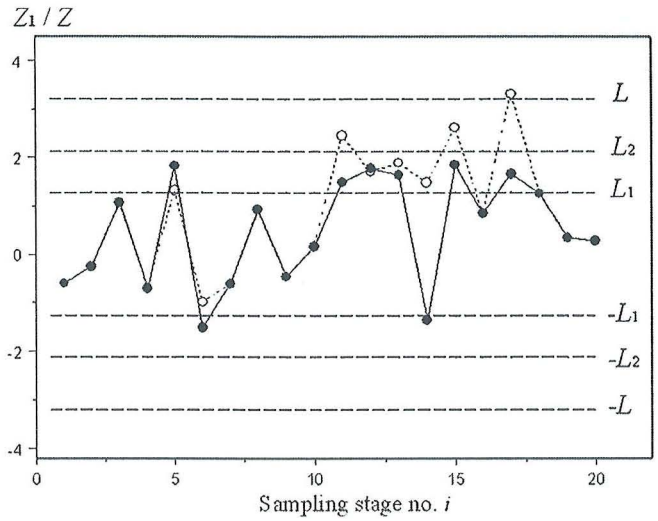

Fig. 5. DS/S sub-chart.

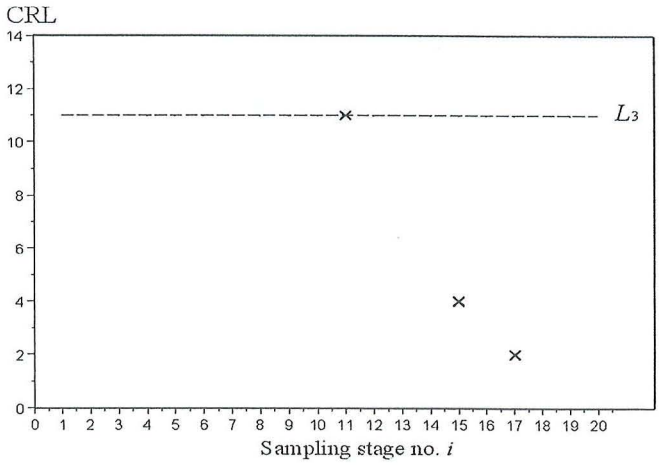

Fig. 7. CRL/S sub-chart.

Figure 5 shows the DS/S sub-chart and Fig. 6 shows the non-conforming sampling stages at $i = 11$, 15, and 17, and their respective CRL values. Note that in Fig. 5, the solid dots represent $Z_{1,i}$ while the hollow dots denote Z_i. Figure 7 gives the CRL/S sub-chart that identifies the out-of-control sampling stages at $i = 11$, 15, and 17 based on the CRL values in Fig. 6.

7.2. *DS $\bar{X}$ chart*

Since $ARL_S(0) = 370$, $n = 5$, and $\delta_{opt} = 0.5$ are desired, the optimal parameters for the DS $\bar{X}$ chart can be found using the optimization program 7 that incorporates the method given in Irianto and Shinozaki (1998) to be $n_1 = 4$, $n_2 = 10$, $L = 3.206\,38$, $L_1 = 1.638\,37$, and $L_2 = 3.003$. The charting procedure described in Section 3.2 is used to construct the DS $\bar{X}$ chart. Table 4 presents the data of the 20 sampling stages and their corresponding $\bar{X}_{1,i}$, $\bar{X}_i$, $Z_{1,i}$, and Z_i values, computed for the DS $\bar{X}$ chart. The DS $\bar{X}$ chart is plotted in Fig. 8. Only one out-of-control signal is detected; i.e., at sampling stage, $i = 19$, as $Z_{19} = 4.4057 > L_2 = 3.003$. In Fig. 8, the solid dots represent $Z_{1,i}$ and the hollow dots denote Z_i.

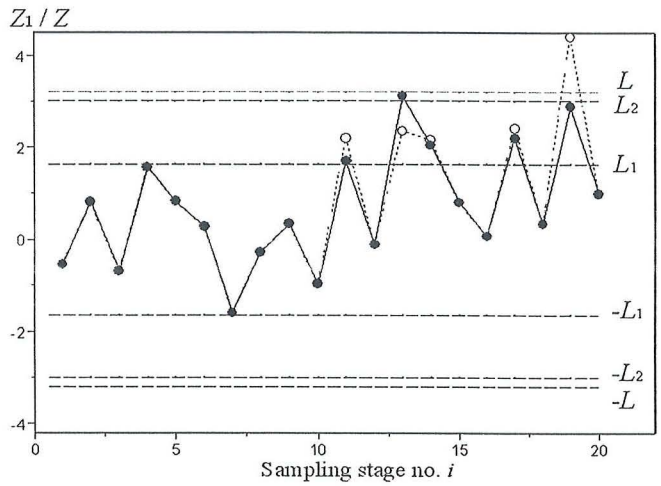

Fig. 8. DS $\bar{X}$ chart.

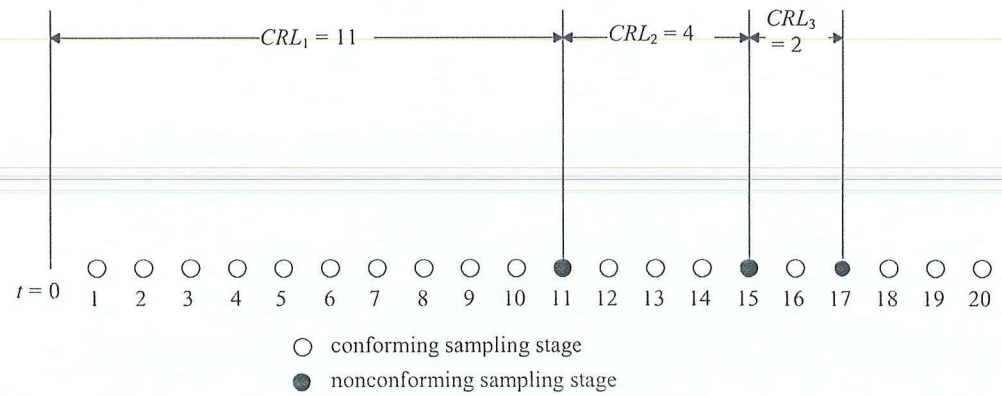

Fig. 6. CRL values for the sampling stages.

Table 5. Optimal parameters for the EWMA, synthetic, DS $\bar{X}$, and synthetic DS charts when $n = 5$ or 10 and $ARL(0) = 370$

	EWMA δ_{opt}		*Synthetic* δ_{opt}		*DS* $\bar{X}$ δ_{opt}		*Synthetic DS* δ_{opt}	
n	*0.5*	*1.0*	*0.5*	*1.0*	*0.5*	*1.0*	*0.5*	*1.0*
5	$\lambda = 0.08$ $m = 2.65$	$\lambda = 0.33$ $m = 2.94$	$L = 15$ $k = 2.454\,87$	$L = 4$ $k = 2.218\,55$	$n_1 = 4$ $n_2 = 10$ $L = 3.206\,38$ $L_1 = 1.638\,37$ $L_2 = 3.003$	$n_1 = 4$ $n_2 = 10$ $L = 3.206\,38$ $L_1 = 1.638\,37$ $L_2 = 3.003$	$n_1 = 3$ $n_2 = 10$ $L = 3.205\,55$ $L_1 = 1.277\,72$ $L_2 = 2.131$ $L_3 = 11$	$n_1 = 4$ $n_2 = 5$ $L = 3.205\,55$ $L_1 = 1.277\,72$ $L_2 = 2.065$ $L_3 = 4$
10	$\lambda = 0.37$ $m = 2.95$	$\lambda = 0.5$ $m = 2.98$	$L = 8$ $k = 2.345\,56$	$L = 2$ $k = 2.084\,59$	$n_1 = 8$ $n_2 = 20$ $L = 3.206\,38$ $L_1 = 1.638\,37$ $L_2 = 3.003$	$n_1 = 8$ $n_2 = 16$ $L = 3.206\,05$ $L_1 = 1.528\,67$ $L_2 = 3.064$	$n_1 = 6$ $n_2 = 19$ $L = 3.205\,51$ $L_1 = 1.248\,43$ $L_2 = 2.043$ $L_3 = 7$	$n_1 = 8$ $n_2 = 7$ $L = 3.2053$ $L_1 = 1.064\,59$ $L_2 = 2.007$ $L_3 = 2$

7.3. *Comparison of results*

By comparing the synthetic DS and DS $\bar{X}$ charts, we found that the synthetic DS chart detects the mean shift quicker than the DS $\bar{X}$ chart. The synthetic DS chart issues an out-of-control signal at sampling stage 11; i.e., at the time when the process mean starts to shift. On the contrary, the DS $\bar{X}$ chart only signals an out-of-control situation at sampling stage 19; i.e., after a lapse of nine sampling stages, from the time the process starts to shift. In terms of the number of observations sampled, the synthetic DS chart requires 63 observations (from $i = 1$, see Table 3), whereas the DS $\bar{X}$ chart needs 126 observations (from $i = 1$, see Table 4) to signal the first out-of-control situation. It is obvious that the former requires only half the number of observations needed by the latter to detect the mean shift.

8. A study on the performance of the synthetic DS chart

The synthetic DS chart is now compared with the DS $\bar{X}$, synthetic, and EWMA charts. Two optimal shifts, i.e., $\delta_{opt} = 0.5$ and 1, are considered. The sample sizes for the EWMA and synthetic charts are set as $n = 5$ and 10. Similarly, the mean sample sizes in each sampling stage of the DS $\bar{X}$ and synthetic DS charts for the in-control process are fixed as $n = 5$ and 10. The in-control ARL considered is $ARL(0) = 370$. The charts are compared based on their zero-state ARL and ANOS performances, where the magnitude of a shift in the mean, $\delta \in \{0.2, 0.4, 0.5, 0.6, 0.8, 1, 1.25, 1.5, 1.75, 2, 2.5, 3, 4, 5\}$ is considered. For the EWMA chart, the charting statistic plotted is

$$W_i = \lambda \bar{X}_i + (1 - \lambda) W_{i-1} \quad \text{for} \quad i = 1, 2, \ldots, \tag{11}$$

where λ is the smoothing constant. The upper/lower limits of the EWMA chart are computed as

$$UCL/LCL = \mu_0 \pm \frac{m\sigma}{\sqrt{n}} \sqrt{\frac{\lambda}{2 - \lambda}}. \tag{12}$$

The software given by Aparisi and Garcia-Diaz (2004), based on a genetic algorithm, is used to determine the optimal parameters λ and m corresponding to $\delta_{opt} = 0.5$ and 1.

The optimization program used in Section 7 is used to determine the optimal parameters of the DS $\bar{X}$ chart using the method given in Irianto and Shinozaki (1998), where for a predefined mean sample size for the in-control process, the optimal parameters give the lowest ARL for a mean shift of size δ_{opt}. The optimal parameters of the synthetic DS chart are obtained from Table 1. The computation of the optimal parameters for the synthetic chart is made using a program, written in Mathematica 7 that incorporates the method given by Wu and Spedding (2000). Table 5 shows the optimal parameters of all the charts.

The ARL and ANOS values of the EWMA chart are computed using SAS 9.2 while that for the synthetic, DS $\bar{X}$, and synthetic DS charts use Mathematica 7.

9. Results and Discussion

Tables 6, 7, 8, and 9 show the ARL and ANOS values of the charts for $(n, \delta_{opt}) = (5, 0.5)$, $(5, 1)$, $(10, 0.5)$, and $(10, 1)$, respectively. From these tables, it is obvious that the synthetic DS chart outperforms the synthetic chart for small and large shifts in the mean ($(\delta leq$ 0.8 and $\delta \geq 2.5$ for $n = 5$) and ($\delta \leq 0.6$ and $\delta \geq 1.75$ for $n = 10$)) and the DS $\bar{X}$ chart for all sizes of shifts where $n = 5$ or 10, except when $\delta_{opt} = 1$ and δ is very large. When $\delta_{opt} = 1$

Table 6. ARL and ANOS values of the various charts for $n = 5$ and $\delta_{opt} = 0.5$

	EWMA		*Synthetic*		*DS* $\bar{X}$		*Synthetic DS*	
δ	*ARL*	*ANOS*	*ARL*	*ANOS*	*ARL*	*ANOS*	*ARL*	*ANOS*
0	370	1850	370	1850	370	1850	370	1850
0.2	32.56	162.81	134.44	672.22	111.50	586.37	67.32	353.91
0.4	11.61	58.06	27.76	138.81	23.16	139.02	10.07	60.19
0.5	8.68	43.39	14.48	72.42	12.02	78.39	5.48	35.45
0.6	6.96	34.80	8.53	42.65	6.88	48.86	3.57	25.00
0.8	5.00	25.00	4.01	20.04	2.98	24.74	2.08	16.89
1	3.95	19.76	2.42	12.10	1.77	16.44	1.53	13.84
1.25	3.18	15.88	1.58	7.90	1.27	12.25	1.23	11.90
1.5	2.66	13.31	1.23	6.13	1.10	9.82	1.10	10.31
1.75	2.30	11.48	1.08	5.39	1.03	7.78	1.04	8.64
2	2.08	10.40	1.02	5.11	1.01	6.10	1.01	6.94
2.5	1.88	9.41	1.00	5.00	1.00	4.36	1.00	4.30
3	1.52	7.61	1.00	5.00	1.00	4.03	1.00	3.23
4	1.02	5.08	1.00	5.00	1.00	4.00	1.00	3.00
5	1.00	5.00	1.00	5.00	1.00	4.00	1.00	3.00

and δ is very large, say $\delta \geq 3$, the synthetic DS and DS $\bar{X}$ charts have comparable performances. In terms of ANOS, the synthetic DS chart outperforms the synthetic and DS $\bar{X}$ charts by close to or more than two times when $0.4 \leq \delta \leq 0.6$ and $n = 5$. The synthetic DS chart also performs better than the EWMA chart for $\delta \geq 0.5$ and $\delta \geq 0.6$ when $\delta_{opt} = 0.5$ and 1, respectively, where $n = 5$, in terms of the ARL and ANOS performances. For $n = 10$, the synthetic DS chart has superior ARL and ANOS performances to the EWMA chart when $\delta \geq 0.4$ and $\delta_{opt} = 1$. However, for the same n and $\delta_{opt} = 0.5$, the former outperforms the latter when $0.4 \leq \delta \leq 0.6$ and $\delta \geq 1.75$. This is an improvement over the synthetic chart, which outperforms the EWMA chart only when $\delta \geq 0.8$ for $n = 5$ and $\delta \geq 0.6$ for $n = 10$.

Note that the average number of observations to signal an out-of-control situation, i.e., ANOS values for the EWMA and synthetic charts, is computed as $ANOS = ARL(\delta) \times n$, where $n = 5$ or 10, since these charts use a fixed sample size, n, in each sampling stage. Here, $ARL(\delta)$ represents the ARL of a chart corresponding to a shift of size δ. However, for the synthetic DS chart, whose sample size in each sampling stage is either n_1 or $n_1 + n_2$, its ANOS value is calculated using Equation (6), where $ARL_S(\delta)$ denotes the ARL of the chart corresponding to a shift of size δ. Similarly, the ANOS value of the DS $\bar{X}$ chart is also computed using Equation (6) but with $ARL_S(\delta)$ being replaced by $ARL(\delta)$, where $ARL(\delta)$ is the ARL of the DS $\bar{X}$ chart for a shift of size δ.

Table 7. ARL and ANOS values of the various charts for $n = 5$ and $\delta_{opt} = 1.0$

	EWMA		*Synthetic*		*DS* $\bar{X}$		*Synthetic DS*	
δ	*ARL*	*ANOS*	*ARL*	*ANOS*	*ARL*	*ANOS*	*ARL*	*ANOS*
0	370	1850	370	1850	370	1850	370	1850
0.2	61.85	309.27	150.33	751.63	111.50	586.37	98.37	508.59
0.4	14.38	71.88	32.83	164.16	23.16	139.02	15.77	88.88
0.5	9.08	45.40	16.61	83.06	12.02	78.39	7.75	46.02
0.6	6.43	32.15	9.20	46.02	6.88	48.86	4.42	27.71
0.8	3.98	19.92	3.73	18.66	2.98	24.74	2.12	14.54
1	2.92	14.59	2.10	10.48	1.77	16.44	1.44	10.46
1.25	2.22	11.08	1.40	7.01	1.27	12.25	1.15	8.33
1.5	1.82	9.11	1.15	5.74	1.10	9.82	1.05	7.02
1.75	1.54	7.68	1.05	5.24	1.03	7.78	1.01	5.94
2	1.32	6.59	1.01	5.06	1.01	6.10	1.00	5.07
2.5	1.05	5.26	1.00	5.00	1.00	4.36	1.00	4.18
3	1.00	5.02	1.00	5.00	1.00	4.03	1.00	4.00
4	1.00	5.00	1.00	5.00	1.00	4.00	1.00	4.00
5	1.00	5.00	1.00	5.00	1.00	4.00	1.00	4.00

Table 8. ARL and ANOS values of the various charts for $n = 10$ and $\delta_{opt} = 0.5$

	EWMA		*Synthetic*		*DS* $\bar{X}$		*Synthetic DS*	
δ	*ARL*	*ANOS*	*ARL*	*ANOS*	*ARL*	*ANOS*	*ARL*	*ANOS*
0	370	3700	370	3700	370	3700	370	3700
0.2	33.30	332.97	72.72	727.20	56.19	619.55	30.64	336.16
0.4	7.34	73.37	10.18	101.80	8.24	113.69	4.15	55.93
0.5	4.87	48.67	5.19	51.90	4.20	65.20	2.53	37.93
0.6	3.61	36.11	3.19	31.90	2.56	43.88	1.86	30.55
0.8	2.44	24.36	1.75	17.50	1.44	27.66	1.33	24.48
1	1.88	18.78	1.26	12.60	1.14	21.17	1.13	20.96
1.25	1.46	14.56	1.06	10.60	1.03	15.29	1.04	16.50
1.5	1.18	11.80	1.01	10.10	1.00	10.96	1.01	12.02
1.75	1.04	10.39	1.00	10.00	1.00	8.81	1.00	8.64
2	1.01	10.06	1.00	10.00	1.00	8.14	1.00	6.86
2.5	1.00	10.00	1.00	10.00	1.00	8.00	1.00	6.03
3	1.00	10.00	1.00	10.00	1.00	8.00	1.00	6.00
4	1.00	10.00	1.00	10.00	1.00	8.00	1.00	6.00
5	1.00	10.00	1.00	10.00	1.00	8.00	1.00	6.00

10. Conclusions

This article proposed a synthetic DS chart, which combines the DS $\bar{X}$ chart of Daudin (1992) and the CRL chart of Bourke (1991). The optimal design, construction, and performance evaluation of the new chart were discussed in detail in this article. On the whole, the synthetic DS chart has been shown to outperform both of its standard counterparts, namely, the synthetic chart of Wu and Spedding (2000) and the DS $\bar{X}$ chart of Daudin (1992). The proposed chart has also been shown to have better zero-state ARL and ANOS performances than the EWMA chart for detecting a shift in the mean of about 0.5σ onwards. Since this size of shift is usually the main concern in process monitoring, the use of the proposed method is pertinent. Although the implementation of the synthetic DS chart is more complicated than using either the synthetic or the DS $\bar{X}$ chart as the synthetic DS chart involves six parameters, the optimization programs enabled the optimal parameters of the proposed chart to be determined easily by the end user. In addition, for ease of implementation, Table 1 provided the optimal parameters of the synthetic DS chart for selected combinations of $ARL_S(0)$, δ_{opt}, and n. The use of a simple spreadsheet will help one in keeping track of the decisions and computations from one sample to another. Furthermore, improvements in terms of the ARL and ANOS performances justify the use of the proposed chart.

Table 9. ARLs and ANOS values of the various charts for $n = 10$ and $\delta_{opt} = 1.0$

	EWMA		*Synthetic*		*DS* $\bar{X}$		*Synthetic DS*	
δ	*ARL*	*ANOS*	*ARL*	*ANOS*	*ARL*	*ANOS*	*ARL*	*ANOS*
0	370	3700	370	3700	370	3700	370	3700
0.2	44.15	441.54	88.78	887.80	62.62	682.95	58.72	616.14
0.4	8.74	87.41	13.06	130.60	9.48	125.93	7.29	84.92
0.5	5.35	53.47	6.25	62.50	4.74	69.79	3.60	44.04
0.6	3.77	37.72	3.50	35.00	2.79	44.79	2.18	27.71
0.8	2.34	23.43	1.67	16.70	1.47	25.74	1.27	16.55
1	1.73	17.34	1.19	11.90	1.13	18.94	1.06	13.03
1.25	1.32	13.21	1.03	10.30	1.02	13.90	1.01	10.63
1.5	1.10	11.03	1.00	10.00	1.00	10.38	1.00	9.05
1.75	1.02	10.19	1.00	10.00	1.00	8.65	1.00	8.28
2	1.00	10.00	1.00	10.00	1.00	8.11	1.00	8.05
2.5	1.00	10.00	1.00	10.00	1.00	8.00	1.00	8.00
3	1.00	10.00	1.00	10.00	1.00	8.00	1.00	8.00
4	1.00	10.00	1.00	10.00	1.00	8.00	1.00	8.00
5	1.00	10.00	1.00	10.00	1.00	8.00	1.00	8.00

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Appendix

Derivation of the $ARL_S(\delta)$ formula in Equation (5) for the synthetic DS chart.

Under the zero-state mode, a process shift is assumed to occur from the beginning or immediately after a non-conforming sample. The synthetic chart's zero-state ARL for a given mean shift, δ, is calculated as (Wu and Spedding, 2000):

$$ARL(\delta) = \frac{1}{r(\delta) \times R(\delta)}, \tag{A1}$$

where $r(\delta)$ is the probability of observing a non-conforming sample and $R(\delta)$ is the probability that the CRL is smaller than or equal to the lower limit, L', of the CRL chart. Let $s(\delta)$ denote the complementary of $r(\delta)$; i.e., $s(\delta) = 1 - r(\delta)$. Since it is usually assumed that the CRL follows a geometric distribution, then

$$\begin{aligned} R(\delta) &= \Pr(CRL \le L') \\ &= \Pr(CRL=1)+\Pr(CRL=2)+\ldots+\Pr(CRL=L') \\ &= r(\delta) + s(\delta)r(\delta) + \ldots + [s(\delta)]^{L'-1} + r(\delta) \\ &= r(\delta)\{1 + s(\delta) + \ldots + [s(\delta)]^{L'-1}\} \\ &= r(\delta)\left\{\frac{1-[s(\delta)]^{L'}}{1-s(\delta)}\right\} \\ &= 1 - [s(\delta)]^{L'} \end{aligned} \tag{A2}$$

It should be noted that $R(\delta)$ is also the probability that the synthetic chart produces an out-of-control signal. Combining Equations (A1) and (A2) leads to

$$ARL(\delta) = \frac{1}{r(\delta)\{1-[s(\delta)]^{L'}\}} = \frac{1}{r(\delta)\{1-[1-r(\delta)]^{L'}\}}. \tag{A3}$$

It follows that to compute $ARL_S(\delta)$ for a synthetic DS chart, $r(\delta)$ is replaced by $P(\delta)$ and L' by L_3 in Equation (A3). Note that $P(\delta)$ is the probability of obtaining a nonconforming sampling stage while L_3 is the lower limit of the CRL/S sub-chart.

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An EWMA Control Chart for Monitoring the Mean of Skewed Populations Using Weighted Variance

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Abstract – **This paper discusses the use of weighted variance (WV) in setting up the limits of the exponentially weighted moving average (EWMA) chart for the monitoring of the mean of a process from a skewed population. This chart, called the WV–EWMA chart hereafter, reduces to the standard EWMA chart when the underlying distribution is symmetric. The Type-I and Type-II errors of the WV–EWMA chart are compared with that of the existing charts for skewed populations. Simulation results show that the new method gives a considerable improvement over the existing methods when the underlying distribution is skewed.**

Keywords – **EWMA chart, $\bar{X}$ chart, skewed populations, Type-I error, Type-II error, weighted variance, weighted standard deviation, skewness correction**

I. INTRODUCTION

A control chart is used to monitor a process. Standard variable charts assume that the quality characteristic is normally distributed. In many situations, this assumption does not hold. For example, chemical, semiconductor and cutting tool wear processes are often skewed [1]. For a skewed population, the Type–I error of standard charts increases with the skewness. Three different approaches that are currently used to deal with skewed populations are transformation, increasing the sample size and heuristic control charts. The heuristic charts that are available include the $\bar{X}$ and R charts based on the weighted variance (WV) [2], weighted standard deviation (WSD) [1] and skewness correction (SC) [3] methods as well as the EWMA chart based on the WSD method [1]. Other works on skewed univariate charts are reported in [4] – [10]. This paper considers the use of weighted variance (WV) to compute the limits of an EWMA chart. This method is found to perform well when the distribution is skewed.

II. LITERATURE REVIEW OF HEURISTIC CHARTS FOR MEAN

A. Weighted Variance (WV) $\bar{X}$ Chart

The WV method is based on the idea that a skewed distribution can be splitted into two segments at its average and each segment is used for creating a new symmetric distribution. The WV method uses both created symmetric distributions in setting up the chart's limits. The limits of the WV $\bar{X}$ chart are [2]

$$UCL_{\mathrm{WV}} = \mu + 3\frac{\sigma}{\sqrt{n}}\sqrt{2P_X} \tag{1a}$$

and

$$LCL_{\mathrm{WV}} = \mu - 3\frac{\sigma}{\sqrt{n}}\sqrt{2(1-P_X)}, \tag{1b}$$

where P_X is the probability that a random variable X will be less than or equal to its mean μ. Note that the WV $\bar{X}$ chart reduces to the standard $\bar{X}$ chart when $P_X = 0.5$. Here, σ denotes the standard deviation of X while n represents the sample size. The notations defined here will also be used in the later sections. When parameters are unknown, the control limits are computed using the following formulae [2]:

$$UCL_{\mathrm{WV}} = \bar{\bar{X}} + \frac{3\bar{R}}{d_2'\sqrt{n}}\sqrt{2\hat{P}_X} \tag{2a}$$

and

$$LCL_{\mathrm{WV}} = \bar{\bar{X}} - \frac{3\bar{R}}{d_2'\sqrt{n}}\sqrt{2\left(1-\hat{P}_X\right)}, \tag{2b}$$

where

$$\hat{P}_X = \frac{\sum_{i=1}^{k}\sum_{j=1}^{n}\delta\left(\bar{\bar{X}} - X_{ij}\right)}{n\times k} \tag{3}$$

with k and n denoting the number of samples and the number of observations in a sample, respectively, and $\delta(x) = 1$ for $x \geq 0$ or $\delta(x) = 0$ for $x < 0$. In Equations (2a) and (2b), $\bar{\bar{X}}$ denotes the grand mean, $\bar{R}$ the mean of the sample ranges and d_2' the constant for a given skewed population corresponding to the constant d_2 for a normal distribution. The constant d_2' can be computed via numerical integration for a given skewed distribution [2].

B. Weighted Standard Deviation (WSD) $\bar{X}$ Chart

The WSD method uses the same approach as the WV method, i.e., by splitting a skewed distribution into two segments. The limits of the WSD $\bar{X}$ chart are [1]

$$UCL_{\mathrm{WSD}} = \mu + 3\frac{\sigma}{\sqrt{n}}2P_X \tag{4a}$$

and

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$$LCL_{\text{WSD}} = \mu - 3\frac{\sigma}{\sqrt{n}}2\left(1-P_X\right). \quad (4b)$$

When parameters are unknown, the control limits are computed using the following formulae [1]:

$$UCL_{\text{WSD}} = \bar{\bar{X}} + 3\frac{\bar{R}}{d_2^{\text{WSD}}\sqrt{n}}\left(2\hat{P}_X\right) \quad (5a)$$

and

$$LCL_{\text{WSD}} = \bar{\bar{X}} - 3\frac{\bar{R}}{d_2^{\text{WSD}}\sqrt{n}}\left[2\left(1-\hat{P}_X\right)\right]. \quad (5b)$$

Here,

$$d_2^{\text{WSD}} \equiv P_X d_2\left(2n\left(1-P_X\right)\right) + \left(1-P_X\right)d_2\left(2nP_X\right), \quad (6)$$

where $d_2\left(n\right)$ is d_2 for the normal distribution when the sample size is n. If P_X is unknown, we can use $\hat{P}_X$ in Equation (3) to compute d_2^{WSD}. Note that $d_2^{WSD} = d_2$ if the underlying distribution is symmetric [1].

C. Skewness Correction (SC) $\bar{X}$ Chart

Reference [3] proposes the SC method based on the Cornish–Fisher expansion. The SC $\bar{X}$ chart is based on the following limits when parameters are known [3]:

$$UCL_{\text{SC}} = \mu + \left(3 + c_4^*\right)\frac{\sigma}{\sqrt{n}} \quad (7a)$$

and

$$LCL_{\text{SC}} = \mu + \left(-3 + c_4^*\right)\frac{\sigma}{\sqrt{n}}. \quad (7b)$$

Here,

$$c_4^* = \frac{\frac{4}{3}\kappa_3\left(\bar{X}\right)}{1 + 0.2\kappa_3^2\left(\bar{X}\right)}, \quad (8)$$

where $\kappa_3\left(\bar{X}\right)$ is the skewness coefficient of the sample mean $\bar{X}$. Note that $\kappa_3\left(\bar{X}\right) = \frac{\kappa_3}{\sqrt{n}}$, where κ_3 is the skewness coefficient of X. When parameters are unknown, the control limits are computed using the following formulae [3]:

$$UCL_{\text{SC}} = \bar{\bar{X}} + \left(3 + \frac{4\kappa_3/\left(3\sqrt{n}\right)}{1 + 0.2\,\kappa_3^2/n}\right)\frac{\bar{R}}{d_2'\sqrt{n}} \quad (9a)$$

and

$$LCL_{\text{SC}} = \bar{\bar{X}} + \left(-3 + \frac{4\kappa_3/\left(3\sqrt{n}\right)}{1 + 0.2\,\kappa_3^2/n}\right)\frac{\bar{R}}{d_2'\sqrt{n}} \quad (9b)$$

D. Weighted Standard Deviation (WSD) EWMA chart

The WSD–EWMA chart statistics are [1]

$$E_i = \lambda\bar{X}_i + \left(1-\lambda\right)E_{i-1}, \text{ for } i = 1, 2,\ldots, \quad (10)$$

where $0 < \lambda \le 1$ is a smoothing constant and $E_0 = \mu$. The limits of the WSD–EWMA chart are [1]

$$UCL_{\text{WSD-EWMA}} = \mu + L\frac{\sigma}{\sqrt{n}}\sqrt{\left(\frac{\lambda}{2-\lambda}\right)}\left(2P_X\right) \quad (11a)$$

and

$$LCL_{\text{WSD-EWMA}} = \mu - L\frac{\sigma}{\sqrt{n}}\sqrt{\left(\frac{\lambda}{2-\lambda}\right)}\left[2\left(1-P_X\right)\right]. \quad (11b)$$

The selection of λ and L is based on the approach discussed in [11]. The WSD–EWMA chart reduces to the standard EWMA chart when $P_X = 0.5$. When parameters are unknown, the control limits are computed using the following formulae [1]:

$$UCL_{\text{WSD-EWMA}} = \bar{\bar{X}} + L\frac{\bar{R}}{d_2^{\text{WSD}}\sqrt{n}}\sqrt{\left(\frac{\lambda}{2-\lambda}\right)}\left(2\hat{P}_X\right) \quad (12a)$$

and

$$LCL_{\text{WSD-EWMA}} = \bar{\bar{X}} - L\frac{\bar{R}}{d_2^{\text{WSD}}\sqrt{n}}\sqrt{\left(\frac{\lambda}{2-\lambda}\right)}\left[2\left(1-\hat{P}_X\right)\right]. \quad (12b)$$

III. A PROPOSED WEIGHTED VARIANCE (WV) EWMA CHART

Let $\bar{X}_i$, for $i = 1, 2,\ldots$, denote a sequence of sample means for a process. The successive EWMA statistics can be described by Equation (10) and the limits of the WV–EWMA chart can be computed as follows when parameters are known:

$$UCL_{\text{WV-EWMA}} = \mu + L\frac{\sigma}{\sqrt{n}}\sqrt{\left(\frac{\lambda}{2-\lambda}\right)}\sqrt{2P_X} \quad (13a)$$

and

$$LCL_{\text{WV-EWMA}} = \mu - L\frac{\sigma}{\sqrt{n}}\sqrt{\left(\frac{\lambda}{2-\lambda}\right)}\sqrt{\left[2\left(1-P_X\right)\right]}. \quad (13b)$$

The WV method decomposes the variance into two parts while the WSD method decomposes the standard deviation into two parts. In the WV method, $\left(2P_X\right)\sigma^2$ and $\left[2\left(1-P_X\right)\right]\sigma^2$ are used in place of σ^2 for computing $UCL_{\text{WV-EWMA}}$ and $LCL_{\text{WV-EWMA}}$, respectively. On the contrary, in the WSD method, $\left(2P_X\right)\sigma$ and $\left[2\left(1-P_X\right)\right]\sigma$ are used in place of σ for computing $UCL_{\text{WSD-EWMA}}$ and $LCL_{\text{WSD-EWMA}}$, respectively. When process parameters are unknown, the limits are computed as follows:

$$UCL_{\text{WV-EWMA}} = \bar{\bar{X}} + L\frac{\bar{R}}{d_2'\sqrt{n}}\sqrt{\left(\frac{\lambda}{2-\lambda}\right)}\sqrt{\left(2\hat{P}_X\right)} \quad (14a)$$

and

$$LCL_{\text{WV-EWMA}} = \bar{\bar{X}} - L\frac{\bar{R}}{d_2'\sqrt{n}}\sqrt{\left(\frac{\lambda}{2-\lambda}\right)}\sqrt{\left[2\left(1-\hat{P}_X\right)\right]} \quad (14b)$$

IV. PERFORMANCE OF THE WV–EWMA CHART

The WV–EWMA chart is compared with the existing heuristic charts for skewed data, the standard $\bar{X}$ chart and the standard EWMA chart, in terms of the Type-I and Type-II error rates, computed via a Monte Carlo simulation using SAS. The Type-I error rate is defined as the probability of signaling an out-of-control even though the process is actually in-control. On the contrary, the Type-II error rate is defined as the probability of failing to signal an out-of-control although the process has shifted. The existing heuristic charts considered are the WV $\bar{X}$, WSD $\bar{X}$, SC $\bar{X}$ and WSD–EWMA charts. In this paper, the WSD–CUSUM chart suggested in [1] is not considered in the performance comparison, because both the WSD–CUSUM and the WSD–EWMA charts have almost the same performance [1]. Hence, we only consider the WSD–EWMA chart.

The skewed distributions considered are Weibull and gamma because they represent a wide variety of shapes from symmetric to highly skewed. For the sake of comparison, the standard normal distribution is also considered. For convenience, a scale parameter of one is used for the Weibull and gamma distributions. Note that P_X for the Weibull [2] and gamma [12] distributions are

$$P_X = 1 - \exp\left[-\left\{\Gamma\left(1+\frac{1}{\beta}\right)\right\}^{\beta}\right] \quad (15)$$

and

$$P_X = F(\eta), \quad (16)$$

respectively, where β and η are both shape parameters. Here, $\Gamma(\cdot)$ is the gamma function and $F(\cdot)$ is the gamma distribution function.

The skewness coefficients considered for the Weibull and gamma distributions are $\alpha_3 \in \{0, 1, 2, 3\}$. The shape parameters corresponding to these values of α_3 are $\beta \in \{3.6286, 1.5688, 0.9987, 0.7637\}$ and $\eta \in \{38000, 3.913, 0.983, 0.442\}$ for the Weibull and gamma distributions, respectively. The in-control means of the Weibull and gamma distributions are

$$\mu = \Gamma\left(1+\frac{1}{\beta}\right) \quad (17)$$

and

$$\mu = \eta, \quad (18)$$

respectively, while their in-control standard deviations are

$$\sigma = \sqrt{\Gamma\left(1+\frac{2}{\beta}\right) - \left[\Gamma\left(1+\frac{1}{\beta}\right)\right]^2} \quad (19)$$

and

$$\sigma = \sqrt{\eta}, \quad (20)$$

respectively. The out-of-control mean is $\mu_1 = \mu + \delta\sigma$, where $\delta \in \{0.25, 0.5, 1, 2\}$.

All the charts considered in this study are designed based on an in-control ARL of 370 when the process follows a normal distribution. The combinations of $(\lambda, L) \in \{(0.1, 2.6952), (0.2, 2.8537), (0.3, 2.9286), (0.4, 2.9614)\}$ for the standard EWMA, WSD–EWMA and WV–EWMA charts are determined using the plots in [11]. The simulated results are tabulated in Tables I and II for the Type-I and Type-II error rates, respectively. Here, the Type-I error rate of a chart is computed as the proportion of sample points (based on 1,000,000 samples) plotting beyond its control limits when the process is in-control. On the contrary, the computation of the Type-II error rate is made based on the proportion of sample points (from 1,000,000 samples) plotting within the limits of a chart when the process is out-of-control. Only the Type-II error rates for the gamma distribution are shown because of space constraint.

The average run length (ARL) performance is not considered because the ARL values can be easily computed from the Type-I and Type-II errors in Tables I and II, respectively. Here, the in-control ARL is $\text{ARL}_0 = 1/\alpha$, while the out-of-control ARL is $\text{ARL}_1 = 1/(1-\beta)$, where α and β are the Type-I and Type-II error rates, respectively. Since the interpretation based on ARL or Type-I and Type-II errors are the same, we only consider the Type-I and Type-II errors in this study.

In general, Table I shows that the proposed WV–EWMA chart with $\lambda \le 0.2$ gives a lower Type-I error rate compared to the other charts when $n = 3$ or 5 (see the boldfaced values). When $n = 1$, the WV–EWMA chart with $\lambda = 0.1$ has the lowest Type-I error rate (also, see the boldfaced values). Since $0.05 < \lambda < 0.25$ is recommended [13] and due to the fact that the WV–EWMA chart provides a fovourable rate of Type-I error compared with the other charts when $0.05 < \lambda < 0.25$, the WV–EWMA chart is recommended. Table II shows that the WV–EWMA chart has the lowest Type-II error rate compared with the WSD–EWMA charts for all δ and $\lambda \in \{0.1, 0.2, 0.3, 0.4\}$. The WV–EWMA chart also has lower Type-II error rates than the $\bar{X}$ charts for skewed populations and the standard $\bar{X}$ chart when $\delta \le 1$. However, the WV–EWMA chart has a slightly higher Type-II error rate than the standard EWMA chart. Since the standard EWMA chart has a very high Type-I error rate compared with the WV–EWMA and WSD–EWMA charts when the skewness is large, the standard EWMA chart is less attractive. Furthermore, when $\lambda = 0.3$ and $\lambda =$

0.4, the standard EWMA chart has a higher Type-I error rate than the $\bar{X}$ charts based on the WSD and SC methods.

V. CONCLUSION

This paper proposes the WV–EWMA chart for the mean. The new chart gives a more favourable performance than all existing heuristic charts for skewed populations, in terms of the Type-I and Type-II errors.

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TABLE I
TYPE-I ERROR RATES FOR THE VARIOUS CHARTS

Sample Size, n	Distribution	Skewness, α_3	EWMA Charts												$\overline{X}$ Charts			
			$\lambda = 0.1$			$\lambda = 0.2$			$\lambda = 0.3$			$\lambda = 0.4$						
			WV Method	WSD Method	Standard Method	WV Method	WSD Method	Standard Method	WV Method	WSD Method	Standard Method	WV Method	WSD Method	Standard Method	WV Method	WSD Method	SC Method	Standard Method
1	Normal	0	0.0027	0.0027	0.0027	0.0027	0.0027	0.0027	0.0027	0.0027	0.0027	0.0027	0.0027	0.0027	0.0026	0.0026	0.0026	0.0026
	Weibull	0	**0.0026**	0.0026	0.0026	0.0025	0.0025	0.0025	0.0021	0.0021	0.0021	0.0019	0.0019	0.0019	0.0007	0.0007	0.0007	0.0007
		1	**0.0026**	0.0029	0.0029	0.0028	0.0024	0.0038	0.0033	0.0023	0.0047	0.0041	0.0027	0.0058	0.0072	0.0050	0.0015	0.0098
		2	**0.0027**	0.0034	0.0037	0.0039	0.0026	0.0062	0.0055	0.0034	0.0086	0.0071	0.0045	0.0108	0.0125	0.0084	0.0042	0.0183
		3	**0.0028**	0.0040	0.0044	0.0047	0.0028	0.0077	0.0068	0.0041	0.0105	0.0087	0.0054	0.0130	0.0144	0.0095	0.0074	0.0213
	Gamma	0	**0.0028**	0.0028	0.0028	0.0027	0.0027	0.0027	0.0027	0.0027	0.0027	0.0026	0.0026	0.0027	0.0027	0.0027	0.0027	0.0027
		1	**0.0027**	0.0028	0.0030	0.0031	0.0026	0.0040	0.0037	0.0028	0.0051	0.0046	0.0033	0.0063	0.0079	0.0059	0.0020	0.0106
		2	**0.0027**	0.0033	0.0037	0.0039	0.0026	0.0063	0.0055	0.0033	0.0087	0.0072	0.0044	0.0110	0.0128	0.0084	0.0043	0.0186
		3	**0.0026**	0.0046	0.0045	0.0047	0.0025	0.0078	0.0067	0.0037	0.0108	0.0086	0.0050	0.0137	0.0148	0.0092	0.0071	0.0224
3	Normal	0	0.0028	0.0028	0.0028	0.0028	0.0028	0.0028	0.0027	0.0027	0.0027	0.0027	0.0027	0.0027	0.0027	0.0027	0.0027	0.0027
	Weibull	0	**0.0027**	0.0027	0.0027	**0.0026**	0.0027	0.0026	0.0025	0.0025	0.0025	0.0024	0.0024	0.0024	0.0020	0.0021	0.0021	0.0021
		1	**0.0028**	0.0032	0.0028	**0.0027**	0.0030	0.0030	0.0027	0.0026	0.0034	0.0028	0.0024	0.0039	0.0038	0.0025	0.0017	0.0059
		2	**0.0029**	0.0046	0.0030	**0.0029**	0.0040	0.0043	0.0033	0.0032	0.0056	0.0039	0.0028	0.0069	0.0071	0.0039	0.0021	0.0118
		3	**0.0031**	0.0065	0.0035	**0.0032**	0.0054	0.0056	0.0042	0.0038	0.0076	0.0054	0.0032	0.0095	0.0097	0.0053	0.0035	0.0163
	Gamma	0	**0.0027**	0.0027	0.0027	**0.0027**	0.0027	0.0027	0.0026	0.0026	0.0026	0.0026	0.0026	0.0026	0.0026	0.0026	0.0026	0.0026
		1	**0.0028**	0.0032	0.0029	**0.0027**	0.0030	0.0031	0.0028	0.0027	0.0035	0.0030	0.0026	0.0040	0.0043	0.0029	0.0026	0.0063
		2	**0.0030**	0.0047	0.0030	**0.0029**	0.0042	0.0042	0.0032	0.0032	0.0055	0.0039	0.0028	0.0070	0.0072	0.0041	0.0023	0.0118
		3	**0.0033**	0.0078	0.0034	**0.0030**	0.0068	0.0055	0.0039	0.0045	0.0075	0.0050	0.0030	0.0096	0.0091	0.0046	0.0033	0.0165
5	Normal	0	0.0027	0.0027	0.0027	0.0027	0.0028	0.0028	0.0027	0.0027	0.0027	0.0027	0.0027	0.0027	0.0027	0.0027	0.0027	0.0027
	Weibull	0	**0.0027**	0.0027	0.0027	**0.0027**	0.0027	0.0027	0.0026	0.0026	0.0026	0.0025	0.0025	0.0025	0.0023	0.0023	0.0023	0.0023
		1	0.0029	0.0033	0.0028	**0.0027**	0.0032	0.0029	0.0026	0.0029	0.0030	0.0026	0.0026	0.0033	0.0031	0.0021	0.0021	0.0048
		2	0.0031	0.0051	0.0029	**0.0029**	0.0049	0.0036	0.0029	0.0042	0.0045	0.0031	0.0035	0.0056	0.0052	0.0026	0.0021	0.0092
		3	0.0034	0.0073	0.0032	**0.0032**	0.0071	0.0047	0.0034	0.0058	0.0063	0.0041	0.0045	0.0078	0.0073	0.0036	0.0027	0.0133
	Gamma	0	**0.0028**	0.0028	0.0028	**0.0028**	0.0028	0.0028	0.0027	0.0027	0.0027	0.0027	0.0027	0.0027	0.0027	0.0027	0.0027	0.0027
		1	**0.0028**	0.0032	0.0028	**0.0027**	0.0032	0.0030	0.0026	0.0029	0.0031	0.0027	0.0028	0.0034	0.0033	0.0024	0.0026	0.0049
		2	0.0032	0.0052	0.0028	**0.0029**	0.0050	0.0036	0.0028	0.0043	0.0046	0.0032	0.0035	0.0056	0.0053	0.0027	0.0022	0.0094
		3	0.0036	0.0088	0.0031	**0.0031**	0.0090	0.0047	0.0031	0.0075	0.0063	0.0037	0.0056	0.0078	0.0069	0.0032	0.0025	0.0134

TABLE II
TYPE-II ERROR RATES FOR THE VARIOUS CHARTS WHEN THE UNDERLYING DISTRIBUTION IS GAMMA

Sample Size, n	Shape Parameter, η	Skewness, α_3	Shift, δ	EWMA Charts												$\overline{X}$ Charts			
				$\lambda = 0.1$			$\lambda = 0.2$			$\lambda = 0.3$			$\lambda = 0.4$						
				WV Method	WSD Method	Standard Method	WV Method	WSD Method	Standard Method	WV Method	WSD Method	Standard Method	WV Method	WSD Method	Standard Method	WV Method	WSD Method	SC Method	Standard Method
1	38000	0	0.25	0.9888	0.9888	0.9888	0.9918	0.9918	0.9917	0.9934	0.9935	0.9934	0.9943	0.9943	0.9943	0.9964	0.9964	0.9964	0.9964
			0.50	0.9645	0.9646	0.9645	0.9723	0.9724	0.9723	0.9789	0.9790	0.9788	0.9833	0.9833	0.9832	0.9935	0.9936	0.9935	0.9935
			1.00	0.8972	0.8973	0.8971	0.8978	0.8980	0.8977	0.9088	0.9090	0.9086	0.9216	0.9218	0.9215	0.9771	0.9772	0.9770	0.9770
			2.00	0.7605	0.7607	0.7603	0.7213	0.7216	0.7211	0.7060	0.7063	0.7058	0.7023	0.7025	0.7020	0.8417	0.8422	0.8412	0.8412
	3.913	1	0.25	0.9898	0.9921	0.9872	0.9906	0.9931	0.9877	0.9904	0.9930	0.9874	0.9900	0.9927	0.9876	0.9885	0.9916	0.9971	0.9847
			0.50	0.9699	0.9745	0.9649	0.9754	0.9806	0.9695	0.9783	0.9835	0.9724	0.9796	0.9848	0.9736	0.9835	0.9879	0.9957	0.9783
			1.00	0.9088	0.9173	0.9000	0.9141	0.9256	0.9022	0.9253	0.9381	0.9116	0.9350	0.9478	0.9208	0.9670	0.9752	0.9911	0.9574
			2.00	0.7771	0.7918	0.7620	0.7460	0.7654	0.7259	0.7393	0.7634	0.7150	0.7456	0.7745	0.7170	0.8799	0.9075	0.9631	0.8486
	0.983	2	0.25	0.9907	0.9942	0.9865	0.9901	0.9939	0.9852	0.9889	0.9934	0.9833	0.9875	0.9921	0.9814	0.9836	0.9892	0.9945	0.9763
			0.50	0.9744	0.9818	0.9662	0.9781	0.9857	0.9689	0.9791	0.9866	0.9697	0.9800	0.9864	0.9694	0.9791	0.9862	0.9930	0.9695
			1.00	0.9194	0.9340	0.9038	0.9286	0.9470	0.9085	0.9391	0.9574	0.9175	0.9460	0.9633	0.9248	0.9655	0.9774	0.9885	0.9501
			2.00	0.7903	0.8164	0.7621	0.7664	0.8024	0.7282	0.7703	0.8139	0.7239	0.7863	0.8351	0.7335	0.9074	0.9392	0.9690	0.8650
	0.442	3	0.25	0.9916	0.9954	0.9863	0.9902	0.9946	0.9841	0.9886	0.9935	0.9816	0.9867	0.9923	0.9791	0.9820	0.9889	0.9909	0.9727
			0.50	0.9783	0.9866	0.9680	0.9807	0.9889	0.9697	0.9806	0.9888	0.9695	0.9798	0.9882	0.9685	0.9782	0.9866	0.9890	0.9667
			1.00	0.9283	0.9471	0.9076	0.9404	0.9615	0.9153	0.9496	0.9693	0.9247	0.9545	0.9727	0.9309	0.9675	0.9802	0.9840	0.9501
			2.00	0.8004	0.8350	0.7621	0.7798	0.8306	0.7274	0.7954	0.8553	0.7237	0.8212	0.8819	0.7476	0.9263	0.9561	0.9648	0.8833
5	38000	0	0.25	0.9573	0.9573	0.9573	0.9655	0.9656	0.9654	0.9732	0.9733	0.9732	0.9787	0.9787	0.9786	0.9925	0.9925	0.9925	0.9925
			0.50	0.8805	0.8805	0.8805	0.8771	0.8772	0.8769	0.8866	0.8868	0.8864	0.9001	0.9003	0.8999	0.9699	0.9699	0.9699	0.9699
			1.00	0.7301	0.7301	0.7301	0.6821	0.6823	0.6818	0.6596	0.6599	0.6593	0.6488	0.6492	0.6485	0.7778	0.7778	0.7778	0.7778
			2.00	0.4999	0.4999	0.4999	0.3822	0.3826	0.3817	0.2640	0.2646	0.2634	0.1819	0.1825	0.1814	0.0707	0.0707	0.0707	0.0707
	3.913	1	0.25	0.9632	0.9686	0.9575	0.9714	0.9778	0.9643	0.9774	0.9835	0.9702	0.9809	0.9866	0.9740	0.9898	0.9932	0.9953	0.9850
			0.50	0.8914	0.9008	0.8817	0.8927	0.9058	0.8791	0.9047	0.9204	0.8885	0.9181	0.9347	0.9004	0.9708	0.9800	0.9854	0.9590
			1.00	0.7471	0.7631	0.7306	0.7050	0.7259	0.6835	0.6880	0.7134	0.6620	0.6842	0.7146	0.6535	0.8344	0.8753	0.9023	0.7876
			2.00	0.5099	0.5282	0.4942	0.4265	0.4556	0.3899	0.3281	0.3758	0.2750	0.2408	0.2960	0.1862	0.0889	0.1377	0.1865	0.0547
	0.983	2	0.25	0.9632	0.9686	0.9575	0.9714	0.9778	0.9643	0.9774	0.9835	0.9702	0.9809	0.9866	0.9740	0.9898	0.9932	0.9953	0.9850
			0.50	0.8914	0.9008	0.8817	0.8927	0.9058	0.8791	0.9047	0.9204	0.8885	0.9181	0.9347	0.9004	0.9708	0.9800	0.9854	0.9590
			1.00	0.7471	0.7631	0.7306	0.7050	0.7259	0.6835	0.6880	0.7134	0.6620	0.6842	0.7146	0.6535	0.8344	0.8753	0.9023	0.7876
			2.00	0.5221	0.5715	0.4889	0.4525	0.4885	0.3988	0.3803	0.4426	0.2879	0.3019	0.3909	0.1891	0.1150	0.2739	0.3730	0.0296
	0.442	3	0.25	0.9721	0.9834	0.9589	0.9790	0.9899	0.9636	0.9820	0.9918	0.9667	0.9831	0.9925	0.9681	0.9861	0.9937	0.9948	0.9732
			0.50	0.9088	0.9304	0.8850	0.9174	0.9461	0.8850	0.9312	0.9610	0.8948	0.9419	0.9700	0.9050	0.9723	0.9872	0.9895	0.9471
			1.00	0.7727	0.8103	0.7313	0.7408	0.7916	0.6861	0.7355	0.7986	0.6688	0.7447	0.8193	0.6662	0.8953	0.9497	0.9584	0.8111
			2.00	0.5376	0.6125	0.4843	0.4663	0.5144	0.4078	0.4153	0.4731	0.3008	0.3526	0.4415	0.1886	0.1501	0.4568	0.5266	0.0001

Proposed $\bar{X}$ and S Control Charts for Skewed Distributions

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Abstract – **This paper proposes a weighted variance method to compute the limits of the $\bar{X}$ and S charts for skewed distributions. The proposed charts extend the weighted variance $\bar{X}$ and R charts in [1] by enabling a process from a skewed distribution with moderate and large sample sizes to be monitored efficiently, hence producing more favourable Type-I and Type-II error rates than the charts in [1]. Note that the charts in [1] are only intended to be used for small sample sizes. The Type-I and Type-II error rates computed show that the proposed charts outperform the existing heuristic charts, as well as those in [1] for moderate and large sample sizes, involving cases with known and unknown parameters, when the distribution of a process is skewed.**

Keywords - **$\bar{X}$ chart, S chart, weighted variance, weighted standard deviation, skewness correction**

I. INTRODUCTION

In process monitoring, the assumption of a normally distributed process is often violated. A Shewhart chart is not applicable for skewed distributions as its Type-I error rate is inflated.

Nonparametric methods that do not rely on the assumption of normality can be used to deal with nonnormality. However, a major setback of nonparametric methods is that these methods are robust or insensitive to outliers [2]. Another popular strategy for dealing with nonnormality is to transform the data to achieve approximate normality. Among the transformation methods commonly used are the power transformation and Box-Cox procedure [2]. A potential drawback of transformation methods is that the samples are no longer plotted in the original scale of measurement.

To overcome the shortcomings faced by the nonparametric and transformation methods, heuristic $\bar{X}$ and R charts are suggested to address the problems of skewed underlying process distributions. These include the $\bar{X}$ and R charts using the weighted variance (WV) [1] and skewness correction (SC) [3] methods, and the $\bar{X}$ chart using the weighted standard deviation (WSD) method [4].

The standard deviation, σ of the heuristic charts in [1], [3] and [4] are estimated using the sample range, instead of the sample standard deviation. Since the range method in estimating σ loses statistical efficiency as the sample size, n increases [5], the use of the heuristic charts in [1], [3] and [4] is not desirable when n is moderate or large. As n increases, the sample standard deviation is more efficient than the sample range as an estimator of standard deviation. Thus, heuristic charts whose standard deviation are estimated from the sample standard deviation must be constructed for moderate or large n.

This paper proposes the $\bar{X}$ and S charts based on the WV approach, for use when $n \geq 10$. Unlike the standard $\bar{X}$ and S charts, the proposed charts provide asymmetric limits in accordance with the direction and degree of skewness by using different variances in computing the upper and lower limits. Thus, the proposed charts have lower false alarm rates than the standard $\bar{X}$ and S charts for a process from a skewed distribution. Contrary to the weighted variance $\bar{X}$ and R charts suggested in [1], the proposed charts are based on the standard deviation estimated from the sample standard deviation and not the sample range, like in the case of the charts in [1]. It will be shown in Section IV that the proposed charts outperform the existing heuristic charts for the various skewed distributions when process parameters are known and unknown by having lower Type-I and Type-II error rates when n is moderate or large.

In Section II, we review the existing heuristic charts for skewed distributions. The proposed weighted variance $\bar{X}$ and S charts are discussed in Section III. The performances of the proposed charts are compared with other heuristic charts for skewed distributions in Section IV. Finally, conclusions are drawn in Section V.

II. AN OVERVIEW OF HEURISTIC CHARTS FOR SKEWED DISTRIBUTIONS

A. WV $\bar{X}$ and R charts

The control limits of the WV – $\bar{X}$ chart are [1]

$$\mathrm{UCL}_{\mathrm{WV}-\bar{X}} = \mu_X + \frac{3\sigma_X}{\sqrt{n}}\sqrt{2P_X} \tag{1a}$$

$$\mathrm{LCL}_{\mathrm{WV}-\bar{X}} = \mu_X - \frac{3\sigma_X}{\sqrt{n}}\sqrt{2(1-P_X)}, \tag{1b}$$

where $P_X = \Pr(X \leq \mu_X)$. When parameters are unknown, the estimators used are $\hat{\mu}_X = \bar{\bar{X}}$, $\hat{\sigma}_X = \dfrac{\bar{R}}{d_2'}$ and $\hat{P}_X = \dfrac{\sum_{i=1}^{k}\sum_{j=1}^{n}\delta\left(\bar{\bar{X}} - X_{ij}\right)}{n \times k}$. Here, $\bar{\bar{X}}$ represents the grand average, $\bar{R}$ the average sample range, d_2' the constant for

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a given skewed population and $\delta(x) = 1$ for $x \geq 0$ or $\delta(x) = 0$ for $x < 0$.

The WV–R chart is based on the following limits [1]:

$$\mathrm{UCL}_{\mathrm{WV}-R} = \mu_R + 3\,\sigma_R\sqrt{2P_X} \quad (2a)$$

$$\mathrm{LCL}_{\mathrm{WV}-R} = \left[\mu_R - 3\sigma_R\sqrt{2(1-P_X)}\right]^+, \quad (2b)$$

where μ_R and σ_R are the mean and standard deviation of the range R while $[a]^+$ denotes $\max\{0, a\}$. When parameters are unknown, μ_R, σ_R and P_X are estimated as $\bar{R}$, $\frac{\bar{R}d_3'}{d_2'}$ and $\hat{P}_X$, respectively, where d_3' is a constant for a given skewed population.

B. SC $\bar{X}$ and R charts

The SC – $\bar{X}$ chart employs the following limits [3]:

$$\mathrm{UCL}_{\mathrm{SC}-\bar{X}} = \mu_X + \left(3 + c_4^*\right)\frac{\sigma_X}{\sqrt{n}} \quad (3a)$$

$$\mathrm{LCL}_{\mathrm{SC}-\bar{X}} = \mu_X + \left(-3 + c_4^*\right)\frac{\sigma_X}{\sqrt{n}}. \quad (3b)$$

Here, $c_4^* = \frac{4\kappa_3(\bar{X})}{3\left[1 + 0.2\kappa_3^2(\bar{X})\right]}$, where $\kappa_3(\bar{X}) = \frac{\kappa_3}{\sqrt{n}}$ with κ_3 representing the skewness coefficient of X. μ_X, σ_X and κ_3 are estimated as $\bar{\bar{X}}$, $\frac{\bar{R}}{d_2^*}$ and $\frac{1}{nr-3}\sum_{i=1}^{r}\sum_{j=1}^{n}\left(\frac{X_{ij} - \bar{\bar{X}}}{S_{nr}}\right)^3$, respectively, when parameters are unknown, with r representing the number of in-control subgroups used in the estimation. Note that $S_{nr} = \sqrt{\frac{1}{nr-1}\sum_{i=1}^{r}\sum_{j=1}^{n}\left(X_{ij} - \bar{\bar{X}}\right)^2}$.

The limits for the SC – R chart are [3]

$$\mathrm{UCL}_{\mathrm{SC}-R} = \mu_R + \left(3 + d_4^*\right)\sigma_R \quad (4a)$$

$$\mathrm{LCL}_{\mathrm{SC}-R} = \left[\mu_R + \left(-3 + d_4^*\right)\sigma_R\right]^+, \quad (4b)$$

where $d_4^* = \frac{4\kappa_3(R)}{3\left[1 + 0.2\kappa_3^2(R)\right]}$. Here, $\kappa_3(R)$ denotes the skewness of R. When parameters are unknown, μ_R and σ_R are estimated as $\bar{R}$ and $\frac{\bar{R}d_3^*}{d_2^*}$, respectively. Values of d_2^* and d_3^* are given in [3].

C. WSD $\bar{X}$ chart

The WSD – $\bar{X}$ chart uses the following limits [4]:

$$\mathrm{UCL}_{\mathrm{WSD}-\bar{X}} = \mu_X + \frac{3\sigma_X}{\sqrt{n}}(2P_X) \quad (5a)$$

$$\mathrm{LCL}_{\mathrm{WSD}-\bar{X}} = \mu_X - \frac{3\sigma_X}{\sqrt{n}}[2(1-P_X)]. \quad (5b)$$

For the case of unknown parameters, μ_X, σ_X and P_X are estimated as $\bar{\bar{X}}$, $\frac{\bar{R}}{d_2^{\mathrm{WSD}}}$ and $\hat{P}_X$, respectively, where $d_2^{\mathrm{WSD}} = P_X d_2\left(2n(1-P_X)\right) + (1-P_X)d_2\left(2nP_X\right)$ and $d_2(n)$ is d_2 for the normal distribution corresponding to n.

III. PROPOSED WV $\bar{X}_S$ AND S CHARTS

The WV procedure splits a skewed distribution into two parts at its mean, where each part is used to create a new symmetric distribution. The two new symmetric distributions are used to set up the limits of the chart [1]. The construction of the proposed WV – $\bar{X}_S$ and WV–S charts are discussed in the following paragraphs.

Let $\bar{X}_i$, for i = 1, 2, …, be a sequence of sample means for a process. The WV – $\bar{X}_S$ chart is constructed by plotting $\bar{X}_i$ based on the limits in Equations (1a) and (1b), when parameters are known. When parameters are unknown, the limits of the WV – $\bar{X}_S$ chart differ from those in [1] and are given as follow:

$$\mathrm{UCL}_{\mathrm{WV}-\bar{X}_S} = \bar{\bar{X}} + \frac{3\bar{S}}{c_4'\sqrt{n}}\sqrt{2\hat{P}_X} = \bar{\bar{X}} + A_U\bar{S} \quad (6a)$$

$$\mathrm{LCL}_{\mathrm{WV}-\bar{X}_S} = \bar{\bar{X}} - \frac{3\bar{S}}{c_4'\sqrt{n}}\sqrt{2\left(1-\hat{P}_X\right)} = \bar{\bar{X}} - A_L\bar{S}. \quad (6b)$$

Here, $\bar{S} = \frac{\sum_{i=1}^{r} S_i}{r}$ is the average of the sample standard deviations, estimated from r preliminary subgroups while c_4' is a constant for a given skewed population.

The WV–S chart is set up by plotting the sample standard deviations, S_i, for i = 1, 2, …, based on limits

$$\mathrm{UCL}_{\mathrm{WV}-S} = \mu_S + 3\sigma_S\sqrt{2P_X} \quad (7a)$$

$$\mathrm{LCL}_{\mathrm{WV}-S} = \left[\mu_S - 3\sigma_S\sqrt{2(1-P_X)}\right]^+, \quad (7b)$$

where μ_S and σ_S are the mean and standard deviation of S, respectively. These limits are different from that of the WV–R chart in Equations (2a) and (2b). For the case of unknown parameters, the limits of the WV–S chart are

$$\mathrm{UCL}_{\mathrm{WV}-S} = \bar{S}\left[1 + \frac{3\sqrt{1-(c_4')^2}}{c_4'}\sqrt{2\hat{P}_X}\right] = B_U\bar{S} \quad (8a)$$

$$\mathrm{LCL}_{\mathrm{WV}-S} = \bar{S}\left[1 - \frac{3\sqrt{1-(c_4')^2}}{c_4'}\sqrt{2\left(1-\hat{P}_X\right)}\right]^+ = B_L\bar{S}. \quad (8b)$$

Here, $c_4' = \frac{E(S)}{\sigma_X}$. Values of c_4', A_U, A_L, B_U and B_L are estimated via simulation using the Statistical Analysis System (SAS) as follows: First, 200000 samples, each of size n, are generated from the Weibull, Burr and lognormal distributions based on a fixed level of κ_3 and

its corresponding value, P_X. The average of the sample standard deviations from these 200000 samples are computed, followed by $\hat{\sigma}_X$ for $200000 \times n$ observations. Next, c_4' is computed for each of the three distributions and the average is taken. Finally, A_U, A_L, B_U and B_L are computed. Table I gives values of c_4', A_U, A_L, B_U and B_L for selected combinations of n and P_X. Note that A_U for $P_X \le 0.5$ is similar to A_L for $1-P_X$.

IV. PERFORMANCE COMPARISON

The WV$-\bar{X}$ and WV–S charts are compared with the heuristic $\bar{X}$ and R charts discussed in Section II, in terms of their Type-I and Type-II error rates. The Type-I error rate is the probability of signaling a false out-of-control while the Type-II error rate is the probability of signalling a false in-control.

The Type-I error rate is computed based on the Weibull, gamma and normal distributions while the Type-II error rate based on an exponential distribution. The Weibull distribution reduces to an exponential distribution when its shape parameter, β is one.

Note that P_X for the Weibull [1] and gamma [6] distributions are

$$P_X = 1-\exp\left[-\left(\Gamma\left(1+1/\beta\right)\right)^{\beta}\right] \tag{9}$$

and

$$P_X = F(\eta), \tag{10}$$

respectively, where β and η are shape parameters, while $\Gamma(\cdot)$ and $F(\cdot)$ are the gamma function and gamma distribution function, respectively.

The in-control means of the Weibull and gamma distributions are

$$\mu_X = \Gamma\left(1+\frac{1}{\beta}\right) \tag{11}$$

and

$$\mu_X = \eta, \tag{12}$$

respectively, and the in-control standard deviations are

$$\sigma_X = \sqrt{\Gamma\left(1+\frac{2}{\beta}\right)-\left[\Gamma\left(1+\frac{1}{\beta}\right)\right]^2} \tag{13}$$

and

$$\sigma_X = \sqrt{\eta}, \tag{14}$$

respectively.

The mean of an out-of-control process is $\mu_1 = \mu_X + a\sigma_X$, where $a \in \{0.25, 0.5(0.25), 1.5, 2\}$. Here, μ_X and σ_X are the in-control mean and standard deviation, respectively. The out-of-control standard deviation is $\sigma_1 = b\sigma_X$, where $b \in \{1.5, 2(0.5), 4.5\}$. For the in-control process, the skewness coefficients used for the Weibull and gamma distributions are $\kappa_3 = \{0, 1, 2, 3\}$.

When parameters are known, the Type-I and Type-II error rates are obtained based on 10000 simulation trials and sample sizes, $n \in \{10, 15, 20\}$. For the case with unknown parameters, first, 30 in-control samples, each of size, n are generated and the limits of the chart estimated. Then to compute the Type-I error rate, another 1000 in-control samples, each of size, n are generated in a phase-II process and the proportion of points falling beyond the limits computed. On the contrary, for computing the Type-II error rate, 1000 out-of-control samples, each containing n observations, are generated in a phase-II process and the proportion of points plotting within the limits computed. The procedure of generating 30 in-control samples followed by 1000 in-control/out-of-control samples is repeated for 10000 simulation trials and the average Type-I/Type-II error rate is recorded.

Tables II and III give the Type-I error rates for the heuristic charts for mean and variance, respectively, while Tables IV and V, show the Type-II error rates for those charts. The WV$-\bar{X}_S$ and WV$-\bar{X}$ charts have similar limits when parameters are known, hence they have the same error rates in Tables II and IV. Table II shows that when parameters are known, the Type-I error rates of the charts are comparable, though that of the WSD$-\bar{X}$ chart tends to be higher when n and κ_3 increase. When parameters are unknown, the WV$-\bar{X}_S$ chart has the lowest Type-I error rate (see Table II). Overall, Table III shows that the WV–S chart has a lower Type-I error rate than the other charts for variance when parameters are known and unknown. When parameters are known, the Type-II error rates of both the WV$-\bar{X}_S$ and WV$-\bar{X}$ charts are lower than that of the WSD$-\bar{X}$ and SC$-\bar{X}$ charts (see Table IV). For the case of unknown parameters in Table IV, the WV$-\bar{X}_S$ chart has a lower Type-II error rate than the WSD$-\bar{X}$ and SC$-\bar{X}$ charts but slightly higher than that of the WV$-\bar{X}$ chart. Table V shows that the WV–S chart has the lowest Type-II error rate when parameters are known and unknown.

V. CONCLUSION

Numerous charts for skewed distributions have been suggested, besides those described in Section II (see [6] – [11], to name a few). The WV$-\bar{X}_S$ and WV–S charts proposed in this paper for skewed populations are found to be superior to their existing counterparts when the sample size is moderate or large.

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TABLE I. VALUES OF c_4', A_U, A_L, B_U AND B_L

	P_X										
	0.50	0.52	0.54	0.56	0.58	0.60	0.62	0.64	0.66	0.68	0.70
						$n = 10$					
c_4'	0.97	0.97	0.97	0.96	0.96	0.94	0.92	0.89	0.87	0.83	0.76
A_U	0.96	0.98	1.01	1.02	1.04	1.10	1.15	1.20	1.22	1.31	1.43
A_L	0.96	0.95	0.93	0.92	0.91	0.90	0.90	0.90	0.89	0.91	0.95
B_U	1.71	1.73	1.81	1.91	2.00	2.16	2.47	2.77	2.95	3.36	4.02
B_L	0.29	0.30	0.25	0.20	0.15	0.05	0.00	0.00	0.00	0.00	0.00
						$n = 15$					
c_4'	0.98	0.98	0.98	0.97	0.97	0.96	0.94	0.91	0.90	0.86	0.80
A_U	0.78	0.79	0.80	0.82	0.85	0.86	0.91	0.95	0.97	1.02	1.12
A_L	0.78	0.76	0.74	0.73	0.72	0.70	0.70	0.71	0.70	0.72	0.74
B_U	1.56	1.58	1.65	1.73	1.82	1.96	2.24	2.52	2.67	3.04	3.67
B_L	0.44	0.44	0.40	0.35	0.30	0.21	0.03	0.00	0.00	0.00	0.00
						$n = 20$					
c_4'	0.99	0.99	0.98	0.98	0.98	0.97	0.95	0.93	0.92	0.88	0.83
A_U	0.68	0.69	0.70	0.71	0.72	0.74	0.77	0.80	0.83	0.85	0.93
A_L	0.68	0.66	0.64	0.64	0.63	0.62	0.61	0.60	0.60	0.61	0.63
B_U	1.48	1.50	1.56	1.64	1.71	1.84	2.10	2.36	2.49	2.84	3.42
B_L	0.52	0.52	0.48	0.44	0.39	0.31	0.14	0.00	0.00	0.00	0.00

TABLE II. TYPE-I ERROR RATES FOR MEAN CHARTS

	Distribution	κ_3	$n = 10$			$n = 15$			$n = 20$		
			WV – $\bar{X}_S$	WV – $\bar{X}$	WSD – $\bar{X}$	WV – $\bar{X}_S$	WV – $\bar{X}$	WSD – $\bar{X}$	WV – $\bar{X}_S$	WV – $\bar{X}$	WSD – $\bar{X}$
Parameters Known	Normal	0.0	0.0027	0.0027	0.0027	0.0027	0.0027	0.0027	0.0027	0.0027	0.0027
	Weibull	0.0	0.0025	0.0025	0.0025	0.0026	0.0026	0.0026	0.0026	0.0026	0.0026
		1.0	0.0025	0.0025	0.0025	0.0027	0.0027	0.0028	0.0026	0.0026	0.0025
		2.0	0.0030	0.0030	0.0027	0.0030	0.0030	0.0038	0.0027	0.0027	0.0027
		3.0	0.0048	0.0048	0.0028	0.0038	0.0038	0.0049	0.0032	0.0032	0.0032
	Gamma	0.0	0.0027	0.0027	0.0027	0.0027	0.0027	0.0027	0.0027	0.0027	0.0027
		1.0	0.0028	0.0028	0.0027	0.0027	0.0027	0.0029	0.0027	0.0027	0.0027
		2.0	0.0034	0.0034	0.0027	0.0029	0.0029	0.0038	0.0027	0.0027	0.0027
		3.0	0.0042	0.0042	0.0027	0.0033	0.0032	0.0068	0.0027	0.0027	0.0026
Parameters Unknown	Normal	0.0	0.0034	0.0034	0.0037	0.0034	0.0036	0.0035	0.0034	0.0033	0.0035
	Weibull	0.0	0.0031	0.0033	0.0035	0.0031	0.0038	0.0037	0.0031	0.0037	0.0038
		1.0	0.0032	0.0035	0.0042	0.0031	0.0039	0.0048	0.0030	0.0040	0.0052
		2.0	0.0040	0.0044	0.0060	0.0034	0.0043	0.0068	0.0031	0.0038	0.0074
		3.0	0.0051	0.0061	0.0085	0.0041	0.0051	0.0095	0.0033	0.0040	0.0099
	Gamma	0.0	0.0034	0.0034	0.0036	0.0034	0.0036	0.0035	0.0033	0.0033	0.0035
		1.0	0.0036	0.0037	0.0042	0.0033	0.0037	0.0043	0.0033	0.0035	0.0042
		2.0	0.0041	0.0045	0.0060	0.0033	0.0042	0.0067	0.0039	0.0037	0.0074
		3.0	0.0039	0.0057	0.0093	0.0028	0.0045	0.0124	0.0023	0.0034	0.0134

	Distribution	κ_3	$n = 10$			$n = 15$			$n = 20$		
			WV – $\bar{X}_S$	WV – $\bar{X}$	SC – $\bar{X}$	WV – $\bar{X}_S$	WV – $\bar{X}$	SC – $\bar{X}$	WV – $\bar{X}_S$	WV – $\bar{X}$	SC – $\bar{X}$
Parameters Known	Normal	0.0	0.0027	0.0027	0.0027	0.0027	0.0027	0.0027	0.0027	0.0027	0.0027
	Weibull	0.0	0.0025	0.0025	0.0025	0.0026	0.0026	0.0026	0.0026	0.0026	0.0026
		1.0	0.0025	0.0025	0.0025	0.0027	0.0027	0.0025	0.0026	0.0026	0.0025
		2.0	0.0030	0.0030	0.0026	0.0030	0.0030	0.0036	0.0027	0.0027	0.0027
		3.0	0.0048	0.0048	0.0039	0.0038	0.0038	0.0030	0.0032	0.0032	0.0032
	Gamma	0.0	0.0027	0.0027	0.0027	0.0027	0.0027	0.0027	0.0027	0.0027	0.0027
		1.0	0.0028	0.0028	0.0028	0.0027	0.0027	0.0027	0.0027	0.0027	0.0027
		2.0	0.0034	0.0034	0.0026	0.0029	0.0029	0.0026	0.0027	0.0027	0.0027
		3.0	0.0042	0.0042	0.0021	0.0033	0.0032	0.0034	0.0027	0.0027	0.0026
Parameters Unknown	Normal	0.0	0.0034	0.0034	0.0034	0.0034	0.0036	0.0039	0.0034	0.0033	0.0034
	Weibull	0.0	0.0031	0.0033	0.0035	0.0031	0.0038	0.0042	0.0031	0.0037	0.0038
		1.0	0.0032	0.0035	0.0042	0.0031	0.0039	0.0048	0.0030	0.0040	0.0052
		2.0	0.0040	0.0044	0.0060	0.0034	0.0043	0.0068	0.0031	0.0038	0.0074
		3.0	0.0051	0.0061	0.0085	0.0041	0.0051	0.0095	0.0033	0.0040	0.0099
	Gamma	0.0	0.0034	0.0034	0.0036	0.0034	0.0036	0.0035	0.0033	0.0033	0.0035
		1.0	0.0036	0.0037	0.0042	0.0033	0.0037	0.0043	0.0033	0.0035	0.0042
		2.0	0.0041	0.0045	0.0060	0.0033	0.0042	0.0067	0.0039	0.0037	0.0074
		3.0	0.0039	0.0057	0.0093	0.0028	0.0045	0.0124	0.0023	0.0034	0.0134

TABLE III. TYPE-I ERROR RATES FOR VARIANCE CHARTS

	Distribution	κ_3	$n = 10$			$n = 15$			$n = 20$		
			WV–S	WV–R	SC–R	WV–S	WV–R	SC–R	WV–S	WV–R	SC–R
Parameters Known	Normal	0.0	0.0030	0.0043	0.0041	0.0029	0.0045	0.0045	0.0027	0.0046	0.0047
	Weibull	0.0	0.0023	0.0026	0.0026	0.0024	0.0028	0.0038	0.0025	0.0029	0.0034
		1.0	0.0043	0.0055	0.0093	0.0037	0.0057	0.0108	0.0034	0.0059	0.0116
		2.0	0.0070	0.0080	0.0092	0.0061	0.0077	0.0112	0.0055	0.0076	0.0115
		3.0	0.0089	0.0095	0.0040	0.0078	0.0091	0.0051	0.0074	0.0088	0.0061
	Gamma	0.0	0.0028	0.0042	0.0023	0.0028	0.0043	0.0027	0.0027	0.0044	0.0027
		1.0	0.0056	0.0069	0.0069	0.0049	0.0071	0.0088	0.0045	0.0073	0.0096
		2.0	0.0069	0.0081	0.0088	0.0061	0.0077	0.0109	0.0055	0.0078	0.0109
		3.0	0.0074	0.0082	0.0041	0.0065	0.0080	0.0071	0.0059	0.0078	0.0088
Parameters Unknown	Normal	0.0	0.0035	0.0047	0.0032	0.0032	0.0047	0.0035	0.0032	0.0049	0.0039
	Weibull	0.0	0.0015	0.0014	0.0013	0.0015	0.0012	0.0013	0.0014	0.0010	0.0012
		1.0	0.0041	0.0041	0.0048	0.0028	0.0041	0.0054	0.0022	0.0037	0.0054
		2.0	0.0060	0.0074	0.0118	0.0040	0.0057	0.0128	0.0031	0.0047	0.0121
		3.0	0.0064	0.0080	0.0167	0.0084	0.0067	0.0177	0.0035	0.0061	0.0160
	Gamma	0.0	0.0032	0.0046	0.0030	0.0032	0.0048	0.0034	0.0030	0.0048	0.0038
		1.0	0.0069	0.0069	0.0064	0.0054	0.0074	0.0073	0.0044	0.0069	0.0074
		2.0	0.0060	0.0075	0.0121	0.0039	0.0055	0.0119	0.0031	0.0046	0.0124
		3.0	0.0032	0.0050	0.0211	0.0019	0.0038	0.0208	0.0012	0.0031	0.0183

TABLE IV. TYPE-II ERROR RATES FOR MEAN CHARTS

	a	$n = 10$				$n = 15$				$n = 20$			
		WV–$\bar{X}_S$	WV–$\bar{X}$	WSD–$\bar{X}$	SC–$\bar{X}$	WV–$\bar{X}_S$	WV–$\bar{X}$	WSD–$\bar{X}$	SC–$\bar{X}$	WV–$\bar{X}_S$	WV–$\bar{X}$	WSD–$\bar{X}$	SC–$\bar{X}$
Parameters Known	0.25	0.9860	0.9860	0.9933	0.9931	0.9827	0.9827	0.9921	0.9899	0.9790	0.9790	0.9907	0.9854
	0.50	0.9492	0.9492	0.9738	0.9733	0.9151	0.9151	0.9562	0.9452	0.8701	0.8701	0.9299	0.9024
	0.75	0.8453	0.8453	0.9123	0.9108	0.7040	0.7040	0.8203	0.7864	0.5377	0.5377	0.6904	0.6146
	1.0	0.6210	0.6210	0.7573	0.7538	0.3308	0.3308	0.5013	0.4453	0.1301	0.1301	0.2610	0.1862
	1.25	0.3030	0.3030	0.4773	0.4724	0.0532	0.0532	0.1427	0.1060	0.0041	0.0041	0.0206	0.0090
	1.5	0.0632	0.0632	0.1674	0.1638	0.0007	0.0007	0.0070	0.0037	0.0000	0.0000	0.0000	0.0000
	2.0	0.0000	0.0000	0.0001	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Parameters Unknown	0.25	0.9838	0.9827	0.9932	0.9884	0.9799	0.9776	0.9866	0.9829	0.9743	0.9725	0.9852	0.9775
	0.5	0.9429	0.9413	0.9743	0.9588	0.9079	0.8991	0.9349	0.9194	0.8555	0.8488	0.9060	0.8708
	0.75	0.8364	0.8332	0.9123	0.8766	0.7000	0.6802	0.7692	0.7289	0.5305	0.5175	0.6460	0.5656
	1.0	0.6241	0.6200	0.7573	0.7000	0.3532	0.3314	0.4486	0.3938	0.1475	0.1397	0.2462	0.1777
	1.25	0.3356	0.3337	0.4773	0.4320	0.0807	0.0721	0.1366	0.1051	0.0095	0.0087	0.0290	0.0153
	1.5	0.1042	0.1056	0.1674	0.1736	0.0051	0.0044	0.0145	0.0095	0.0000	0.0000	0.0005	0.0002
	2.0	0.0007	0.0009	0.0001	0.0041	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

TABLE V. TYPE-II ERROR RATES FOR VARIANCE CHARTS

	b	$n = 10$			$n = 15$			$n = 20$		
		WV–S	WV–R	SC–R	WV–S	WV–R	SC–R	WV–S	WV–R	SC–R
Parameters Known	1.5	0.8778	0.9193	0.9586	0.8720	0.9086	0.9952	0.8588	0.9007	0.9993
	2.0	0.7273	0.7611	0.9889	0.6435	0.7152	0.9968	0.5593	0.6793	0.9962
	2.5	0.5252	0.5704	0.9883	0.3853	0.4874	0.9852	0.2721	0.4250	0.9796
	3.0	0.3535	0.4009	0.9705	0.2057	0.2991	0.9542	0.1129	0.2312	0.9375
	3.5	0.2298	0.2721	0.9357	0.1039	0.1728	0.8992	0.0437	0.1150	0.8642
	4.0	0.1496	0.1819	0.3003	0.0527	0.0973	0.1935	0.0171	0.0543	0.1263
	4.5	0.0960	0.1203	0.2138	0.0262	0.0536	0.1182	0.0064	0.0251	0.0667
Parameters Unknown	1.5	0.9289	0.9310	0.9487	0.9266	0.9334	0.9492	0.9194	0.9334	0.9488
	2.0	0.7820	0.7962	0.8350	0.7358	0.7810	0.8182	0.6820	0.7637	0.8038
	2.5	0.5998	0.6274	0.6785	0.5008	0.5820	0.6312	0.4055	0.5383	0.5943
	3.0	0.4329	0.4678	0.5202	0.3064	0.3974	0.4460	0.2070	0.3364	0.3911
	3.5	0.3025	0.3373	0.3843	0.1769	0.2558	0.2964	0.0969	0.1939	0.2372
	4.0	0.2080	0.2389	0.2776	0.0992	0.1593	0.1896	0.0437	0.1062	0.1357
	4.5	0.1419	0.1677	0.1985	0.0549	0.0970	0.1185	0.0197	0.0562	0.0753

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A STUDY ON THE PERFORMANCES OF MEWMA AND MCUSUM CHARTS FOR SKEWED DISTRIBUTIONS

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ABSTRACT

A multivariate chart, instead of separate univariate charts is used for a joint monitoring of several correlated variables. Two time weighted multivariate charts that are commonly used for a quick detection of small shifts in the mean vector are the multivariate exponentially weighted moving average (MEWMA) and multivariate cumulative sum (MCUSUM) charts. The MEWMA and MCUSUM charts use information from past data, which make them sensitive to small shifts. These charts require the assumption that the underlying process follows a multivariate normal distribution. This paper studies the robustness of the MEWMA and MCUSUM charts toward nonnormality by considering the multivariate Weibull and multivariate gamma distributions based on different sample sizes and correlation coefficients.

1. INTRODUCTION

In most process monitoring situations, the quality of a process is determined by two or more quality characteristics (Woodall and Montgomery, 1999). Process monitoring problems involving several related variables of interest are called multivariate statistical process control. The most useful tool used in the monitoring of a multivariate process is a multivariate control chart. The first step in constructing a multivariate chart involves the analysis of a preliminary set of data that is assumed to be in statistical control. This analysis is known as a Phase-I analysis and it is conducted to estimate process parameters that will be used for the monitoring of a future process, a.k.a., a Phase-II process.

Numerous multivariate charts and their extensions are presently available. These charts can be grouped into 3 broad categories, namely, the Hotelling's T^2, multivariate EWMA (MEWMA) and multivariate CUSUM (MCUSUM) charts. The Hotelling's T^2 chart was proposed by Hotelling (1947) for the detection of a large sustained shift. The MCUSUM chart was first suggested by Woodall and Ncube (1985) while the MEWMA chart was introduced by Lowry et al. (1992). However, the MCUSUM charts suggested by Crosier (1988) will be discussed in this paper as they are more widely used.

This paper is organized as follows: Section 2 reviews the MEWMA chart while Section 3 reviews the MCUSUM chart. In Section 4, a simulation study is conducted to

compare the performances of MEWMA and MCUSUM charts for skewed distributions. Finally, conclusions are drawn in Section 5.

2. MEWMA CONTROL CHART

The MEWMA chart proposed by Lowry et al. (1992) is based on the following statistic:

$$\boldsymbol{Z}_t = \lambda \boldsymbol{X}_t + (1-\lambda)\boldsymbol{Z}_{t-1}, \text{ for } t = 1, 2, \ldots, \quad (1)$$

where $\boldsymbol{Z}_0 = \mu_0$ and $0 < \lambda \leq 1$. $\boldsymbol{X}_1, \boldsymbol{X}_2, \ldots,$ are assumed to be independent multivariate normal random vectors, each with p quality characteristics. The control charting statistic of a MEWMA chart is (Lowry et al., 1992)

$$T_t^2 = \boldsymbol{Z}_t' \Sigma_{Z_t}^{-1} \boldsymbol{Z}_t. \quad (2)$$

The chart signals a shift in the mean vector when $T_t^2 > h_1$, where h_1 is the limit chosen to achieve a desired in-control ARL (ARL_0) and

$$\Sigma_{Z_t} = \frac{\lambda}{2-\lambda}\left[1-(1-\lambda)^{2t}\right]\Sigma_X \quad (3)$$

is the variance-covariance matrix for $\boldsymbol{Z}_t$. Lowry et al. (1992) showed that the run length performance of the MEWMA chart depends on the off-target mean vector μ_1 and the covariance matrix of $\boldsymbol{X}_t$, i.e., Σ_{X_t} only through the value of the non-centrality parameter,

$$\delta = \left\{(\mu_1 - \mu_0)' \Sigma_X^{-1} (\mu_1 - \mu_0)\right\}^{1/2}, \quad (4)$$

where μ_0 denotes the in-control mean vector.

Lee and Khoo (2006a) provide a method based on the Markov chain approach for the selection of the optimal parameters, λ and h_1, which produce the minimum out-of-control ARL (ARL_1) for a desired size of a shift of interest based on a fixed ARL_0.

3. MCUSUM CONTROL CHART

Crosier (1988) suggested two multivariate CUSUM charts. The one with the better ARL performance is based on the following statistics:

$$C_t = \left\{(\boldsymbol{S}_{t-1} + \boldsymbol{X}_t - \boldsymbol{a})' \Sigma_X^{-1} (\boldsymbol{S}_{t-1} + \boldsymbol{X}_t - \boldsymbol{a})\right\}^{1/2}, \text{ for } t = 1, 2, \ldots, \quad (5)$$

where

$$\boldsymbol{S}_t = \begin{cases} \boldsymbol{0}, & \text{if } C_t \leq k \\ (\boldsymbol{S}_{t-1} + \boldsymbol{X}_t - \boldsymbol{a})\left(1 - \dfrac{k}{C_t}\right), & \text{if } C_t > k \end{cases}. \quad (6)$$

Note that $\boldsymbol{S}_0 = \boldsymbol{0}$, $k > 0$ is the reference value and $\boldsymbol{a}$ is the aim point or target value for the mean vector. The control charting statistic for the MCUSUM chart is (Crosier, 1988)

$$Y_t = \left(\boldsymbol{S}_t' \Sigma_X^{-1} \boldsymbol{S}_t\right)^{1/2} \qquad (7)$$

A shift in the mean vector is signalled when $Y_t > h_2$, where h_2 represents the limit of the chart. The MCUSUM procedure assumes that the multivariate observations $\boldsymbol{X}_t$, for t = 1, 2, ..., follow an independently and identically distributed (i.i.d.) multivariate normal distribution.

Lee and Khoo (2006b) give an approach based on the Markov chain method in determining the optimal parameters, k and h_2 that give the minimum out-of-control ARL $\left(\mathrm{ARL}_1\right)$ for a size of shift of interest based on a fixed ARL_0.

4. A SIMULATION STUDY

The assumption of the underlying process having i.i.d. multivariate normal random variates is required for both the MEWMA and MCUSUM charts. Since many multivariate processes, such as chemical processes come from populations that are skewed, it is difficult to satisfy the multivariate normality assumption. In this section, the performances of the MEWMA and MCUSUM charts will be studied when the multivariate normality assumption is violated.

The performances of the MEWMA and MCUSUM charts are compared based on the false alarm rates when the process is in-control for multivariate skewed distributions, such as the Lee's multivariate Weibull (Lee, 1979) and Cheriyan and Ramabhadran's multivariate gamma distributions (Cheriyan, 1941 and Ramabhadran, 1951). For the sake of comparison, the multivariate normal distribution is also considered. For convenience, the bivariate case, i.e., the number of quality characteristics, p = 2 is considered. Note that the bivariate Weibull distribution can represent various skewnesses and correlations but the bivariate gamma can only represent some positive correlations (Kotz et al., 2000).

SAS programs are used to compute the false alarm rates for the three multivariate distributions considered. Each false alarm rate is computed based on 5000 simulation trials. The nominal false alarm rate is assumed to be α = 0.0027 when the underlying distribution is bivariate normal. The MEWMA and MCUSUM charts are designed for a quick detection of a shift in the mean vector of size δ = 1. The optimal smoothing constant, λ = 0.13 and limit h_1 = 10.55 are found for the MEWMA chart using the approach described in Lee and Khoo (2006a). Similarly, using the procedure given in Lee and Khoo (2006b), the optimal parameters are found to be k = 0.5 and h_2 = 6.227 for the MCUSUM chart.

The correlation coefficients, ρ = 0.3, 0.5 and 0.8 are considered for the bivariate distributions. For ease of computation, the sacle parameters of (1,1) for $\left(X_1, X_2\right)$ are selected for the Weibull and gamma distributions. The shape parameters for $\left(X_1, X_2\right)$ are

chosen so that the desired skewnesses (γ_1, γ_2) = {(1,1), (1,2), (1,3), (2,2), (2, 3), (3,3)} for these parameters are attained. The sample sizes, n = 3, 5 and 7 are considered.

The false alarm rates for the MEWMA and MCUSUM charts are given in Tables 1 and 2, respectively. Note that the false alarm rates, marked as "*" in Tables 1 and 2 for the Cheriyan and Ramabhadran's bivariate gamma distribution cannot be computed because the corresponding shape parameters of one of the gamma distributed components, used in the transformation to compute variate X_2 have negative values.

From Tables 1 and 2, it is found that for the multivariate Weibull and gamma distributions, the false alarm rates of the MEWMA and MCUSUM charts increase as the level of skewness and correlation coefficient increase. This is because the covariance matrix of the multivariate observation, $\boldsymbol{X}$ is inflated as the skewness and correlation coefficient increase, hence making it easier for the MEWMA and MCUSUM charts to issue out-of-control signals. Also note that the false alarm rate decreases as the sample size increases. This is consistent with the multivariate central limit theorem, where the sample mean vector of a multivariate skewed distribution approaches multivariate normality as the sample size increases. A comparison of the false alarm rates of the two charts show that generally the MEWMA chart has lower false alarm rates than the MCUSUM chart for various levels of skewnesses. Thus, the MEWMA chart is more robust than the MCUSUM chart.

5. CONCLUSIONS

In this paper, we have studied the performance of the MEWMA and MCUSUM charts for multivariate normal and multivariate skewed distributions. We found that the false alarms of both charts are affected by the skewness of the underlying distribution. Also, the sample size and correlation of the quality characteristics have an impact on the false alarm rates of the charts. The simulation results show that the MEWMA chart has a lower false alarm rate than the MCUSUM chart when the underlying distribution is skewed. Since it is known that both the MEWMA and MCUSUM charts have equal performances in the detection of small shifts when the underlying process is multivariate normally distributed, the use of the MEWMA chart in process monitoring is recommended because the MEWMA chart is more robust towards skewed populations.

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Table 1. False alarm rates for the MEWMA chart when $\lambda = 0.13$ and $h_1 = 10.55$

Correlation coefficient	Multivariate distribution	Skewness coefficient (γ_1, γ_2)	Sample size, n		
			3	5	7
$\rho = 0.3$	Normal	(0,0)	0.0025830	0.0026450	0.0026600
	Weibull	(1,1)	0.0029040	0.0027250	0.0026130
		(1,2)	0.0035350	0.0031600	0.0030100
		(1,3)	0.0043070	0.0037940	0.0035010
		(2,2)	0.0041670	0.0035550	0.0033540
		(2,3)	0.0049280	0.0041320	0.0038270
		(3,3)	0.0056780	0.0047290	0.0043020
	Gamma	(1,1)	0.0029980	0.0028110	0.0027850
		(1,2)	0.0034710	0.0031680	0.0030860
		(1,3)	0.0039560	0.0035100	0.0033090
		(2,2)	0.0040630	0.0036180	0.0032550
		(2,3)	0.0047300	0.0040400	0.0036070
		(3,3)	0.0053240	0.0043790	0.0039400
$\rho = 0.5$	Normal	(0,0)	0.0025830	0.0026450	0.0026600
	Weibull	(1,1)	0.0028690	0.0026110	0.0024610
		(1,2)	0.0037170	0.0032160	0.0030640
		(1,3)	0.0047470	0.0040950	0.0038720
		(2,2)	0.0044490	0.0037830	0.0035030
		(2,3)	0.0053900	0.0045610	0.0042110
		(3,3)	0.0062970	0.0052450	0.0048160
	Gamma	(1,1)	0.0030340	0.0029600	0.0027750
		(1,2)	0.0036180	0.0031160	0.0030910
		(1,3)	*	*	*
		(2,2)	0.0041920	0.0035860	0.0033770
		(2,3)	0.0047770	0.0039130	0.0036190
		(3,3)	0.0055110	0.0046630	0.0042070
$\rho = 0.8$	Normal	(0,0)	0.0025830	0.0026450	0.0026600
	Weibull	(1,1)	0.0033460	0.0027720	0.0025900
		(1,2)	0.0049340	0.0041010	0.0036990
		(1,3)	0.0070200	0.0060950	0.0057130
		(2,2)	0.0057440	0.0046370	0.0041530
		(2,3)	0.0071680	0.0059480	0.0054110
		(3,3)	0.0081140	0.0068870	0.0062570
	Gamma	(1,1)	0.0032220	0.0030090	0.0029240
		(1,2)	*	*	*
		(1,3)	*	*	*
		(2,2)	0.0048070	0.0040280	0.0036410
		(2,3)	*	*	*
		(3,3)	0.0064080	0.0053320	0.0047210

Table 2. False alarm rates for the MCUSUM chart when $k = 0.5$ and $h_2 = 6.227$

Correlation coefficient	Multivariate distribution	Skewness coefficient (γ_1, γ_2)	Sample size, n		
			3	5	7
$\rho = 0.3$	Normal	(0,0)	0.0026930	0.0027220	0.0027220
	Weibull	(1,1)	0.0029440	0.0027420	0.0026800
		(1,2)	0.0035020	0.0031400	0.0030080
		(1,3)	0.0042500	0.0037290	0.0034920
		(2,2)	0.0036650	0.0033290	0.0031400
		(2,3)	0.0048150	0.0040180	0.0037430
		(3,3)	0.0055340	0.0045740	0.0041640
	Gamma	(1,1)	0.0026930	0.0027220	0.0027220
		(1,2)	0.0029440	0.0027420	0.0026800
		(1,3)	0.0035020	0.0031400	0.0030080
		(2,2)	0.0042500	0.0037290	0.0034920
		(2,3)	0.0036650	0.0033290	0.0031400
		(3,3)	0.0048150	0.0040180	0.0037430
$\rho = 0.5$	Normal	(0,0)	0.0026930	0.0027220	0.0027220
	Weibull	(1,1)	0.0028180	0.0025440	0.0024610
		(1,2)	0.0036490	0.0031830	0.0030670
		(1,3)	0.0046620	0.0040150	0.0038180
		(2,2)	0.0043290	0.0036840	0.0034060
		(2,3)	0.0052570	0.0044200	0.0040660
		(3,3)	0.0061670	0.0050810	0.0046200
	Gamma	(1,1)	0.0031260	0.0028970	0.0028610
		(1,2)	0.0034620	0.0032160	0.0029970
		(1,3)	*	*	*
		(2,2)	0.0039720	0.0035540	0.0033110
		(2,3)	0.0047190	0.0039000	0.0035950
		(3,3)	0.0055250	0.0045230	0.0040310
$\rho = 0.8$	Normal	(0,0)	0.0026930	0.0027220	0.0027220
	Weibull	(1,1)	0.0030700	0.0026280	0.0024780
		(1,2)	0.0047220	0.0039330	0.0036050
		(1,3)	0.0068540	0.0059950	0.0056410
		(2,2)	0.0054640	0.0044190	0.0039710
		(2,3)	0.0068990	0.0056910	0.0051910
		(3,3)	0.0078760	0.0065890	0.0059620
	Gamma	(1,1)	0.0032610	0.0029770	0.0028680
		(1,2)	*	*	*
		(1,3)	*	*	*
		(2,2)	0.0045670	0.0038680	0.0035700
		(2,3)	*	*	*
		(3,3)	0.0062740	0.0052470	0.0044940

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A Comparison of the Performances of Various Single Variable Charts

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ABSTRACT

Control charts are used for process monitoring and improvement in industries. Two charts are usually used in the monitoring of both the mean and variance separately. In the past 20 years, numerous control charting approaches that enable a joint monitoring of both the mean and variance on a single chart have been suggested. A joint monitoring of both the mean and variance is more meaningful in a real situation as both the mean and variance may shift simultaneously. Although numerous single variable control charts are available in the literature, not much research is made to compare these charts, in terms of their detection power. This paper compares the performances of several single variable charts, such as the semicircle, MaxEWMA and single MA charts, in terms of their average run length (ARL) results, via a Monte Carlo simulation. The Statistical Analysis System (SAS) software is employed in the simulation study. This comparison serves as a guide to practitioners by helping them to select a suitable single variable chart for process monitoring.

1. INTRODUCTION

Control charts are used for the purpose of detecting assignable causes that affect process stability. Two control charts, one for monitoring the process mean, such as the $\bar{X}$ chart and the other for monitoring the process variance, such as a R chart or a S chart, are usually run simultaneously. Most charts for variable data found in the literature monitor the process mean and variance separately. As shown by Reynolds and Stoumbos (2004) and mentioned again by Costa and Rahim (2006) that running two charts, one for the mean and the other for the variance, may not always be reliable in identifying the nature of the change. Recently, control charts that can simultaneously monitor both the process mean and the procecss variance have been proposed. These charts are called single variables control charts, and are classified as the Shewhart-type charts, CUSUM-type charts and EWMA-type charts.

For the Shewhart-type single variable chart, White and Schroeder (1987) first introdused the use of one control chart to monitor both process mean and variance on the same chart. This chart was designed using resistant measure and a modified box plot display. Chao and Cheng (1996) proposed a single control chart, called the semicircle (SC) control chart. This chart uses a semicircle to plot a single plotting statistic to indicate the position of the mean and standard deviation, by plotting the standard deviation on the y-axis and the mean on the x-axis. When a point plots ouside of the semicircle indicating an out-of-control signal, the chart shows whether the mean, the

variance or both parameters have shifted. The disadvantage of this chart is that it loses track of the time sequence of the plotted points. Chen and Cheng (1998) proposed a single Shewhart-type control chart, called the Max chart. This Max chart plots the maximum absolute value of the standardized mean and standard deviation. This chart performs like the combined Shewhart charts for the mean and standard deviation, i.e., the combined $\bar{X} - S$ charts. Spiring and Cheng (1998) developed a single variable chart that monitors both the process mean and standard deviation. This chart also plots two variables at the same time and has the advantage of performing equally well for both large and small subgroup sizes. Gan et al. (2004) proposed a single control chart based on the interval approach that combines both $\bar{X}$ and S charts into one scheme. Wu and Tian (2006) suggested a single weighted loss function chart (WL chart) for a simultaneous monitoring of the process mean and variance. İt was shown that the WL chart is significantly more effective than the unadjusted loss function chart and joint $\bar{X} - S$ charts, as well as the other charts.

For the CUSUM-type single chart, the CUSUM M-chart and CUSUM V-chart for detecting small shifts in the process mean and process variance, respectively, were proposed by Yeh et al. (2004). Because these charts have the same distribution when the process is in-control, they can be effectively combined into a single chart, thus, enabling a simultaneous monitoring of the mean and variance to be made on the same chart. A weighted loss function CUSUM (WLC) scheme with variable sampling interval (VSI) that enables a simultaneous monitoring of both the mean shift and an increasing variance shift by using a CUSUM chart was suggested by Zhang and Wu (2006).

For the EWMA-type single chart, numerous single EWMA charts for a simultaneous monitoring of the process mean and variance have been proposed. Domangue and Patch (1991) suggested some omnibus EWMA schemes based on the exponentiation of the absolute value of the standardized sample mean of the observations for a joint monitoring of the mean and variance. Gan (2000) proposed a simultaneous EWMA chart that was developed by combining a chart for the mean and a chart for the variance into one chart by plotting the EWMA of $\log(S^2)$ against the EWMA of $\bar{X}$. The control limit of this chart is formed by either using a rectangle or an ellipse. Morais and Pacheco (2000) considered a joint monitoring of the process mean and variance using a combined EWMA (CEWMA) scheme, where the average run length, percentage points of the run length and probability of a misleading signal were investigated. By using a two dimensional Markov chain approximation, these three performance measures are obtained. The MaxEWMA chart which combines the EWMA charts for the process mean and process variance into a single chart was developed by Chen et al. (2001). This chart extends and improves upon the earlier work of Chen and Cheng (1998) on the Max chart. Chen et al. (2004) proposed the EWMA-SC chart by applying the EWMA technique to the statistics employed in the semicircle chart. This proposed chart provides a better detection ability with regards to small shifts in the mean and/or variance in comparison to the SC chart. Costa and Rahim (2004) suggested the use of a single non-central chi-square chart to monitor both the process mean and variance simultaneously. Costa and Rahim (2004) also found that the EWMA chart based on the non-central chi-square statistic has a similar performance to the MaxEWMA chart proposed by Chen et al. (2001). A single EWMA chart which is an extension of the EWMA-SC chart studied by Chen et al. (2004) was suggested by Costa and Rahim (2006).

This paper compares the performances of three single control charts, namely the semicircle (SC), MaxEWMA and single moving average (MA) charts, in terms of thier average run lengths (ARLs), via a Monte Carlo simulation. A simulation study conducted using the Statistical Analysis System (SAS) software shows that the MaxEWMA chart gives the best performance, while the SC chart has the poorest performance. This comparison assist practitioners in selecting a suitable single variable chart for process monitoring.

This paper is organized as follows: Section 2 reviews the semicircle (SC) chart. A review of the MaxEWMA chart is made in Section 3, while Section 4 reviews the single moving average (MA) chart. In Section 5, a performance comparison is made to compare the performances of the SC, MaxEWMA and single MA charts, in terms of their average run length (ARL) profiles. Finally, conclusions are drawn in Section 6.

2. SEMICIRCLE (SC) CHART

Chao and Cheng (1996) proposed a semicircle control chart, where a semicircle is used to plot a single plotting statistic to represent the position of the mean and standard deviation, where the standard deviation is plotted on the *y*-axis and the mean on the *x*-axis. When a point plots outside of the simicircle indicating an out-of-control signal, the chart can easily tell whether the mean, the variance or both the parameters have changed. With its straight forward calculations, this chart can be considerd as a new alternative to the combination of the $\overline{X}$ and R charts.

The plotting statistic proposed by Chao and Cheng (1996) is

$$T = \left(\overline{X} - \mu\right) + S^{*2}. \tag{1}$$

The plotting statistic is based on the sample mean, $\overline{X}$, and the root mean square, $S^* = \left(\frac{n-1}{n}\right)^{\frac{1}{2}} S$, where $S = \sqrt{\frac{\sum_{i=1}^{n}\left(X_i - \overline{X}\right)^2}{n-1}}$ is the sample standard deviation. Under the normality assumption, $\left(\frac{n}{\sigma^2}\right)T$ is distributed exactly as a χ^2_n random variable for any sample size n. The statistic T in Equation (1) defines a circle, but because $S^* \geq 0$, when plotting $\left(\overline{X}, S_i^*\right)$ for each sample on the $\left(\overline{X}, S^*\right)$ plane, a semicircle will be sufficient.

İn order to construct the semicircle chart, the following formula can be used in determining the radius r, where α is the size of the Type-I error (Chao and Cheng, 1996):

$$P\left(T < r^2\right) = P\left(\frac{n}{\sigma^2}T < \frac{n}{\sigma^2}r^2\right)$$
$$= 1 - \alpha,$$

Since $\frac{n}{\sigma^2}T \sim \chi^2_n$, we have $\frac{n}{\sigma^2}r^2 \sim \chi^2_{n,(1-\alpha)}$, i.e.,

$$r = \left(\frac{\chi^2_{n,(1-\alpha)}}{n} \right)^{1/2} \sigma. \quad (2)$$

Here, $\chi^2_{n,(1-\alpha)}$ is a 100(1−α)% percentile of the χ^2_n distribution. Note that if parameters are unknown, $\overline{\overline{X}}$ is used to estimate μ and $\overline{S}^*$ to estimate σ.
Then we have (Chao and Cheng, 1996)

$$\hat{r} = q\overline{S}^* \quad (3)$$

where $q = \left(\frac{\chi^2_{n,(1-\alpha)}}{n} \right)^{1/2}$.

3. MaxEWMA CHART

An exponentially weighted moving average (EWMA) chart is a control chart for variable data. İt plots weighted moving averages. A weighting factor is chosen by the user to determine how older data points affect the mean value compared to more recent ones. Because the EWMA chart uses information from all samples, it detects smaller process shifts quicker than the Shewhart control chart. The MaxEWMA chart is constructed as follows (Chen et al., 2001):

Assume that a sequence of individual measurements, X_{ij}, in sample i, follow a $N(\mu,\sigma)$ distribution, for i = 1, 2, ... and j = 1, 2, ..., n_i. Let μ_0 be the nominal process mean and σ_0 be a known value of the process standard deviation. Assume that the process parameters μ and σ can be expressed as $\mu = \mu_0 + a\sigma_0$ and $\sigma = b\sigma_0$, where a and b (>0) are constants. The process is in-control when a = 0 and b = 1; otherwise the process has changed.

Let $\overline{X}_i = \frac{\sum_{j=1}^{n_i} X_{ij}}{n_i}$ be the i^{th} sample mean and $S_i^2 = \frac{\sum_{j=1}^{n_i} (X_{ij} - \overline{X}_i)^2}{(n_i - 1)}$ be the i^{th} sample variance. Define (Chen et al., 2001)

$$U_i = \frac{(\overline{X}_i - \mu_0)}{\sigma_0 / \sqrt{n_i}} \quad (4)$$

and

$$V_i = \Phi^{-1}\left\{ H\left(\frac{(n_i - 1)S_i^2}{\sigma_0^2}; n_i - 1 \right) \right\}, \quad (5)$$

where $\Phi(z) = P(Z \le z)$, for $Z \sim N(0, 1)$, $\Phi^{-1}(\cdot)$ is the inverse function of $\Phi(\cdot)$ and $H(\omega;\nu) = P(W \le \omega)$, where W follows a chi-square distribution with ν degrees of freedom.

It is known that U_i and V_i are independent when a = 0 and b = 1 because $\overline{X}_i$ and S_i^2 are independent. İt can be shown that $U_i \sim N(0,1)$ and $V_i \sim N(0,1)$ (Chen et al.,

2001). The distributions of U_i and V_i are both independent of the sample size n_i, when $a = 0$ and $b = 1$, therefore the variable sample size problem can be handled easily by the MaxEWMA chart. Since both U_i and V_i have the same distribution, a single variable chart to monitor both the process mean and process variability can be constructed (Chen et al., 2001).

First, define

$$Y_i = \lambda U_i + (1-\lambda) Y_{i-1}, \quad 0 < \lambda \leq 1, \quad \text{for } i = 1, 2, \ldots \tag{6a}$$

and

$$Z_i = \lambda V_i + (1-\lambda) Z_{i-1}, \quad 0 < \lambda \leq 1, \quad \text{for } i = 1, 2, \ldots, \tag{6b}$$

with Y_0 and Z_0 as the starting values, respectively. Then, the above two EWMA statistics are combined into a single chart by defining a new statistic M_i given by

$$M_i = \max\{|Y_i|, |Z_i|\}. \tag{7}$$

The statistic M_i will be large when the process mean has shifted away from μ and/or when the process variability has increased or decreased. On the other hand, the statistic M_i will be small when the process mean and process variabiity stay close to their respective targets.

Since M_i is non-negative, only a UCL is needed. The UCL is given by

$$\text{UCL} = E(M_i) + K\sqrt{Var(M_i)}, \tag{8}$$

where $E(M_i)$ is the mean of M_i and $Var(M_i)$ is the variance of M_i, when $a = 0$ and $b = 1$. Here, K is a multiplier, which together with λ, controls the performance of the new chart. Because this chart is based on M_i, the maximum of $|Y_i|$ and $|Z_i|$, it is called the MaxEWMA chart.

4. SINGLE MOVING AVERAGE (MA) CHART

Khoo and Yap (2005) suggested the use of a joint moving average control chart for a simultaneous monitoring of the process mean and variance. Besides being efficient in detecting increases and decreases in the process mean and/or variability, the joint MA chart is also able to indicate the source and direction of a shift.

Let X_{ij} for $i = 1, 2, \ldots$, and $j = 1, 2, \ldots, n_i$, be observations from subgroups of size n_i, with i representing the subgroup number. It is assumed that $X_{ij} \sim N(\mu + a\sigma, b^2\sigma^2)$, where $a = 0$ and $b = 1$ indicate that the process is in-control; otherwise, the process has shifted.

Let $\bar{X}_i = \dfrac{(X_{i1} + X_{i2} + \cdots + X_{in_i})}{n_i}$ be the i^{th} sample mean and let $S_i^2 = \dfrac{\sum_{j=1}^{n_i}(X_{ij} - \bar{X}_i)^2}{n_i - 1}$ be the i^{th} sample variance. Khoo and Yap (2005) define the following statistics:

$$U_i = \frac{(\bar{X}_i - \mu)}{\sigma/\sqrt{n_i}} \sim N(0,1), \quad \text{for } i = 1, 2, \ldots \tag{9}$$

and

$$V_i = \Phi^{-1}\left\{H\left(\frac{(n_i-1)S_i^2}{\sigma^2}; n_i-1\right)\right\} \sim N(0,1), \quad \text{for } i=1, 2, \ldots, \tag{10}$$

where $\Phi^{-1}(\cdot)$ and $H(\cdot)$ denote the inverse standard normal distribution function and the chi-square distribution function with $n_i - 1$ degrees of freedom, respectively. Because the sample mean, $\bar{X}_i$ and sample variance, S_i^2, are independent, U_i and V_i are also independent. The sample grand average, $\bar{\bar{X}}$ and R/d_2 or $\bar{S}/c_4$ are substituted for μ and σ, respectively, if the target values of these parameters are unknown, where d_2 and c_4 are the control chart constants.

The plotting statistic, K_i, of the chart can then be defined as (Khoo and Yap, 2005)

$$K_i = \max\{|L_i|, |M_i|\}, \tag{11}$$

where

$$L_i = \frac{U_i + U_{i-1} + \cdots + U_{i-w+1}}{w}$$

and

$$M_i = \frac{V_i + V_{i-1} + \cdots + V_{i-w+1}}{w},$$

for $i \geq w$, while for $i < w$, say $w = 3$,

$$L_1 = \frac{U_1}{1},\ L_2 = \frac{U_1 + U_2}{2},\ L_3 = \frac{U_1 + U_2 + U_3}{3}$$

and

$$M_1 = \frac{V_1}{1}, M_2 = \frac{V_1 + V_2}{2}, M_3 = \frac{V_1 + V_2 + V_3}{3}.$$

Note that w is the span of the moving average statistic. The statistic K_i will be large when the process mean has shifted away from its target value and/or when the process variance has increased or decreased. Only the upper control limit, UCL is applied on the joint MA chart as K_i is non-negative.

The density function of K_i, for the in-control case is (Khoo and Yap, 2005)

$$f(k) = 4\sqrt{w}\phi\left(k\sqrt{w}\right)\left\{2\Phi\left(k\sqrt{w}\right) - 1\right\}, \quad \text{for } k \geq 0. \tag{12}$$

Here, $\phi(\cdot)$ and $\Phi(\cdot)$ are the density and distribution functions of a standard normal random variable, respectively. Suppose that the desired Type-I error set by management based on some predetemined factors is α, then UCL is obtained by solving the following definite integral:

$$\int_{UCL}^{\infty} f(k)\,dk = \alpha \tag{13}$$

For a MA control chart, the control limits for periods $i < w$ are wider than their steady-state value. Besides having the desirable properties of the moving average chart, the variable sample size problem can also be handled automatically with the application of

the joint MA chart. Therefore, the joint MA chart can be considered as an attractive alternative to the combined $\bar{X} - R$ or $\bar{X} - S$ charts (Khoo and Yap, 2005).

5. PERFORMANCE COMPARISON

A simulation study is conducted using SAS version 9 to study the performances of the SC, MaxEWMA and single MA charts. The sample size of $n = 5$ is considered. The shifts in the mean and variance considered are $\mu_1 = \mu_0 + a\sigma_0$ and $\sigma_1 = b\sigma_0$, respectively, where $a \in \{0, 0.25, 0.5, 1, 2\}$ and $b \in \{0.25, 0.5, 1, 1.5, 2\}$. Note that when the process is in-control, $a = 0$ and $b = 1$. The in-control ARL (ARL_0) is fixed as 185. For the SC chart, the radius of the chart for $n = 5$ is $r = 1.8174$. For the MaxEWMA chart, $\lambda \in \{0.05, 0.1, 0.15, 0.2, 0.25, 0.3, 0.4, 0.5, 0.8, 1\}$ are considered and their corresponding K values are determined. The values of the moving span $w \in \{2, 3, 4, 5\}$ are employed and their corresponding UCLs are determined for the single MA chart.

The ARL profiles for the SC, single MA and MaxEWMA charts are given in Tables 1, 2 and 3, respectively. Generally, the results show that the MaxEWMA chart is superior to the other two charts. The SC chart is found to have the poorest performance. Note that for an arbitrary combination of (a, b), where $a > 0$ and $b \neq 1$, the MaxEWMA chart has the lowest out-of-control ARL, followed by the single MA chart. On the contrary, the SC chart always has the highest out-of-control ARL among the three charts.

6. CONCLUSIONS

İn this paper, the performances of three single variable control charts are compared based on their ARLs. Overall, the MaxEWMA chart provides a better performance than the other two charts, while the SC chart gives the poorest performance.

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Table 1. ARL profiles for the SC chart with ARL_0 =185 and n =5

	a				
b	0.00	0.25	0.50	1.00	2.00
r = 1.8174	0.25	1.3	1.5	1.0	1.0	1.0
	0.50	8.7	1.1	1.0	1.0	1.0
	1.00	185.0	123.7	52.0	8.3	1.2
	1.50	5.0	4.7	4.0	2.4	1.2
	2.00	1.9	1.8	1.7	1.5	1.1

Table 2. ARL profiles for the single MA chart with ARL_0 =185 and n =5

	a				
(w, UCL)	b	0.00	0.25	0.50	1.00	2.00
w = 2	0.25	1.3	1.3	1.3	1.1	1.0
UCL= 2.1233	0.50	11.5	11.5	11.0	1.8	1.0
	1.00	185.0	64.7	14.3	2.2	1.0
	1.50	5.0	4.5	3.4	1.8	1.1
	2.00	1.7	1.7	1.6	1.4	1.1
w = 3	0.25	1.0	1.0	1.0	1.0	1.0
UCL= 1.7358	0.50	5.2	5.2	4.5	1.1	1.0
	1.00	185.0	46.7	9.0	1.6	1.0
	1.50	3.8	3.5	2.7	1.5	1.0
	2.00	1.5	1.4	1.4	1.2	1.0
w = 4	0.25	1.0	1.0	1.0	1.0	1.0
UCL= 1.5066	0.50	3.3	3.3	2.6	1.0	1.0
	1.00	185.0	36.9	6.4	1.4	1.0
	1.50	3.1	2.8	2.2	1.3	1.0
	2.00	1.3	1.3	1.2	1.1	1.0
w =5	0.25	1.0	1.0	1.0	1.0	1.0
UCL= 1.3540	0.50	2.4	2.4	1.9	1.0	1.0
	1.00	185.0	29.8	5.0	1.3	1.0
	1.50	2.6	2.4	1.8	1.2	1.0
	2.00	1.2	1.2	1.2	1.1	1.0

Table 3. ARL profiles for the MaxEWMA chart with ARL_0 =185 and n =5

	a					
	b	0.00	0.25	0.50	1.00	2.00
λ=0.05 K=3.3833	0.25	2.0	2.0	2.0	1.9	1.0
	0.50	3.5	3.5	3.4	2.2	1.0
	1.00	185.5	12.1	5.1	2.3	1.2
	1.50	4.1	3.9	3.3	2.2	1.2
	2.00	2.1	2.1	2.0	1.7	1.2
λ=0.10 K=3.4028	0.25	1.8	1.8	1.8	1.8	1.0
	0.50	3.3	3.3	3.1	2.0	1.0
	1.00	185.0	12.3	4.8	2.1	1.1
	1.50	3.7	3.5	3.0	2.0	1.2
	2.00	1.9	1.9	1.8	1.6	1.2
λ=0.15 K=3.2254	0.25	1.8	1.8	1.8	1.8	1.0
	0.50	3.3	3.3	3.1	2.0	1.0
	1.00	185.0	14.7	4.0	2.1	1.1
	1.50	3.7	3.5	2.9	1.9	1.2
	2.00	1.9	1.8	1.8	1.5	1.2
λ=0.20 K=3.0996	0.25	1.8	1.8	1.8	1.8	1.0
	0.50	3.5	3.5	3.3	2.0	1.0
	1.00	185.1	18.4	5.4	2.1	1.1
	1.50	3.9	3.6	3.0	2.0	1.1
	2.00	1.9	1.9	1.8	1.5	1.2
λ=0.25 K=3.0415	0.25	1.8	1.8	1.8	1.8	1.0
	0.50	3.7	3.7	3.5	2.1	1.0
	1.00	185.1	23.1	6.0	2.2	1.1
	1.50	4.1	3.8	3.1	2.0	1.2
	2.00	1.9	1.9	1.8	1.6	1.2
λ=0.30 K=3.0278	0.25	1.8	1.8	1.8	1.8	1.0
	0.50	4.1	4.0	3.8	2.1	1.0
	1.00	185.0	28.5	6.6	2.2	1.1
	1.50	4.3	4.0	3.2	2.0	1.2
	2.00	1.9	1.9	1.8	1.6	1.2
λ=0.40 K=3.0499	0.25	1.9	1.9	1.9	1.8	1.0
	0.50	4.9	4.9	4.7	2.2	1.0
	1.00	185.0	38.3	8.3	2.3	1.1
	1.50	4.7	4.3	3.4	2.1	1.2
	2.00	2.0	1.9	1.8	1.6	1.2
λ=0.50 K=3.0884	0.25	1.9	1.9	1.9	1.8	1.0
	0.50	6.5	6.5	6.2	2.3	1.0
	1.00	185.0	47.7	10.25	2.5	1.1
	1.50	5.0	4.6	3.6	2.1	1.2
	2.00	2.0	2.0	1.9	1.6	1.2
λ=0.80 K=3.0019	0.25	2.6	2.6	2.6	2.4	1.0
	0.50	21.4	21.4	21.1	4.2	1.0
	1.00	185.0	76.8	19.9	3.3	1.1
	1.50	6.5	5.9	4.5	2.4	1.2
	2.00	2.2	2.1	2.0	1.7	1.2
λ=1.00 K=2.6918	0.25	4.8	4.8	4.8	4.7	1.0
	0.50	50.4	50.4	50.2	12.1	1.0
	1.00	185.0	96.8	30.3	4.4	1.1
	1.50	7.3	6.7	5.1	2.7	1.2
	2.00	2.3	2.2	2.1	1.7	1.2

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A comparison of multivariate control charts for skewed distributions using weighted standard deviations

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ABSTRACT

The quality of a manufacturing process usually depends on more than one quality characteristic. Thus, most process monitoring data are multivariate in nature. The assumption that the underlying process follows a multivariate normal distribution is usually required by most multivariate quality control charts. However, in most process monitoring situations, the multivariate normality assumption is often violated. Multivariate control charts for skewed distributions have been suggested to enable process monitoring to be made when the underlying process distribution is skewed. Among the recent heuristic multivariate charts for skewed distributions suggested in the literature are those based on the weighted standard deviation (WSD) approach. This paper compares the performances of three multivariate charts for skewed distributions incorporating the WSD method, i.e., the WSD T^2 , WSD multivariate cumulative sum (WSD MCUSUM) and WSD multivariate exponentially weighted moving average (WSD MEWMA) charts. These heuristic charts are compared based on the multivariate lognormal, gamma and Weibull distributions. The charts' performances are evaluated using the false alarm rates, computed via a Monte-carlo simulation. The chart with the best performance, i.e., the one having the lowest false alarm rate for most of the skewness levels and sample sizes will be identified.

| Multivariate control charts | Skewed distributions | Weighted standard deviations | WSD T^2 | WSD MEWMA| WSD MCUSUM|

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1. INTRODUCTION

In process monitoring, we often deal with two or more related variables [1]. An individual might have a false understanding that by applying univariate control charts to each of the variables, instead of a multivariate chart, the same results can be obtained. Previous studies have shown that using separate univariate charts are not only inefficient and time consuming, but they also lead to misleading or erroneous conclusions. Multivariate statistical quality control charts which consider different variables simultaneously are required [2].

Three common types of multivariate control charts are the Hotelling's T^2, multivariate exponentially weighted moving average (MEWMA) and multivariate cumulative sum (MCUSUM) charts. The Hotelling's T^2 chart is the most common multivariate process monitoring procedure in controlling the mean vector of a process. This control chart is actually an analog of the univariate Shewhart mean chart [2]. The MEWMA chart has been developed to overcome the disadvantages of the Hotelling's T^2 chart, such as its insensitivity to small and moderate shifts. A MEWMA chart is a logical extension of the univariate EWMA chart. Like MEWMA, the MCUSUM chart is also a useful chart in detecting small shifts. There are two versions of the MCUSUM charts, namely the MCUSUM#1 ($MC1$) chart and the MCUSUM#2 ($MC2$) chart. Pignatiello and Runger [2] stated that comparatively for both $MC1$ and $MC2$ charts, the $MC1$ chart seems to be more efficient in detecting small shifts than the $MC2$ chart. As for the $MC2$ chart, it appears to be more efficient in detecting large shifts compared to the $MC1$ chart.

The multivariate charts that are mentioned above are confined to multivariate normal populations. However, the normality assumption is found to be difficult to justify and is often violated [3].

Chang and Bai [3] proposed a heuristic method for constructing the multivariate T^2 control chart for skewed populations. Chang [4] also suggested a heuristic method of constructing the MEWMA and MCUSUM charts for skewed distributions.

The layout of this paper is as follows: Section 2 explains the weighted standard deviation method, Sections 3, 4 and 5 discuss the WSD T^2 chart, WSD MEWMA chart and WSD MCUSUM chart, respectively. Section 6 compares the performances of the three types of multivariate charts for skewed distributions. Conclusions are drawn in Section 7.

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2. WEIGHTED STANDARD DEVIATION (WSD) METHOD

Chang and Bai [3] proposed the WSD method for constructing the multivariate T^2 control chart for skewed populations. The WSD method approximates the probability density function (pdf) of a v-variate skewed distribution with segments from 2^v multivariate normal distributions by modifying the variance-covariance matrix according to the estimated degree and direction of the skewness.

The proposed WSD method assumes that a v-variate random vector $X=(X_1,...,X_v)^T$ is distributed with a multivariate skewed distribution having a mean vector

$$\boldsymbol{\mu}=(\mu_1,...,\mu_v)^T \tag{1}$$

and variance-covariance matrix,

$$\Sigma=\begin{bmatrix}\sigma_1^2 & \rho_{12}\sigma_1\sigma_2 & \cdots & \rho_{1v}\sigma_1\sigma_v\\ & \sigma_2^2 & \cdots & \rho_{2v}\sigma_2\sigma_v\\ & & \ddots & \vdots\\ & & & \sigma_v^2\end{bmatrix}, \tag{2}$$

where σ_j is the standard deviation of X_j and ρ_{ij} is the correlation coefficient of X_i and X_j.

The WSD method works by adjusting the variance-covariance matrix. Hence, the variance-covariance matrix of the 2^v multivariate normal distributions for the approximation is [3]

$$\boldsymbol{\Sigma}^{\mathrm{W}}=\mathbf{W}\cdot\Sigma\cdot\mathbf{W}=\begin{bmatrix}\left(\sigma_1^W\right)^2 & \rho_{12}\sigma_1^W\sigma_2^W & \cdots & \rho_{1v}\sigma_1^W\sigma_v^W\\ & \left(\sigma_2^W\right)^2 & \cdots & \rho_{2v}\sigma_2^W\sigma_v^W\\ & & \ddots & \vdots\\ & & & \left(\sigma_v^W\right)^2\end{bmatrix}, \tag{3}$$

where $\boldsymbol{W}=\mathrm{diag}\left\{W_1(X_1-\mu_1),...,W_v(X_v-\mu_v)\right\}$, and

$$W_j(x)=\begin{cases}2P_j & \text{if } x>0\\ 2(1-P_j) & \text{otherwise}\end{cases} \tag{4}$$

Note that $\sigma_j^W=W_j(X_j-\mu_j)\cdot\sigma_j$ and $P_j=\Pr(X_j\le\mu_j)$. This means that if a skewed distribution is found to be skewed to the right, then $P_j>\frac{1}{2}$ and $2P_j\sigma_j>2(1-P_j)\sigma_j$. However, the correlation matrix $\boldsymbol{\rho}=\{\rho_{ij}\}$ does not change.

3. THE WSD T^2 CHART

Chang and Bai [3] proposed the T^2 statistic based on the WSD method by defining $\boldsymbol{Z}^W=\boldsymbol{W}^{-1}\boldsymbol{Z}$, where the j^{th} element of $\boldsymbol{Z}^W$ is

$$Z_j^W=\frac{X_j-\mu_j}{\sigma_j^W}=\frac{X_j-\mu_j}{W_j\sigma_j}=\begin{cases}\dfrac{1}{2P_j}Z_j & \text{if } X_j>\mu_j\\ \dfrac{1}{2(1-P_j)}Z_j & \text{, otherwise}\end{cases}. \tag{5}$$

Then the WSD T^2 statistic is defined as

$$T_{W,i}^2=\left(n\bar{\boldsymbol{Z}}_i^W\right)^T\rho^{-1}\bar{\boldsymbol{Z}}_i^W, \tag{6}$$

where $\bar{\boldsymbol{Z}}_i^W=\left(\bar{Z}_{1i}^W,...,\bar{Z}_{vi}^W\right)^T$. An out-of-control signal is detected when $T_{W,i}^2>\chi_\alpha^2(v)$, where $\chi_\alpha^2(v)$ is defined as the $100(1-\alpha)^{\text{th}}$ percentile of the χ^2 distribution with v degree of freedom.

4. THE WSD MEWMA CHART

The performance of the MEWMA chart for skewed populations can be improved by applying the WSD method. The WSD MEWMA statistic is defined as [4]:

$$\boldsymbol{M}_i^W = \lambda \boldsymbol{Z}_i^W + (1-\lambda)\boldsymbol{M}_{i-1}^W, \tag{7}$$

where $i = 1, 2, \ldots$ and $\boldsymbol{M}_0^W = \mathbf{0}$.

Since $\boldsymbol{Z}_i^W$ follows a multivariate normal distribution with mean vector $\mathbf{0}$ and variance-covariance matrix, $\boldsymbol{\rho}$ approximately, then the asymptotic variance-covariance matrix of $\boldsymbol{M}_i^W$ is

$$\Sigma_M = \left[\frac{\lambda}{(2-\lambda)}\right]\boldsymbol{\rho}, \tag{8}$$

which is the same as that of the standard MEWMA statistic. Hence, when the charting statistic exceeds h_E, that is

$$E_i^W = \left(\boldsymbol{M}_i^W\right)^T \Sigma_M^{-1} \boldsymbol{M}_i^W > h_E, \tag{9}$$

the WSD MEWMA chart issues an out-of-control signal. When the distribution is symmetric, the proposed WSD MEWMA chart reduces to the standard MEWMA chart [4].

5. THE WSD MCUSUM CHART

The WSD MCUSUM chart modifies the MCUSUM charting statistic with the degree and direction of the skewness. Based on the WSD method, the sample observations are first standardized as $\boldsymbol{Z}^W = \boldsymbol{W}^{-1}\boldsymbol{Z}$, of which the jth element is defined as [4]

$$Z_j^W = \frac{X_j - \mu_j}{\sigma_j^W} = \frac{X_j - \mu_j}{W_j \sigma_j} = \begin{cases} \dfrac{1}{2P_j} Z_j & \text{if } X_j > \mu_j \\ \dfrac{1}{2(1-P_j)} Z_j & \text{, otherwise} \end{cases}. \tag{10}$$

Note that $\boldsymbol{Z}^W$ follows a multivariate normal distribution, approximately. The cumulative statistic at i is defined as

$$\boldsymbol{A}_i^W = \boldsymbol{S}_{i-1}^W + \boldsymbol{Z}_i^W, \tag{11}$$

where the length of $\boldsymbol{A}_i^W$ is $C_i^W = \left[\left(\boldsymbol{A}_i^W\right)^T \rho^{-1} \boldsymbol{A}_i^W\right]^{1/2}$, and $\boldsymbol{S}_i^W$ is the cumulative sum at i. The size of the mean shift $d(\mu_1)$ is also modified according to the adjusted variance-covariance matrix, using the WSD method as follows:

$$d^W(\mu_1) = \left[(\mu_1 - \mu)^T \left(\Sigma^W\right)^{-1} (\mu_1 - \mu)\right]^{1/2}. \tag{12}$$

Let $\Sigma^W = \boldsymbol{W} \cdot \Sigma \cdot \boldsymbol{W}$ (see Equation (3)) and $k^W = \dfrac{d^W(\mu_1)}{2}$ be the reference value. Chang [4] mentioned that when $C_i^W > k^W$, the cumulative sum will shrink towards $\mathbf{0}$. He pointed out that the same direction and size cannot be applied to all the variables because the upper and lower WSDs of each of the variables are not the same. Hence, the j^{th} element of the cumulative sum $\boldsymbol{S}_i^W$ is defined as [4]

$$S_{ji}^W = \begin{cases} A_{ji}^W \left(1 - \dfrac{k}{2P_j C_i^W}\right) & \text{if } C_i^W > k^W \text{ and } A_{ji}^W > 0 \\ A_{ji}^W \left(1 - \dfrac{k}{2(1-P_j) C_i^W}\right) & \text{if } C_i^W > k^W \text{ and } A_{ji}^W \le 0 \\ 0 & \text{otherwise} \end{cases}. \tag{13}$$

If $Y_i^W = \left[\left(\boldsymbol{S}_i^W\right)^T \rho^{-1} \boldsymbol{S}_i^W\right]^{1/2}$ exceeds h_C, the WSD MCUSUM chart will issue an out-of-control alarm. Here, h_C is determined based on a desired in-control ARL. When the distribution is symmetric, the proposed WSD MCUSUM chart reduces to the MCUSUM chart [4].

6. A COMPARISON OF THE PERFORMANCES OF THREE TYPES OF MULTIVARIATE CHARTS FOR SKEWED DISTRIBUTIONS

This section compares the performances of three different types of multivariate control charts for skewed distributions, namely the WSD T^2, WSD MEWMA and WSD MCUSUM charts. The skewed distributions considered are the lognormal, gamma and Weibull distributions. The *Statistical Analysis System* (*SAS*) version 9 is used to conduct the simulation studies of the charts based on the bivariate case, which involves two quality characteristics ($p = 2$).

In general, all the three bivariate WSD charts for skewed distributions are compared based on the skewness coefficients of $(\gamma_1, \gamma_2) = (1,1), (1,2), (1,3), (2,2), (2,3)$ and $(3,3)$ and correlation coefficients of $\rho = 0.3$, 0.5 and 0.8. The sample size, n, for all the three WSD charts are fixed as $n = 1$, 5 and 10.

From the simulation results in Tables 1 – 3, it is found that the false alarm rates for all the three types of the bivariate WSD charts, based on the bivariate lognormal, gamma and Weibull distributions increase as the level of skewness increases. For example, the false alarm rate for $(\gamma_1, \gamma_2) = (2,2)$ is higher than that of $(\gamma_1, \gamma_2) = (1,1)$. Note that for all three types of the bivariate WSD charts, based on gamma distribution (see Tables 1 – 3), the symbol "*" is used to represent the false alarm rate that cannot be computed by SAS as there exists a negative value in the shape parameter of the gamma distribution.

The results in Table 1 shows that the false alarm rate of the WSD T^2 chart decreases as the sample size increases. In contrast, the results in Tables 2 and 3 show that, generally, the false alarm rates of the WSD MEWMA and WSD MCUSUM charts increase with the sample size.

In general, the WSD MCUSUM chart has the lowest false alarm rate while the WSD T^2 chart has the highest, when the same level of skewness, sample size and underlying distribution are considered.

7. CONCLUSION

In statistical quality control, the assumption that the underlying distribution is normal is usually made. However, this leads to erroneous conclusion if the normality assumption is violated. In practice, many processes come from skewed populations with multivariable or quality characteristics. In this paper, three multivariate control charts for skewed populations, namely the Hotelling's T^2, MCUSUM and MEWMA charts using the weighted standard deviation (WSD) method, based on the bivariate lognormal, gamma and Weibull distributions are considered. The false alarm rates of the charts computed using the SAS program are compared, for various levels of skewness and sample sizes. The WSD MCUSUM chart is found to have the lowest false alarm rate for the various levels of skewness considered. This study helps practitioners in deciding the type of chart to be used in process monitoring.

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Table 1: False alarm rates for the WSD T^2 chart, based on the bivariate lognormal, gamma and Weibull distributions for $\rho \in \{0.3, 0.5, 0.8\}$ and $n \in \{1, 5, 10\}$

ρ	n	(γ_1, γ_2)	Distribution		
			lognormal	gamma	Weibull
0.3	1	(1,1)	0.008700	0.008185	0.007923
		(1,2)	0.010960	0.010507	0.010959
		(1,3)	0.011739	0.011331	0.012168
		(2,2)	0.013349	0.013173	0.013951
		(2,3)	0.014127	0.014343	0.015145
		(3,3)	0.014899	0.015441	0.016313
	5	(1,1)	0.003317	0.003122	0.002835
		(1,2)	0.004250	0.003459	0.003576
		(1,3)	0.005221	0.004100	0.004567
		(2,2)	0.005278	0.004210	0.004330
		(2,3)	0.006267	0.004840	0.005392
		(3,3)	0.007314	0.005696	0.006623
	10	(1,1)	0.002992	0.002832	0.002749
		(1,2)	0.003376	0.002992	0.003174
		(1,3)	0.003994	0.003762	0.003778
		(2,2)	0.003834	0.003184	0.003590
		(2,3)	0.004487	0.003841	0.004240
		(3,3)	0.005129	0.004274	0.004884
0.5	1	(1,1)	0.008949	0.009151	0.011196
		(1,2)	0.011290	0.010907	0.014008
		(1,3)	0.011868	*	0.014971
		(2,2)	0.013750	0.014903	0.016789
		(2,3)	0.014400	0.015704	0.017804
		(3,3)	0.015158	0.016691	0.018735
	5	(1,1)	0.003345	0.003398	0.003444
		(1,2)	0.004376	0.003963	0.004472
		(1,3)	0.005391	*	0.005900
		(2,2)	0.005459	0.005159	0.005667
		(2,3)	0.006499	0.006036	0.007189
		(3,3)	0.007531	0.007221	0.008770
	10	(1,1)	0.002986	0.002985	0.002934
		(1,2)	0.003381	0.003579	0.003569
		(1,3)	0.004034	*	0.004668
		(2,2)	0.003915	0.004026	0.004308
		(2,3)	0.004556	0.005182	0.005470
		(3,3)	0.005271	0.005997	0.006650
0.8	1	(1,1)	0.009382	0.013133	0.022012
		(1,2)	0.011369	*	0.023218
		(1,3)	0.011167	*	0.023560
		(2,2)	0.014180	0.020178	0.023732
		(2,3)	0.014583	*	0.023662
		(3,3)	0.015405	0.021604	0.023395
	5	(1,1)	0.003489	0.004908	0.007392
		(1,2)	0.004689	*	0.010716
		(1,3)	0.005761	*	0.014355
		(2,2)	0.005835	0.020178	0.013566
		(2,3)	0.006929	*	0.016718
		(3,3)	0.008033	0.021604	0.019283
	10	(1,1)	0.003058	0.004048	0.004894
		(1,2)	0.003580	*	0.007581
		(1,3)	0.004417	*	0.011791
		(2,2)	0.004094	0.008394	0.009848
		(2,3)	0.004917	*	0.013451
		(3,3)	0.005676	0.015986	0.016788

Table 2: False alarm rates for the WSD MEWMA chart, based on the bivariate lognormal, gamma and Weibull distributions for $\rho \in \{0.3, 0.5, 0.8\}$ and $n \in \{1, 5, 10\}$

ρ	n	(γ_1, γ_2)	Distribution		
			lognormal	gamma	Weibull
0.3	1	(1,1)	0.003042	0.003009	0.003215
		(1,2)	0.003434	0.004167	0.004212
		(1,3)	0.003700	0.006534	0.005158
		(2,2)	0.0036990	0.004740	0.004967
		(2,3)	0.003925	0.006530	0.005877
		(3,3)	0.004098	0.008636	0.006882
	5	(1,1)	0.004071	0.004202	0.004419
		(1,2)	0.005698	0.007949	0.007847
		(1,3)	0.007511	0.015924	0.012554
		(2,2)	0.00711	0.010586	0.010526
		(2,3)	0.008732	0.01773	0.014738
		(3,3)	0.010366	0.023721	0.018382
	10	(1,1)	0.004340	0.004594	0.004820
		(1,2)	0.006409	0.008839	0.008820
		(1,3)	0.008862	0.018181	0.014634
		(2,2)	0.008114	0.012147	0.011973
		(2,3)	0.010297	0.010446	0.017103
		(3,3)	0.012359	0.027005	0.021457
0.5	1	(1,1)	0.002909	0.002886	0.002988
		(1,2)	0.003264	0.004288	0.003931
		(1,3)	0.003518	*	0.005133
		(2,2)	0.003256	0.004092	0.004303
		(2,3)	0.003355	0.006037	0.005111
		(3,3)	0.003413	0.006870	0.005702
	5	(1,1)	0.003853	0.003989	0.003977
		(1,2)	0.005425	0.007994	0.007359
		(1,3)	0.007305	*	0.012743
		(2,2)	0.006356	0.009752	0.009478
		(2,3)	0.007796	0.016576	0.013414
		(3,3)	0.009052	0.021121	0.016431
	10	(1,1)	0.004158	0.004436	0.004281
		(1,2)	0.006215	0.009018	0.008543
		(1,3)	0.008715	*	0.014895
		(2,2)	0.007420	0.010989	0.010870
		(2,3)	0.009406	0.019249	0.015809
		(3,3)	0.010991	0.024673	0.019531
0.8	1	(1,1)	0.002773	0.003283	0.003464
		(1,2)	0.003240	*	0.005210
		(1,3)	0.003787	*	0.008147
		(2,2)	0.002826	0.004385	0.004642
		(2,3)	0.002878	*	0.005537
		(3,3)	0.002755	0.006216	0.005498
	5	(1,1)	0.003647	0.004062	0.003579
		(1,2)	0.005659	*	0.008457
		(1,3)	0.008696	*	0.018611
		(2,2)	0.005619	0.009255	0.008604
		(2,3)	0.007042	*	0.013355
		(3,3)	0.007583	0.019161	0.014879
	10	(1,1)	0.003971	0.004240	0.003739
		(1,2)	0.006592	*	0.009785
		(1,3)	0.010563	*	0.021683
		(2,2)	0.006646	0.010338	0.009895
		(2,3)	0.008785	*	0.015942
		(3,3)	0.009539	0.021977	0.017833

Table 3: False alarm rates for the WSD MCUSUM chart, based on the bivariate lognormal, gamma and Weibull distributions for $\rho \in \{0.3, 0.5, 0.8\}$ and $n \in \{1, 5, 10\}$

ρ	n	(γ_1, γ_2)	Distribution		
			lognormal	gamma	Weibull
0.3	1	(1,1)	0.002567	0.002527	0.002264
		(1,2)	0.002557	0.002266	0.002219
		(1,3)	0.002617	0.002188	0.002304
		(2,2)	0.002563	0.002088	0.002130
		(2,3)	0.002692	0.002114	0.002274
		(3,3)	0.002802	0.002073	0.002479
	5	(1,1)	0.002812	0.002793	0.002725
		(1,2)	0.002859	0.003089	0.003026
		(1,3)	0.002858	0.003388	0.003167
		(2,2)	0.002791	0.003092	0.003191
		(2,3)	0.002768	0.003392	0.003223
		(3,3)	0.002721	0.003501	0.003236
	10	(1,1)	0.002981	0.002967	0.002880
		(1,2)	0.003203	0.003590	0.003549
		(1,3)	0.003366	0.004765	0.004127
		(2,2)	0.003355	0.003971	0.003983
		(2,3)	0.003508	0.004973	0.004473
		(3,3)	0.003598	0.005735	0.004858
0.5	1	(1,1)	0.002572	0.002467	0.002369
		(1,2)	0.002599	0.002325	0.002455
		(1,3)	0.002620	*	0.002750
		(2,2)	0.002629	0.002319	0.002491
		(2,3)	0.002744	0.002392	0.003071
		(3,3)	0.002922	0.002370	0.003069
	5	(1,1)	0.002752	0.002969	0.002446
		(1,2)	0.002780	0.003138	0.002851
		(1,3)	0.002770	*	0.003189
		(2,2)	0.002672	0.003012	0.002934
		(2,3)	0.002605	0.003233	0.003071
		(3,3)	0.002495	0.003069	0.003069
	10	(1,1)	0.002903	0.002977	0.002627
		(1,2)	0.003105	0.003634	0.003343
		(1,3)	0.003237	*	0.004156
		(2,2)	0.003150	0.003803	0.003660
		(2,3)	0.003215	0.004616	0.004167
		(3,3)	0.003231	0.005043	0.004478
0.8	1	(1,1)	0.002563	0.002908	0.003248
		(1,2)	0.002683	*	0.003811
		(1,3)	0.002877	*	0.004795
		(2,2)	0.002769	0.003107	0.003887
		(2,3)	0.002963	*	0.004337
		(3,3)	0.003186	0.003445	0.004601
	5	(1,1)	0.002711	0.002786	0.002361
		(1,2)	0.002818	*	0.003385
		(1,3)	0.002921	*	0.005311
		(2,2)	0.002549	0.003136	0.003153
		(2,3)	0.002459	*	0.003725
		(3,3)	0.002373	0.003378	0.003725
	10	(1,1)	0.002827	0.002989	0.002277
		(1,2)	0.003165	*	0.003832
		(1,3)	0.003483	*	0.006800
		(2,2)	0.002976	0.003756	0.003521
		(2,3)	0.003026	*	0.004559
		(3,3)	0.002910	0.004942	0.004712

A Study on Q Chart for Short Runs Production

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ABSTRACT

The Shewhart control chart is an important tool in process monitoring in manufacturing industries. The Shewhart chart is only suitable in mass production, where data for parameter estimation are readily available. In recent years, due to just-in-time (JIT) techniques, synchronous productions, job-stop settings and the reduction of in-process inventories and costs, there is a trend among manufacturers to produce items in smaller lot sizes. The Shewhart chart is not suitable for process monitoring in such an environment. Such low volume production is referred to as "short runs production" or simply "short runs". A major problem faced in a short runs environment is the need to chart a large number of different processes and the consequent large number of charts needed. Numerous research have been conducted to modify current control charts so that they can be employed in short runs. Among the short runs charts that have been suggested are the Q, Z, difference and deviation from nominal (DNOM) charts. This paper studies the average run length (ARL) performance of the Q chart for the mean for cases where parameters are known and unknown.

Keywords: *Average run length; process mean; Q chart; Q statistics; short runs production.*

INTRODUCTION

Short production runs involve processes that produce products to fulfil customers specific needs and requirements. Now, short production runs is a necessity in most manufacturing industries and it will become more important in the future. Short runs involve low-volume manufacturing in smaller lot sizes.

The main problem faced in a low-valume manufacturing is to estimate the process parameters with the limited data available and to compute the control limits before the start of a production run. Thus, many control charts are needed due to the existence of many different processes. Two main requirements in short runs are as follow (Quesenberry, 1991):

i) The use of the first few units of production to compute the process parameters.
ii) Plotting of all statistics on a standard scale so that the different variables in a process can all be plotted on the same control chart to simplify the chart's management program.

Traditional statistical quality control methods should be modified so that they can be used in short runs.

To date, numerous works on the use of short runs control charting procedures have been suggested. Bothe (1989) and Burr (1989) suggested using target specification values for process parameters to construct charts. However, the choice of control limits based on this method produce higher false alarm rates. To solve this problem, Quesenberry (1991) introduced the Q charts for attribute and variable data. The Q charts based on attribute data are the binomial, Poisson and geometric Q charts. The Q charts for variable data are based on individual measurements and subgrouped data. The Q charts for variable data are used to control the mean and variance. Using some transformation methods, the Q statistics are all independent and identically distributed standard normal variables. This transformation maintains the information of the original statistics but all the Q statistics can be plotted on a standard scale. Wheeler (1991) suggested a few short runs variable control charts. They are the Zed, Z^*, difference and Zed-bar charts. This paper compares the performance of the Q chart for the mean, based on individual measurements using several tests or runs rules.

Q STATISTICS FOR THE PROCESS MEAN

İn computing the Q statistics for monotoring the mean, individual measurements will be transformed into the following four different cases (Quesenberry, 1995):
Case KK: $\mu = \mu_0, \sigma = \sigma_0$, both known

$$Q_r(X_r) = \frac{X_r - \mu_0}{\sigma_0}, \; r = 1, 2, \ldots \quad (1)$$

Case UK: μ unknown, $\sigma = \sigma_0$ known

$$Q_r(X_r) = \left(\frac{r-1}{r}\right)\left(\frac{X_r - \bar{X}_{r-1}}{\sigma_0}\right), \; r = 2, 3, \ldots \quad (2)$$

Case KU: $\mu = \mu_0$ known, σ unknown

$$Q_r(X_r) = \Phi^{-1}\left\{G_{r-1}\left(\frac{X_r - \mu_0}{S_{0,r-1}}\right)\right\}, \; r = 2, 3, \ldots, \quad (3)$$

where

$$S_{0,r-1} = \frac{1}{r}\sum_{j=1}^{r}(X_j - \mu_0)^2.$$

Case UU: μ and σ both unknown

$$Q_r(X_r) = \Phi^{-1}\left\{G_{r-2}\left[\left(\frac{r-1}{r}\right)^{1/2}\left(\frac{X_r - \bar{X}_{r-1}}{S_{r-1}}\right)\right]\right\}, \; r = 3, 4, \ldots \quad (4)$$

The statistic for Case KK is similar to that of the conventional control chart since the values of the mean and variance are both known. The first value of the Q statistic for Cases UK and KU cannot be obtained. Similarly, the first and second values of the Q statistics for Case UU cannot be computed. This is due to the existence of unknown parameters for these three cases. The result of Case UU is very important since it can be used to control the mean of a normally distributed process at the beginning of a process, where both the mean and standard deviation are unknown. Case UU solves the short runs control charting problem.

A SIMULATION STUDY

We will now study the performance of the Q chart for individual measurements, based on the Q statistics in Equations (1), (2), (3) and (4), for cases with known and unknown mean and variance. The performance of the Q chart is evaluated using the 1-of-1, 3-of-3, 9-of-9 and 4-of-5 tests.

We consider a one step permanent shift in the mean from a nominal value, $\mu = \mu_0$ to a new value, $\mu = \mu_0 + \delta\sigma$. By examining the formulae of the Q statistics for cases KK, UK and KU, we observe that a shift in the mean will increase the mean of the Q statistic when δ is positive and decrease the mean of the Q statistic when δ is negative, for all values of r. However, for case UU, the effect of a mean shift will not only cause the numerator $\left(X_r - \bar{X}_{r-1}\right)$ to shift due to the direction of the

mean shift, but it will also affect the denominator, S_{r-1}. From the formulae, $\bar{X} = \frac{1}{r}\sum_{j=1}^{r} X_j$ and $S_r^2 = \frac{1}{r-1}\sum_{j=1}^{r}\left(X_j - \bar{X}\right)^2$, for r = 2, 3, ... If the shift occurs at $r = c$, it can be seen that the values of S_r^2 for $r \geq c$, will tend to be inflated due to the shift and the inflation increases as r increases. This tendency to increase the denominator of the argument in Equation (4) in turn decreases the Q statistic itself and decreases the probability of detecting the shift by the 1-of-1 test, after the point on which the shift first occurs (Quesenberry, 1995).

After a shift in μ, the Q statistics for the 3 cases with at least one unknown parameter (i.e., Cases UK, KU and UU) are no longer independent standard normal statistics. İf μ shifts from $\mu = \mu_0$ to $\mu = \mu_0 + \delta\sigma$, immediately after $r = c$, the distribution of these 3 cases are dependent on c. For each value of $c \in \{5, 20, 100\}$ and $\delta \in \{0, 0.5, 1.0, 2.0, 3.0, 4.0\}$, c observations were generated from the $N(0, 1)$ distribution and 30 subsequent observations, i.e., from c+1 onwards were generated from a $N(\delta, 1)$ distribution. This was repeated 5000 times and the number of observations, from c+1 onwards, required to signal the first out-of-control will be used as a performance assessment for the Q chart. The average number of observations that are needed by the Q chart to signal an out-of-control signal is called the average run length (ARL) of the chart. This simulation study discusses the effect of an increase in the process mean, i.e., when $\delta > 0$. The ARL values of the Q chart are computed using the SAS program based on 5000 simulation trials.

Table 1 shows the computed ARLs for the Q chart, used in the monitoring of the process mean, based on individual measurements. From this table, note that for a fixed value of δ, the ARL values for Case KK are approximately the same and do not depend on c. For instance, for the 1-of-1 test with δ = 1, the ARLs for c = 5, 20 and 100 are 43.8 43.7 and 44.4, respectively. Clearly, these ARLs are all estimates of the same value from different sets of 5000 samples. The case with known values of parameters performs better than the case when parameters are unknown. However, the performance for each of the tests for cases with at least one unknown parameter improve as c increases. For a large value of c, the performances of Cases UK, KU and UU will approach the performance of Case KK.

For all cases of $\delta < 2$, the classical 1-of-1 test is not competitive compared to the other tests in detecting a one step permanent shift in the process mean. Overall, the out-of-control ARL for Case KU is lower than that of Case UK, except for the 1-of-1 test, which means that Case UK has a lower power in detecting a shift in the mean than Case KU. This shows that it is more important to know the nominal value μ_0 of the mean than to know the nominal value of the standard deviation, σ_0, in the detection of a shift in the process mean. Also, Case UU always has the lowest sensitivity in detecting a shift in the mean, even though this case resembles a real situation, where both values of the parameters are unknown before a production run begins.

COMMENTS AND CONCLUSION

Short runs production is a new trend in most manufacturing industries. Since Shewhart control charts are not suitable to be used in a low volume manufacturing, short runs control charts are becoming more important. However, research in this area is still lacking. Topics for further research will be discussed in this section.

Most short runs control charts assume that the process data are normally distributed. However, in many situations, the normality assumption does not hold. For instance, data from chemical processes are usually nonnormal. Even though increasing the sample size is one method of solving this problem, but it is not suitable in short runs since the data are limited. İn addition, this method will also increase cost. Thus, research in short runs for nonnormal data is needed.

Until now, most research in short runs are focusing on using control charts for process monitoring. On the other hand, research in process capability involving short runs is still limited. Hence, research in this area is deemed necessary.

Table 1. ARL profiles for the Q chart based on individual measurements when c = 5, 20, 100

	δ	0.0				0.5				1.0				2.0				3.0				4.0			
	Cases / Tests	KK	UK	KU	UU	KK	UK	KU	UU	KK	UK	KU	UU	KK	UK	KU	UU	KK	UK	KU	UU	KK	UK	KU	UU
c = 5	1-of-1	747.5	744.5	740.5	735.4	160.5	701.5	439.6	724.7	43.8	641.4	1506.7	745.7	6.34	423.0	53263.7	868.1	2.0	128.1	1.12	1013.3	1.2	12.5	1.0	1099.3
	9-of-9	1022.5	1027.3	1023.0	1028.8	83.9	937.2	84.0	939.4	22.0	801.5	22.1	805.8	9.1	445.4	9.2	450.9	8.1	158.8	8.1	161.5	8.0	50.1	8.1	50.0
	3-of-3	295.3	293.4	293.9	293.0	48.1	251.1	65.2	258.6	13.6	191.9	27.3	230.4	4.1	66.9	8.8	189.4	3.0	11.1	4.4	166.5	2.8	3.2	3.3	163.1
	4-of-5	576.5	578.1	571.9	580.4	61.8	509.8	89.8	527.0	14.8	397.2	33.7	478.7	4.7	144.6	9.9	414.1	3.7	21.8	5.2	372.1	3.6	4.7	4.1	64.43
c = 20	1-of-1	746.5	744.6	737.9	733.5	160.4	623.0	401.7	677.7	43.7	453.6	1187.0	708.0	6.2	99.1	21056.8	884.8	2.0	3.5	1.8	808.0	1.2	1.2	1.3	390.3
	9-of-9	1024.6	1025.3	1024.6	1025.3	83.5	742.1	83.5	742.1	21.8	385.6	21.8	385.6	9.2	26.1	9.2	26.1	8.1	8.6	8.1	8.6	8.0	8.1	8.0	8.1
	3-of-3	294.7	292.3	291.9	292.3	47.7	187.6	8.85	200.0	13.6	76.3	20.0	122.5	4.0	6.1	4.8	27.6	3.0	3.1	3.1	3.1	2.8	2.8	2.9	3.0
	4-of-5	580.0	578.2	573.6	578.0	60.4	381.7	80.0	416.9	14.7	165.4	22.7	259.1	4.7	7.8	5.5	52.2	3.7	3.9	3.9	8.0	3.6	3.7	3.7	3.7
c = 100	1-of-1	746.7	746.7	742.0	736.4	162.3	432.9	309.5	535.9	44.4	168.4	447.3	495.7	6.2	8.1	1597.1	369.7	2.0	2.0	2.4	46.3	1.2	1.2	1.2	1.3
	9-of-9	1031.3	1034.9	1031.3	1034.9	82.6	341.0	82.6	341.0	22.2	46.7	22.2	46.7	9.1	9.5	9.1	9.5	8.0	8.1	8.0	8.1	8.0	8.0	8.0	8.0
	3-of-3	292.4	288.2	290.5	289.9	46.2	92.4	51.4	104.2	13.8	19.2	14.7	25.3	4.1	4.3	4.2	4.4	3.0	3.0	3.0	3.0	2.8	2.8	2.8	2.8
	4-of-5	576.1	574.8	574.1	574.5	60.0	173.8	68.7	203.1	14.9	22.9	16.5	36.5	4.7	4.8	4.7	5.0	3.7	3.8	3.8	3.9	3.6	3.6	3.6	3.6

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Comparing the performances of the optimal SS-DEWMA and Max-DEWMA control charts

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ABSTRACT

The single variable control charts, i.e., the SS-DEWMA (called the Sum of Squares Double Exponentially Weighted Moving Average) and Max-DEWMA (called the Maximum Double Exponentially Weighted Moving Average) charts have the advantage of enabling a simultaneous monitoring of the process mean and variance. In this study, the performance of the SS-DEWMA chart is compared with that of the Max-DEWMA chart via a Monte Carlo simulation. The optimal SS-DEWMA chart is generally shown to give the most favorable results, in terms of average run length.

Keywords: *Average run length (ARL), DEWMA chart, single control chart.*

Introduction

The exponentially weighted moving average (EWMA) chart, first introduced by Roberts [14] has a fairly long history. During the last decade, the application of the EWMA statistic as a process monitoring tool has become increasingly popular in the field of statistical process control (SPC). However, two EWMA charts are required to run concurrently to monitor the process mean and the process variance. Using two charts to monitor the process is inconvenient, besides consuming more resources, time, manpower and cost.

According to Gan [7], by referring to either the mean or the variance chart alone without making reference to the other one might mislead quality control engineers into making a wrong decision. In real life (especially in industrial processes), the process variance tends to increase with the process mean. An ideal situation is a decrease in the variance when the mean is in control, but the situation is undesirable if a decrease in the variance is accompanied by a decrease in the mean. Gan [6] and Gan, Ting and Chang [8] gave an example of a process with a decrease in the variance accompanied by a change in the mean and showed that the state of this process is difficult to detect. For this case, the mean chart becomes insensitive to the change in the process mean because the variance of the sample mean has decreased. Any detection of the mean with a decrease in the variance could lead to a false conclusion that the process mean has improved. In other words, the problem of monitoring the process mean and variance is a bivariate one, and both the mean and variance charts need to be looked at jointly, in order to make meaningful inferences. This led to the development of single EWMA-type charts to simultaneously monitor the process mean and/or the process variance. Many researchers have contributed to the development of single EWMA-type charts (see for example, [1, 2, 4-7, 10-13]). A comprehensive review of single EWMA-type control charts has been discussed in Cheng and Thaga [3] and Khoo, Teh and Wu [9].

Recently, the single Max-DEWMA (called the Maximum Double Exponentially Weighted Moving Average) chart proposed by Khoo et al. [9] which simultaneously detects shifts in the process mean and/or variability was shown to outperform the Max-EWMA chart, proposed by Chen, Cheng and Xie [2], for small and moderate shifts. More recently, Teh, Khoo and Wu [15] proposed another single SS-DEWMA (called the Sum of Squares Double Exponentially Weighted Moving Average)

chart which outperforms the SS-EWMA chart proposed by Xie [16], in detecting shifts of all sizes in the mean and/or variance.

The primary objective of this study is to conduct a comparative study of the performances of two weighted control charts, i.e., the SS-DEWMA and the Max-DEWMA charts, for monitoring the process mean and/or variance of a normally distributed process. An efficient control chart is the one that detects instantaneously a process shift and adopts the necessary corrective actions to improve process quality. The average run lengths (ARLs) are obtained through a Monte Carlo simulation to measure the speed of the charts in detecting the occurrence of assignable cause(s). Since the process mean and variance are allowed to change for various sizes of shifts, the performances of each of the charts based on ARL can be compared, each run ends when the respective chart's statistic exceeds the chart's limits. Each ARL value is an average of 10,000 simulation trials. The rest of this paper is organized as follows: Section 2 reviews the Max-DEWMA and SS-DEWMA charts. The simulation conditions in this study are presented in Section 3. A simulation study is conducted to evaluate and compare the performances of the SS-DEWMA and Max-DEWMA charts in Section 4. Finally, concluding remarks are drawn in the last section.

A Review of the Max-DEWMA and SS-DEWMA charts

Khoo et al. [9] developed a single EWMA-type control chart, called the Max-DEWMA chart. Assume that a series of random observations, $X_{ij} \sim N\left(\mu + a\sigma, b^2\sigma^2\right)$, for $i = 1, 2, 3, \ldots$ and $j = 1, 2, 3, \ldots, n_i$, where a and b are constants and $b \geq 1$. The process is in-control when $a = 0$ and $b = 1$; otherwise the process has shifted. Let

$$\bar{X}_i = \frac{X_{i1} + X_{i2} + \ldots + X_{in_i}}{n_i} \tag{1}$$

and

$$S_i^2 = \frac{\sum_{j=1}^{n_i}\left(X_{ij} - \bar{X}_i\right)^2}{n_i - 1} \tag{2}$$

be the mean and variance of sample i, respectively. The following two independent statistics are defined:

$$U_i = \frac{\bar{X}_i - \mu}{\sigma/\sqrt{n_i}} \tag{3}$$

and

$$V_i = \Phi^{-1}\left\{H\left[\frac{(n_i - 1)S_i^2}{\sigma^2}; n_i - 1\right]\right\}. \tag{4}$$

Note that in Equations (3) and (4), $\Phi^{-1}(\cdot)$ denotes the inverse standard normal distribution function and $H(w; v)$ the chi-square distribution function with v degrees of freedom. When the process is in-control, both U_i and V_i in Equations (3) and (4), respectively, are independent statistics having a common standard normal distribution. The two EWMA statistics computed from U_i and V_i are defined as follow:

$$Y_i = (1-\lambda)Y_{i-1} + \lambda U_i, \quad \text{for } i = 1, 2, \ldots \tag{5}$$

and

$$Z_i = (1-\lambda)Z_{i-1} + \lambda V_i, \quad \text{for } i = 1, 2, \ldots . \tag{6}$$

Here, $0 < \lambda \leq 1$ is the smoothing constant, while $Y_0 = 0$ and $Z_0 = 0$ are the starting values of Y_i and Z_i, respectively. From the two EWMA statistics of Y_i and Z_i, given in Equations (5) and (6), respectively, two corresponding DEWMA statistics can be computed as follow:

$$W_i = (1-\lambda)W_{i-1} + \lambda Y_i, \quad \text{for } i = 1, 2, \ldots \tag{7}$$

and

$$Q_i = (1-\lambda)Q_{i-1} + \lambda Z_i, \quad \text{for } i = 1, 2, \ldots \tag{8}$$

Here, W_0 and Q_0 $(W_0 = Q_0 = 0)$ are the starting values of W_i and Q_i, respectively. Khoo et al. [9] uses the simple approach of setting the smoothing constants the same for Y_i, Z_i, W_i and Q_i, in Equations (5), (6), (7) and (8), respectively. Then the two DEWMA statistics in Equations (7) and (8) are combined into the following single statistic for the proposed Max–DEWMA chart.

$$M_i = max\{|W_i|, |Q_i|\}. \tag{9}$$

Because the proposed chart is based on M_i, i.e., the maximum of $|W_i|$ and $|Q_i|$, it is called the Max-DEWMA chart. If the process mean has shifted away from its target value and/or the variance has increased or decreased, the statistic M_i, will be large. If both the process mean and variance stay close to their respective target values, then M_i will be small. Note that M_i is the maximum of the absolute values of the two DEWMA statistics and it is non-negative. Therefore, the Max-DEWMA chart needs only an upper control limit (UCL_{MD}), which is given in Khoo et al. (in press).

$$UCL_{MD} = E(M_i) + K_{MD}\sqrt{V(M_i)}$$

$$= (1.128379 + 0.602810\, K_{MD}) \times$$

$$\sqrt{\frac{\lambda^4}{\left[1-(1-\lambda)^2\right]^3}\left\{1+(1-\lambda)^2-(i^2+2i+1)(1-\lambda)^{2i}+(2i^2+2i-1)(1-\lambda)^{2i+2}-i^2(1-\lambda)^{2i+4}\right\}}, \tag{10}$$

for $i = 1, 2, \ldots$, where $E(M_i)$ and $V(M_i)$ are the mean and variance of M_i, respectively, when the process is in-control, while K_{MD} is a multiplier that controls the width of UCL_{MD}. Khoo et al. [9] show that the Max–DEWMA chart performs better than the Max–EWMA chart in detecting small and moderate shifts in the mean and/or variance. A quick detection of small and moderate shifts in the mean and/or variance can be made by choosing λ in the interval $0.05 \leq \lambda \leq 0.30$.

More recently, instead of taking the maximum of $|W_i|$ and $|Q_i|$, Teh et al. [15] also explored the idea of using the sum of squares of W_i and Q_i in Equations (7) and (8). They introduced the following single statistic for the proposed SS–DEWMA chart:

$$L_i = W_i^2 + Q_i^2 \tag{11}$$

Similar to the Max-DEWMA chart, the values of L_i will be large if the mean and/or variance has shifted from their target values. However, L_i will be small if both the mean and variance stay close to their respective target values.

Since L_i is non-negative, the SS–DEWMA chart only has an upper control limit, given by

$$\mathrm{UCL_{SD}} = E(L_i) + K_{\mathrm{SD}}\sqrt{Var(L_i)}\,, \tag{12}$$

where $E(L_i)$ and $Var(L_i)$ are the in-control mean and variance of L_i, respectively. Note that K_{SD} is a constant controlling the width of $\mathrm{UCL_{SD}}$. A formula which enables a quick computation of $\mathrm{UCL_{SD}}$ for the initial state SS–DEWMA chart, based on desired λ and K_{SD} values is given in Equation (13).

$$\mathrm{UCL_{SD}} = 2(1+K_{\mathrm{SD}})\times\frac{\lambda^4}{\left[1-(1-\lambda)^2\right]^3}\left\{1+(1-\lambda)^2-(i^2+2i+1)(1-\lambda)^{2i}+(2i^2+2i-1)(1-\lambda)^{2i+2}-i^2(1-\lambda)^{2i+4}\right\} \tag{13}$$

The derivation of $\mathrm{UCL_{SD}}$ is shown in Teh et al. [15]. As a rule of thumb, λ in the interval $0.05 \le \lambda \le 0.30$ is usually considered for a quick detection of small and moderate shifts in the mean and/or variance. Teh et al. [15] showed that the SS-DEWMA chart outperforms the SS-EWMA chart proposed by Xie [16], in detecting shifts of all sizes in the mean and/or variance.

Evaluation and Comparison of the Performances of the SS-DEWMA and Max-DEWMA Charts

One of the most common measures of a control chart's performance is the ARL, which is defined as the average number of points that must be plotted on the chart before an out-of-control signal is observed. When the process is in-control, the ARL should be sufficiently large to avoid false alarms. When the process is out-of-control, the chart should have a small ARL, so that an out-of-control condition can be quickly detected. Without loss of generality, we set $\mu_0 = 0$ and $\sigma^2 = 1$ for the in-control process in this study. The out-of-control condition is represented by a shift in the mean and/or variance. When an assignable cause is detected, the process goes out-of-control which indicates that either the mean and/or the variance have shifted. The performances of the SS-DEWMA and Max-DEWMA charts are compared by designing charts with a common in-control ARL, $\mathrm{ARL_0} = 250$ and then compare the out-of-control ARLs, $\mathrm{ARL_1}$s of the charts, for a given shift in the process. The sample size, $n = 5$ is considered here. The exact limits of both charts are considered.

Monte Carlo simulations are performed herein to estimate the ARLs. The simulations are conducted using the Statistical Analysis Software (SAS) version 9.2 to compare the performances of the optimal SS-DEWMA and optimal Max-DEWMA charts for detecting shifts in the process mean and/or variance. Each simulation run comprises 100,000 iterations, where the normal random variates are generated using the RANNOR function in the SAS library. Without loss of generality, in comparing the performances of the charts, the smallest $\mathrm{ARL_1}$ is computed for each combination of the shift (a, b) for the two charts. For example, for the optimal SS-DEWMA chart, this is achieved by computing $\mathrm{ARL_1}$s corresponding to a shift (a, b) for the various combinations of $(\lambda, K_{\mathrm{SD}})$, where $\lambda \in \{0.050, 0.055, 0.060, \ldots, 1\}$, followed by identifying the smallest $\mathrm{ARL_1}$ and its corresponding optimal $(\lambda, K_{\mathrm{SD}})$ combination. A similar approach is applied for the optimal Max-DEWMA chart. Each trial ends if an out-of-control point is detected by the optimal SS–DEWMA chart, i.e., when $L_i > \mathrm{UCL_{SD}}$ and by the optimal Max-DEWMA chart if $M_i > \mathrm{UCL_{MD}}$.

Table 1: (λ, K_{SD}) combinations and the corresponding ARLs for the optimal **SS-DEWMA** control schemes, based on an initial state mode with a sample size of $n = 5$ and $ARL_0 = 250$

						a					
b		0.00	0.05	0.10	0.25	0.50	1.00	1.50	2.00	2.50	3.00
0.25	λ	0.050	0.050	0.050	0.050	0.050	0.055	0.190	0.530	0.530	0.530
	K_{SD}	1.677	1.677	1.677	1.677	1.677	1.759	3.060	4.248	4.248	4.248
	ARL	1.30	1.31	1.30	1.25	1.13	1.00	1.00	1.00	1.00	1.00
0.50	λ	0.050	0.050	0.050	0.050	0.050	0.050	0.055	0.130	0.475	0.475
	K_{SD}	1.677	1.677	1.677	1.677	1.677	1.677	1.759	2.634	4.217	4.217
	ARL	3.28	3.26	3.21	2.90	2.16	1.21	1.00	1.00	1.00	1.00
0.75	λ	0.050	0.050	0.050	0.050	0.050	0.050	0.050	0.050	0.180	0.530
	K_{SD}	1.677	1.677	1.677	1.677	1.677	1.677	1.677	1.677	2.999	4.248
	ARL	12.60	12.28	11.45	7.92	3.98	1.52	1.06	1.00	1.00	1.00
0.80	λ	0.050	0.050	0.050	0.050	0.050	0.050	0.050	0.055	0.120	0.530
	K_{SD}	1.677	1.677	1.677	1.677	1.677	1.677	1.677	1.759	2.549	4.248
	ARL	18.26	17.69	16.00	9.94	4.36	1.58	1.07	1.00	1.00	1.00
0.90	λ	0.050	0.050	0.050	0.050	0.050	0.050	0.050	0.055	0.150	0.560
	K_{SD}	1.677	1.677	1.677	1.677	1.677	1.677	1.677	1.759	2.797	4.293
	ARL	55.12	47.80	36.32	14.30	5.96	1.66	1.10	1.00	1.00	1.00
0.95	λ	0.050	0.050	0.050	0.050	0.050	0.050	0.050	0.050	0.055	0.465
	K_{SD}	1.677	1.677	1.677	1.677	1.677	1.677	1.677	1.677	1.759	4.101
	ARL	133.86	98.82	55.04	15.87	5.13	1.69	1.11	1.01	1.00	1.00
1.00	λ	0.080	0.055	0.050	0.050	0.050	0.050	0.050	0.055	0.060	0.605
	K_{SD}	2.219	1.759	1.677	1.677	1.677	1.677	1.677	1.759	4.347	4.352
	ARL	250.0	143.61	64.33	18.76	5.19	2.11	1.12	1.01	1.00	1.00
1.05	λ	0.050	0.050	0.050	0.050	0.050	0.050	0.050	0.055	0.055	0.225
	K_{SD}	1.677	1.677	1.677	1.677	1.677	1.677	1.677	1.759	1.759	3.250
	ARL	107.15	80.62	48.59	15.17	5.14	1.73	1.13	1.01	1.00	1.00
1.10	λ	0.050	0.050	0.050	0.050	0.050	0.050	0.050	0.050	0.060	0.175
	K_{SD}	1.677	1.677	1.677	1.677	1.677	1.677	1.677	1.677	4.347	2.971
	ARL	44.63	39.67	30.84	13.13	4.94	1.73	1.14	1.02	1.00	1.00
1.25	λ	0.050	0.050	0.050	0.055	0.050	0.050	0.055	0.050	0.050	0.060
	K_{SD}	1.677	1.677	1.677	1.759	1.677	1.677	1.759	1.677	1.677	1.854
	ARL	10.92	10.81	10.25	7.48	4.03	1.70	1.16	1.02	1.000	1.00
1.50	λ	0.050	0.050	0.050	0.050	0.050	0.050	0.050	0.050	0.055	0.050
	K_{SD}	1.677	1.677	1.677	1.677	1.677	1.677	1.677	1.677	1.759	1.677
	ARL	3.65	3.62	3.63	3.31	2.59	1.57	1.17	1.04	1.01	1.00
2.00	λ	0.055	0.055	0.050	0.050	0.050	0.050	0.060	0.050	0.050	0.080
	K_{SD}	1.759	1.759	1.677	1.677	1.677	1.677	1.854	1.677	1.677	2.219
	ARL	1.58	1.57	1.58	1.55	1.47	1.28	1.13	1.05	1.01	1.00
2.50	λ	0.050	0.050	0.050	0.050	0.050	0.050	0.060	0.060	0.050	0.050
	K_{SD}	1.677	1.677	1.677	1.677	1.677	1.677	1.854	1.854	1.677	1.677
	ARL	1.19	1.19	1.19	1.19	1.17	1.27	1.08	1.04	1.02	1.00
3.00	λ	0.050	0.050	0.050	0.050	0.050	0.050	0.060	0.070	0.060	0.050
	K_{SD}	1.677	1.677	1.677	1.677	1.677	1.677	4.347	1.997	4.347	1.677
	ARL	1.08	1.08	1.08	1.08	1.08	1.06	1.04	1.03	1.01	1.01
4.00	λ	0.055	0.055	0.055	0.060	0.050	0.055	0.055	0.085	0.085	0.085
	K_{SD}	1.759	1.759	1.759	4.347	1.677	1.759	1.759	2.181	2.181	2.181
	ARL	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.01	1.01	1.00

Table 2: (λ, K_{MD}) combinations and the corresponding ARLs for the optimal **Max-DEWMA** control schemes, based on an initial state mode with a sample size of $n = 5$ and $ARL_0 = 250$

						a					
b		0.00	0.05	0.10	0.25	0.50	1.00	1.50	2.00	2.50	3.00
0.25	λ	0.050	0.050	0.050	0.050	0.050	0.050	0.525	0.525	0.525	0.525
	K_{MD}	1.638	1.638	1.638	1.638	1.638	1.638	3.108	3.108	3.108	3.108
	ARL	1.18	1.18	1.18	1.18	1.18	1.06	1.00	1.00	1.00	1.00
0.50	λ	0.050	0.050	0.050	0.050	0.050	0.050	0.050	0.525	0.525	0.525
	K_{MD}	1.638	1.638	1.638	1.638	1.638	1.638	1.638	3.108	3.108	3.108
	ARL	2.93	2.93	2.93	2.92	2.63	1.35	1.00	1.00	1.00	1.00
0.75	λ	0.050	0.050	0.050	0.050	0.050	0.050	0.050	0.105	0.525	0.525
	K_{MD}	1.638	1.638	1.638	1.638	1.638	1.638	1.638	2.110	3.108	3.108
	ARL	11.75	11.68	11.46	9.18	4.37	1.54	1.05	1.00	1.00	1.00
0.80	λ	0.050	0.050	0.050	0.050	0.050	0.050	0.050	0.070	0.525	0.525
	K_{MD}	1.638	1.638	1.638	1.638	1.638	1.638	1.638	1.849	3.108	3.108
	ARL	17.50	17.31	16.60	11.39	4.63	1.58	1.06	1.00	1.00	1.00
0.90	λ	0.050	0.050	0.050	0.050	0.050	0.050	0.050	0.055	0.525	0.525
	K_{MD}	1.638	1.638	1.638	1.638	1.638	1.638	1.638	1.701	3.108	3.108
	ARL	53.20	49.03	39.23	15.23	5.00	1.64	1.09	1.00	1.00	1.00
0.95	λ	0.050	0.050	0.050	0.050	0.050	0.050	0.055	0.055	0.480	0.525
	K_{MD}	1.638	1.638	1.638	1.638	1.638	1.638	1.701	1.701	3.066	3.108
	ARL	131.77	100.56	57.06	16.00	5.10	1.67	1.11	1.00	1.00	1.00
1.00	λ	0.655	0.050	0.050	0.050	0.050	0.050	0.050	0.070	0.335	0.525
	K_{MD}	3.198	1.638	1.638	1.638	1.638	1.638	1.638	1.849	2.859	3.108
	ARL	249.99	138.98	64.08	15.96	5.11	1.69	1.12	1.01	1.00	1.00
1.05	λ	0.055	0.050	0.050	0.050	0.050	0.050	0.050	0.050	0.195	0.525
	K_{MD}	1.701	1.638	1.638	1.638	1.638	1.638	1.638	1.638	2.509	3.108
	ARL	106.43	82.75	50.56	15.42	5.06	1.72	1.13	1.01	1.00	1.00
1.10	λ	0.050	0.050	0.050	0.050	0.050	0.050	0.050	0.050	0.160	0.525
	K_{MD}	1.638	1.638	1.638	1.638	1.638	1.638	1.638	1.638	2.387	3.108
	ARL	44.51	40.90	32.44	13.90	4.99	1.73	1.14	1.01	1.00	1.00
1.25	λ	0.050	0.050	0.050	0.055	0.050	0.050	0.050	0.055	0.075	0.525
	K_{MD}	1.638	1.638	1.638	1.701	1.638	1.638	1.638	1.701	1.898	3.108
	ARL	11.17	11.02	10.51	8.02	4.31	1.73	1.17	1.03	1.00	1.00
1.50	λ	0.050	0.050	0.050	0.050	0.050	0.050	0.050	0.060	0.085	0.200
	K_{MD}	1.638	1.638	1.638	1.638	1.638	1.638	1.638	1.756	1.968	2.528
	ARL	3.73	3.73	3.69	3.45	2.73	1.62	1.18	1.05	1.01	1.00
2.00	λ	0.050	0.050	0.050	0.050	0.050	0.050	0.050	0.055	0.065	0.055
	K_{MD}	1.638	1.638	1.638	1.638	1.638	1.638	1.638	1.701	1.806	1.701
	ARL	1.62	1.61	1.61	1.58	1.51	1.32	1.15	1.06	1.02	1.00
2.50	λ	0.050	0.050	0.050	0.050	0.050	0.050	0.055	0.060	0.060	0.100
	K_{MD}	1.638	1.638	1.638	1.638	1.638	1.638	1.701	1.756	1.756	2.082
	ARL	1.21	1.21	1.21	1.20	1.19	1.14	1.09	1.04	1.02	1.01
3.00	λ	0.050	0.050	0.050	0.050	0.050	0.055	0.060	0.050	0.065	0.085
	K_{MD}	1.638	1.638	1.638	1.638	1.638	1.701	1.756	1.638	1.806	1.968
	ARL	1.09	1.09	1.09	1.09	1.08	1.07	1.05	1.03	1.02	1.01
4.00	λ	0.055	0.055	0.055	0.055	0.060	0.070	0.075	0.080	0.065	0.080
	K_{MD}	1.701	1.701	1.701	1.701	1.756	1.849	1.898	1.932	1.806	1.932
	ARL	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.01	1.01	1.01

Tables 1 and 2 give the ARL results for the optimal SS-DEWMA and Max-DEWMA charts, respectively. Generally, it shows that for a shift combination (a, b), the two charts are optimal at about the same value of λ. It is seen that small values of λ are optimal in detecting most of the shift combinations (a, b). However, for some cases with larger a, say $a > 2$, larger optimal values of λ are observed. About 41.33% of the ARL_1s in Table 1 are lower than the corresponding ARL_1s in Table 2, indicating that the optimal SS-DEWMA chart is more sensitive than the optimal Max-DEWMA chart, in the detection of shifts. As an example, for $\lambda = 0.05$ in Table 2, when $b = 0.90$ and $0.05 \le a \le 0.25$, the ARLs for the optimal SS-DEWMA chart are 47.80, 36.32 and 14.30, while the corresponding

ARLs for the optimal Max-DEWMA chart are 49.03, 39.23 and 15.23, for a = 0.05, 0.10 and 0.10, respectively. In other words, the speed to signal an out-of-control increases when the SS-DEWMA chart is adopted. As for the remaining ARL_1s, in about one third of them, both Tables 1 and 2, have almost the same values. It should be noted that considering other values of ARL_0s will also give similar conclusions.

Table 3: A comparison of the diagnostic abilities of the SS-DEWMA and Max-DEWMA charts

b		SS-DEWMA a					Max-DEWMA a				
		0.00	0.25	0.50	1.00	2.00	0.00	0.25	0.50	1.00	2.00
	m+	0	0	0	49	577	0	0	0	6	1000
	m-	3	0	0	0	0	0	0	0	0	0
	v+	0	0	0	0	0	0	0	0	0	0
	v-	997	905	715	385	0	1000	1000	1000	524	0
0.25	++	0	0	0	0	0	0	0	0	0	0
	+-	0	95	285	566	423	0	0	0	470	0
	-+	0	0	0	0	0	0	0	0	0	0
	--	0	0	0	0	0	0	0	0	0	0
	m+	0	0	14	465	937	0	0	35	940	1000
	m-	55	0	0	0	0	0	0	0	0	0
	v+	0	0	0	0	0	6	6	5	0	0
	v-	945	579	246	65	0	994	994	882	4	0
0.50	++	0	0	0	0	0	0	0	0	0	0
	+-	0	420	740	470	63	0	0	78	56	0
	-+	0	0	0	0	0	0	0	0	0	0
	--	0	1	0	0	0	0	0	0	0	0
	m+	256	902	939	977	988	239	852	919	990	1000
	m-	249	4	1	0	0	255	120	76	10	0
	v+	248	38	27	6	0	236	11	1	0	0
	v-	247	56	33	11	1	270	16	3	0	0
1.00	++	0	0	0	3	4	0	1	0	0	0
	+-	0	0	0	3	7	0	0	1	0	0
	-+	0	0	0	0	0	0	0	0	0	0
	--	0	0	0	0	0	0	0	0	0	0
	m+	97	135	258	559	831	5	72	310	843	997
	m-	93	13	4	1	0	11	18	51	36	2
	v+	791	409	277	118	14	884	797	476	48	0
	v-	3	2	4	3	1	95	82	64	8	0
1.50	++	14	366	427	304	150	0	18	80	52	1
	+-	0	4	5	13	4	2	0	8	10	0
	-+	2	68	25	2	0	3	11	11	3	0
	--	0	3	0	0	0	0	2	0	0	0
	m+	90	115	173	336	531	2	14	52	319	892
	m-	95	35	17	4	1	7	4	12	26	7
	v+	730	546	468	321	73	933	915	839	443	22
	v-	1	2	2	2	1	52	48	46	20	0
2.00	++	36	203	274	321	392	0	9	41	165	73
	+-	0	3	6	2	1	0	2	0	7	6
	-+	47	96	60	14	1	5	7	9	20	0
	--	1	0	0	0	0	1	1	1	0	0

Next, we study the diagnostic abilities of the SS-DEWMA and Max-DEWMA charts. For each pair of shift combination (a, b), 1000 out-of-control signals are simulated for each of the charts. The rules used in counting the number of out-of-control signals detected by the SS-DEWMA and Max-DEWMA charts are described in Khoo et al. [9] and Teh et al. [15], respectively. The charts

incorporating the exact limits, where $\lambda = 0.1$ and $n = 5$ are considered. Here, $K_{SD} = 2.348$ and $K_{MD} = 2.082$ are employed for the SS-DEWMA and Max-*D*EWMA charts, respectively, so that $ARL_0=250$. The results for the diagnostic abilities of the charts are presented in Table 3. Here, the numerical values for a shift combination (a, b) represents the number of times an out-of-control signal is detected by the respective rules. For example, when $(a, b) = (0, 1)$, the SS-DEWMA chart detects 256, 249, 248 and 247 out-of-control signals as an increase in only the mean, decrease in only the mean, increase in only the variance and decrease in only the variance, respectively, based on a total of 1000 out-of-control signals. Overall, it is shown that the SS–DEWMA chart has better diagnostic abilities for the out-of-control cases compared to Max-DEWMA chart. The Max-DEWMA chart has better diagnostic abilities than the SS-DEWMA chart only when an increase in only the variance or a decrease in only the variance occur, while the mean is in control.

From the above discussion, it is found that the optimal SS-DEWMA chart is quicker in detecting shifts compared to the optimal Max-DEWMA chart. It is found that the SS-DEWMA chart has better diagnostic abilities for the out-of-control cases compared to the Max-DEWMA chart. Thus, the SS-DEWMA chart is superior to the Max-DEWMA chart, in terms of the out-of-control detection speed and diagnostic abilities.

Concluding Remarks

An efficient control chart is the one that requires less time to detect shifts in the process mean and/or variance and has superior diagnostic ability. A comparative study of the weighted control charts, i.e., the SS-DEWMA and Max-DEWMA charts shows that the SS-DEWMA chart is superior for a simultaneous monitoring of the process mean and/or variance, for a normally distributed process. Like other EWMA type single variable charts, the SS-DEWMA control chart is not only able to monitor both the process mean and/or process variability simultaneously, but it also has the desirable properties of an EWMA chart. The SS-DEWMA chart allows one to monitor both the process mean and/or process variability by looking at only one chart with a single statistic. Another advantage is that the charting technique is efficient and easy to implement, so that one can quickly identify the source and direction of an out-of-control signal, based on the out-of-control point. The SS-DEWMA chart is best implemented on a common PC or notebook, so that all the computations and graphing aspects can be made quickly. The development of simple quality-control software tools and the use of online computers, have all led to the practical utilization of statistical process monitoring techniques.

Future works on this topic that are worthy of pursuing are as follow:

(i) To compare the performances of the SS-DEWMA and Max-DEWMA charts for skewed populations, in terms of ARL, median run length (MRL), standard deviation of the run length (SDRL) and percentage points of the run length distribution.

(ii) To investigate the performances of SS-DEWMA and Max-DEWMA charts for the steady state mode, as we only consider the initial state mode in this study.

(iii) To measure the performances of the SS-DEWMA and Max-DEWMA charts when their smoothing constants, λ's, have different weights.

(iv) To study the performances of the SS-DEWMA and Max-DEWMA charts, based on an autocorrelated process, i.e., when the independence assumption is violated.

(v) To compare the performances of the SS-DEWMA and Max-DEWMA charts, based on heavy tailed distributions, such as the Student-*t* or Cauchy distributions.

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BIL : USM/KP 2649 TARIKH : 3 November 2008

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3.	**Tarikh Lahir**	:	**14 Ogos 1984**
4.	**Taraf Perkahwinan**	:	**Bujang**
5.	**Jawatan**	:	**Pegawai Penyelidik Sambilan**
6.	**Gred Gaji**	:	**N41**
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	(b) No.Akuan Projek	:	**RU : 1001/PMATHS/811024**
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d/a Prof. Madya Dr. Khoo Boon Chong
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	(b) No.Akuan Projek	:	RU : 1001/PMATHS/811024
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16.	Keterangan Lain	:	-

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EFH/Liza/PE lanjutan (28/5/2009)

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KENYATAAN GAJI KAKITANGAN PROJEK SAMBILAN
(HENDAKLAH DIISI DALAM *5 SALINAN*)

Bil: 599-2010 Tarikh: 1/4/2010

Kepada : BENDAHARI

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☐ **Pelanjutan Pelantikan**
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4. Sila kembalikan borang yang lengkap sebelum 6hb (untuk lantikan berdasarkan gaji harian) dan 20hb (untuk lantikan berdasarkan gaji bulanan) kepada RCMO untuk proses pelantikan dan pembayaran gaji.
5. Kegagalan mengemukakan maklumat yang lengkap dan tepat pada tarikh yang ditetapkan akan menjejaskan proses pembayaran gaji untuk bulan berkenaan.

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3. PAS KESELAMATAN : Tuan/puan dikehendaki memohon Pas Keselamatan dari Jabatan Keselamatan dalam tempoh 7 hari dari tarikh lantikan, sekiranya tuan/puan seorang pelajar Universiti.

1. MAKLUMAT PEMOHON

Nama Penuh : Lim Poh Ying

Jantina : L/P

No.K/P (Baru) : 831117075370

Tarikh Lahir : 17/11/1983

Jawatan & Gred : RO.

No.Matrik Pelajar* : S-MM0025107

Alamat Tetap : 31, Jln Buah Limau, Paya Terubong, 11060, Penang

Alamat e-mail : pohying_my@yahoo.com

Taraf Perkahwinan : (Bujang) / Berkahwin

Pusat Pengajian : Matematik

No. Tel (Pejabat) : - H/P : 0125888713

No. PERKESO :

Faedah Persaraan/KWSP 11% : 12% No. KWSP : 16204351

Keputusan Peperiksaan *(Sila lampirkan satu salinan sijil)* : 3.11

Sila sertakan salinan Kad Pengenalan (yang disahkan) bersama

2. MAKLUMAT BANK

[/] **CIMB BANK**

[ ] **Bank Islam Malaysia Berhad**

[ ] **Bank Muamalat Malaysia Berhad**

[ ] **Malayan Banking Berhad (Maybank)**

[ ] **Lain-lain (sila nyatakan)**

Nombor Akaun Bank : 0 7 1 6 - 0 0 6 2 9 9 6 - 5 2 - 5

3. MAKLUMAT GERAN PENYELIDIKAN

Nama Ketua Projek : Prof. Madya Michael Khoo Boon Chong

Pusat Pengajian : Sains Matematik

Taraf Persaudaraan :

(Sila pastikan tuan/puan tidak melantik suami/isteri/anak)

Jangkamasa Perlantikan : 1 April 2010 Hingga : 30 Jun 2010

Tajuk Projek : A Neural Network Approach to Synthetic Control Chart for the Process Mean

No. Akaun Projek : 1001/PMATHS/811024

Sumber Pembiayaan Projek :

[ ]	JANGKA PENDEK	[ ]	FUNDAMENTAL (FRGS)	[ ]	FUNDAMENTAL (TOP DOWN)
[ ]	GERAN LUAR	[ ]	SCIENCE FUND	[✓]	RU
[ ]	LAIN-LAIN GERAN :				

[ ]	Gaji Harian			[ ]	Gaji Bulanan		
Gaji Sehari	:	RM		Gaji Sebulan	:	RM	1871-18
ITKA	:	RM		ITKA	:	RM	300·00
Bantuan Sara Hidup	:	RM		Bantuan Sara Hidup	:	RM	300·00

BAHAGIAN B : Untuk pelanjutan lantikan, sila lengkapkan bahagian ini.

I. **UNTUK DIISI OLEH CALON LANJUTAN**

1. Tempoh Lantikan :

a) Lantikan Pertama : Hingga :

b) Alasan Lanjutan :

II. **UNTUK DIISI OLEH PENYELIA/KETUA PROJEK**

1. Sila tanda (/) di petak yang berkenaan :-

	Kurang Memuaskan	Sederhana	Memuaskan	Baik	Sangat Baik
Kemajuan Calon					
Prestasi Calon					

(Tandatangan/Cop Ketua Projek)

s.k Dekan/Pengarah/Penyelaras
Pusat Pengajian/Pusat/Unit

Pusat Pengajian Sains Matematik
Universiti Sains Malaysia
11800 PULAU PINANG

* **Sila pastikan pelajar dibayar gaji harian sahaja -**

---

2. **UNTUK KEGUNAAN PEJABAT**

DILULUSKAN OLEH:

HASANAB HAMID
Pentadbir Pegawai Tadbir
Unit Pengurusan Geran & Kontrak
Pejabat Pengurusan Penyelidikan
Universiti Sains Malaysia

(Cop Pegawai RCMO)

6/4/2010

Tarikh

Bil. USM/SSP 913 Tarikh: 9 Jun 2008

UNIVERSITI SAINS MALAYSIA
KENYATAAN BANTUAN KEWANGAN

Kepada: BENDAHARI

LANJUTAN PERKHIDMATAN - SKIM SISWAZAH PEMBANTU (PENYELIDIKAN)

1. NAMA : ABDU MOHAMMED ATTA
2. NO. KAD PENGENALAN/NO. PASPORT: 01111426 JANTINA: LELAKI
3. TARAF PERKAHWINAN: BUJANG
4. JANGKAMASA PELANTIKAN: 9 MEI 2008 hingga 31 OKTOBER 2008
5. JABATAN/P.PENGAJIAN: PUSAT PENGAJIAN SAINS MATEMATIK
6. NAMA PROJEK/AKAUN : A NEURAL NETWORK APPROACH TO SYNTHETIC CONTROL CHART FOR THE PROCES MEAN (NO. AKAUN: 1001/PMATHS/811024)
7. ELAUN: RM 1,500.00 SEBULAN No. Akaun Bank BBMB: 0706-0000111-20-1
8. TARIKH PERLAKSANAAN: 9 MEI 2008
9. KETERANGAN LAIN: *SILA BATALKAN KENYATAAN BANTUAN KEWANGAN BERTARIKH 4/6/2008.....*

bp. DATIN HAJAH FAZIA ALI
Timbalan Pendaftar Kanan
(Pejabat Pengurusan & Kreativiti Penyelidikan]

s.k.

Prof. Madya Dr. Michael Khoo Boon Chong
Ketua Projek
Pusat Pengajian Sains Matematik

Encik Abdu Mohammed Ali Atta
d/a Prof. Madya Michael Khoo Boon Chong
Pusat Pengajian Sains Matematik

mo/ni..D:Lantikan Baru (SSP) 2007

UNIVERSITI SAINS MALAYSIA
PERMOHONAN SKIM SISWAZAH PEMBANTU [PENYELIDIKAN]

Kepada: BENDAHARI

(Terbuka kepada calon warganegara Malaysia dan Bukan Warganegara Malaysia tanpa pembiayaan dan penik masa sahaja. Sila kemukakan salinan transkrip akademik rasmi yang menunjukkan gred purata / CGPA bersama permohonan).

BAHAGIAN A (untuk diisi oleh calon)

1. Nama: ABDU MOHAMMED ALI ATTA [HURUF BESAR] Jantina [L/P]: MALE
2. No. Pasport/K/P [Baru/Lama]: 02946639 Tarikh Lahir: 3/4/1972
3. Alamat Rumah Tetap: DESA AMAN K-18 - C11 USM
   Taraf Perkahwinan: Sudah Kahwin/Belum
4. Ijazah: P.h.D Tarikh berdaftar: 1-12-2005
   [Lampirkan 1 salinan Ijazah]
5. Pusat Pengajian/Pusat: School of Mathemtical Sciences No Akaun Bank BBMB (staf): 0706-000111-20-1
6. Jenis Rancangan: full time
7. Bidang Pengajian/Penyelidikan: Quality Control
8. Tajuk Tesis/Disertasi: To be detemined soon
9. Nama Ketua Projek: Assoc. Prof. Michael Khoo Boon Chong
   Tajuk Projek: A Neural Network Approach to Synthetic Control Chart for the Process Mean.
10. Nama Penyelia Pengajian (jika berbeza daripada NO. 9) : -
    Sumber Pembiayaan Projek: [IRPA/GERAN LUAR] sekiranya GERAN LUAR, sila nyatakan nama penuh agensi luar tersebut.: Geran Universiti Penyelidikan (RU)
    No Akaun Projek: 1001/PMATHS/811024
11. Bantuan Kewangan: Sarjana RM - Doktor Falsafah: RM 625
12. Tarikh Perlaksanaan: Mula 1 Nov 2008 Hingga 30 April 2009

13. Biasiswa/Bantuan yang pernah diterima (jika ada):-

Biasiswa/Pembiayaan	Nilai	Tempoh (Tahun)	
		Mula	Hingga
Scholarship from Al-Hodaidah University (Republic of Yemen)	USD 520	Oct 2005	Feb 2008
Pembantu Siswazah		9 Mei 2008	31 Okt 2008

14. Bila dijangka tesis boleh diserah untuk pemeriksaan/menduduki peperiksaan akhir untuk pengijazahan
    April 2009.

Bil. USM/SSP 913 Tarikh: 28 April 2008

UNIVERSITI SAINS MALAYSIA
KENYATAAN BANTUAN KEWANGAN

Kepada: BENDAHARI

LANJUTAN PERKHIDMATAN - SKIM SISWAZAH PEMBANTU (PENYELIDIKAN)

1. NAMA : ABDU MOHAMMED ALI ATTA
2. NO. KAD PENGENALAN/NO. PASPORT: 01111426 JANTINA: LELAKI
3. TARAF PERKAHWINAN: BUJANG
4. JANGKAMASA PELANTIKAN: 1 MEI 2009 hingga 31 JULAI 2009
5. JABATAN/P.PENGAJIAN: PUSAT PENGAJIAN SAINS MATEMATIK
6. NAMA PROJEK/AKAUN : A NEURAL NETWORK APPROACH TO SYNTHETIC CONTROL CHART FOR THE PROCES MEAN (NO. AKAUN: 1001/PMATHS/811024)
7. ELAUN: RM 625.00 SEBULAN No. Akaun Bank BBMB: 0706-0000111-20-1
8. TARIKH PERLAKSANAAN: 1 MEI 2009
9. KETERANGAN LAIN:

ELLY FATMAWATEE HUSSEIN
Pegawai Kewangan
(Pejabat Pengurusan & Kreativiti Penyelidikan]

s.k.

Prof. Madya Dr. Michael Khoo Boon Chong
Ketua Projek
Pusat Pengajian Sains Matematik

Encik Abdu Mohammed Ali Atta
d/a Prof. Madya Michael Khoo Boon Chong
Pusat Pengajian Sains Matematik

/ni...D:Lanjutan (SSP) 2009

Bil. USM/SSP 913 Tarikh: 14 Disember 2009

UNIVERSITI SAINS MALAYSIA
KENYATAAN BANTUAN KEWANGAN

Kepada: BENDAHARI

LANJUTAN PERKHIDMATAN - SKIM SISWAZAH PEMBANTU (PENYELIDIKAN)

1. NAMA : ABDU MOHAMMED ALI ATTA
2. NO. KAD PENGENALAN/NO. PASPORT: 01111426 JANTINA: LELAKI
3. TARAF PERKAHWINAN: BUJANG
4. JANGKAMASA PELANTIKAN: 1 NOVEMBER 2009 HINGGA 31 JANUARI 2010
5. JABATAN/P.PENGAJIAN: PUSAT PENGAJIAN SAINS MATEMATIK
6. NAMA PROJEK/AKAUN : A NEURAL NETWORK APPROACH TO SYNTHETIC CONTROL CHART FOR THE PROCESS MEON (NO. AKAUN: 1001/PMATHS/811024)
7. ELAUN: RM 625.00 SEBULAN No. Akaun Bank BBMB: 0706-0000111-20-1
8. TARIKH PERLAKSANAAN: 1 NOVEMBER 2009
9. KETERANGAN LAIN:

ELLY FATMAWATEE HUSSEIN
Pegawai Kewangan
(Pejabat Pengurusan & Kreativiti Penyelidikan]

s.k.

Prof. Madya Dr. Michael Khoo Boon Chong
Ketua Projek
Pusat Pengajian Sains Matematik

Encik Abdu Mohammed Ali Atta
d/a Prof. Madya Michael Khoo Boon Chong
Pusat Pengajian Sains Matematik

/ni...D:Lanjutan (SSP) 2009