UNIVERSITI SAINS MALAYSIA

1st. Semester Examination
2005/2006 Academic Session

November 2005

EAS 663/4 – Dynamics and Stability of Structural

Duration: 3 hours

Instructions to Candidates:

1. Ensure that this paper contains **EIGHT (8)** printed pages before you start your examination.

2. This paper contains **FIVE (5)** questions. Answer **ALL (5)** questions.

3. Each question carry equal marks.

4. All questions **CAN BE** answered in English or Bahasa Malaysia or combination of both languages.

5. Each question **MUST BE** answered on a new sheet.

6. Write the answered question numbers on the cover sheet of the answer script.

1. (a) Define viscous damping. Sketch the displacement response, (v) versus (t) of undamped and damped SDOF systems for free vibration. Does the natural period of vibration, T, change with the present of damping?

(6 marks)

(b) Figure 1 shows one-storey building idealized as a rigid girder to support a rotating machine. A horizontal force, F(t) = 800 cos 5.3 t N is exerted on the girder. Assuming the damping of the system is equal to 5% of critical damping and the value of E = 200 x 10^3 MPa, determine:

(i) the natural circular frequency
(ii) the frequency ratio, *r*
(iii) the static deflection, *Vo*
(iv) the steady state amplitude of vibration, given $V = Ds\ Vo$ where $Ds = \dfrac{1}{\sqrt{(1-r^2)+(2\zeta r)^2}}$;
(v) the maximum shear force in the column
(vi) the maximum bending moment in the column

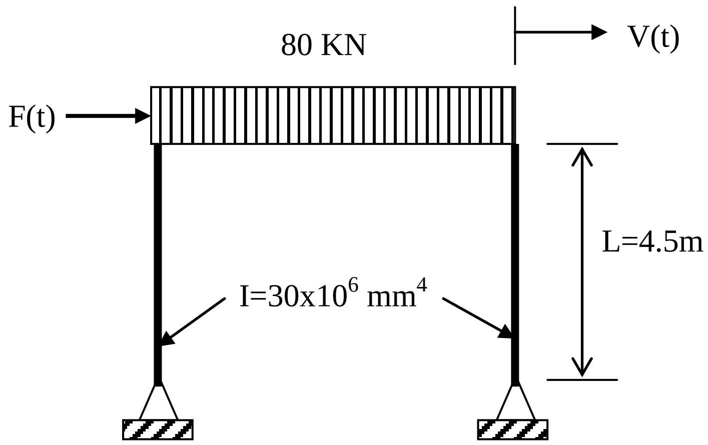


Figure 1

(8 marks)

(c) The frame in Figure 1 is subjected to sinusoidal ground motion Vs(t) = 5.0 x 10^{-3} cos 5.3 t N, instead of F(t), on the girder. Assuming that the damping of the system is equal to 5% of critical damping and the value of E = 200 x 10^3 MPa, determine:

(i) the maximum shear force in the column; given $u_{\max} = \dfrac{r^2\ Vo}{\sqrt{(1-r^2)+(2\zeta r)^2}}$;
(ii) the maximum bending moment in the column

(6 marks)

2. (a) Define response spectra in structural dynamic problems.

(4 marks)

(b) Figure 2(a) shows a model of column-mass SDOF system, subjected to two triangular blast loads, p(t) – Figure 2(b). The weight of the mass block is 2500 kN and the column stiffness, k = 2000 kN/mm. Assume it is an undamped system, predict the maximum displacement response, $v_{max} = R_{max}(\frac{Po}{K})$ and the maximum total elastic forces developed in the system for both p(t). The value of the maximum response ratio, *Rmax* can be obtained from the displacement response spectra as shown in Figure 2(c). Give comments on your observations of the results for both blast loads.

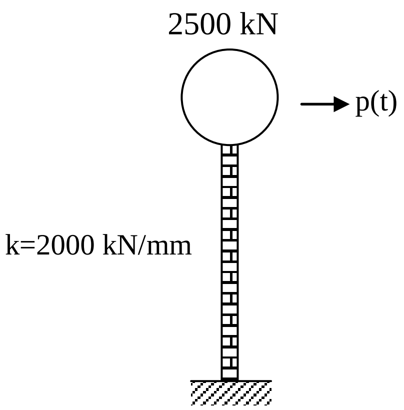


Figure 2(a)

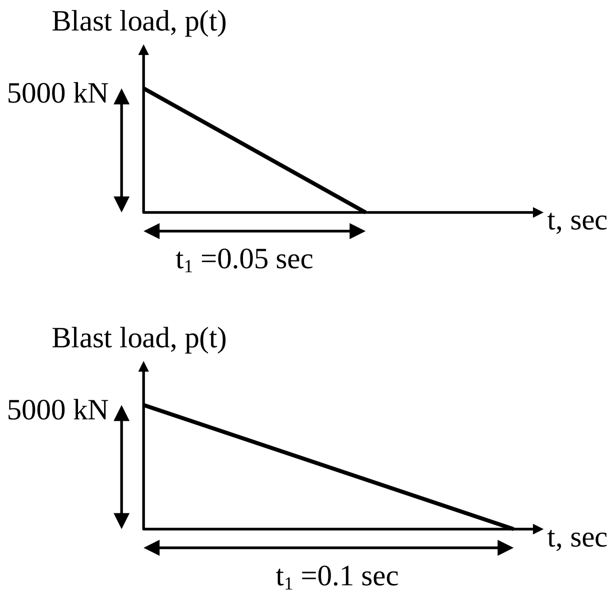


Figure 2(b)

(6 marks)

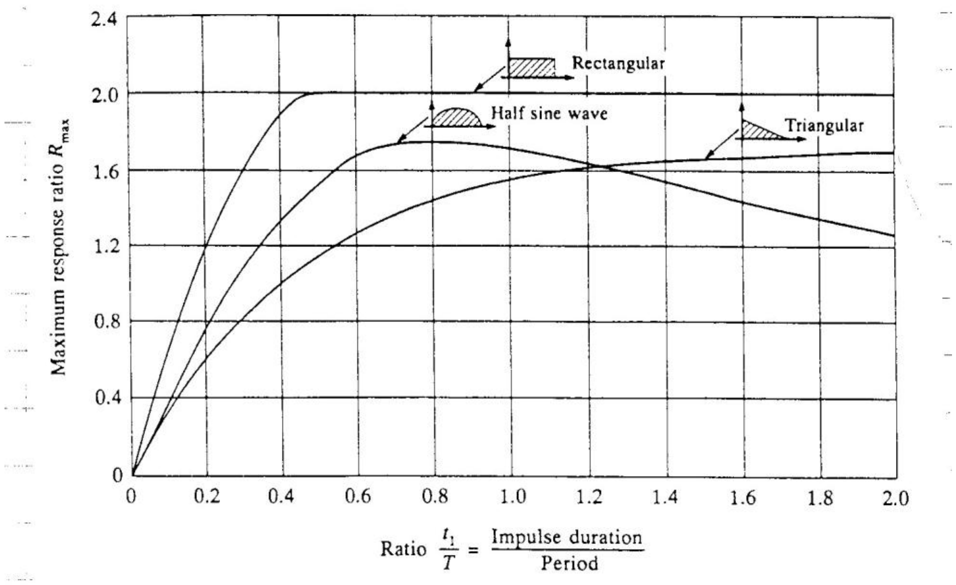


Figure 2(c)

2. (c) Duhamel Integral is normally used for the evaluation of a linear SDOF system subjected to an arbitrary time varying force. Define the underlined term with the help of the graph Force, (P) versus (t).

(5 marks)

(d) Figure 2(d) shows a spring-mass model for 2DOF system under free vibration. Derive the equations of motion for the system.

(5 marks)

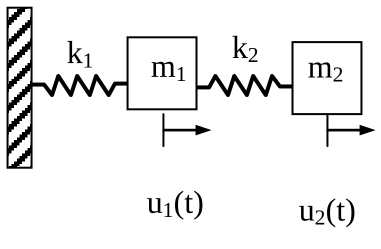


Figure 2(d)

3. (a) Explain the concepts of stable, unstable and neutral equilibrium by using an axially loaded perfectly straight column with both ends pinned. Include suitable sketches in your explanation.

(6 marks)

(b) Figure 3 shows an initially straight column subjected to an axial load P which acts with an eccentricity e from the centroidal axis of the column. Obtain the following relation between mid-height deflection δ and ratio P/P_E where P_E : Euler buckling load = $\pi^2 EI/L^2$:

$$\delta = e\left[\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_E}}\right) - 1\right]$$

Sketch on a plot of P/P_E versus δ, curves for three different values of *e*. Show also on the same plot, the curve representing the behaviour of an initially straight column with *e* = 0. Based on the curves, discuss the effect of imperfection of load on the behaviour of an axially loaded column.

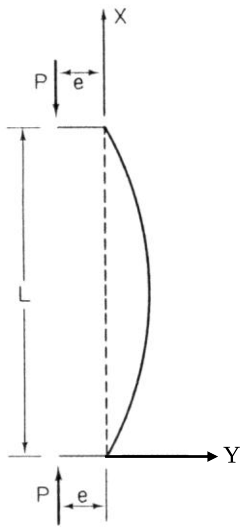


Figure 3

(14 marks)

4. (a) Derive the following fourth order differential equation for beam-column :

$$y^{iv} + k^2 y^{"} = 0 \ , \ k^2 = \frac{P}{EI}$$

where y : lateral displacement of beam-column, P : axial force acting at both ends of beam-column, EI : flexural rigidity, (…)'= $d^2(\ldots)/dx^2$ and ( )^{iv} = $d^4(\ldots)/dx^4$ Next, explain how the above fourth order differential equation is used to determine the critical load of beam-column with different end conditions. You are required to specifically point out in your explanation how starting from the fourth order differential equation, one can arrive at the eigenvalue problem which can be used to solve the critical load of beam-column with different end conditions.

(10 marks)

(b) A simple two-bar frame is shown in Figure 4. A load P acts at end B of vertical member AB. Both supports A and C are fixed. Obtain the effective length L_e for the two-bar frame by using the following equation for an elastically retrained column:

$$(1 - \lambda_1 - \lambda_2 - \lambda_1\lambda_2\Phi^2)\Phi \sin\Phi + (2 + \lambda_1\Phi^2 + \lambda_2\Phi^2)\cos\Phi - 2 = 0$$

where λ_1=EI/(α_1L), λ_2=EI/(α_2L), Φ=kL, k^2=P/EI, EI : flexural rigidity , L : length of column, α_1, α_2 : rotational stiffness of end 1 and 2 of column being studied, respectively. Comment the effect of change in length of member BC on the effective length of column AB.

(10 marks)

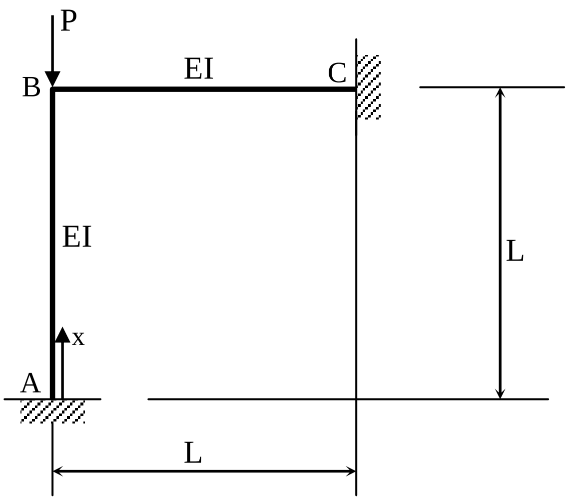
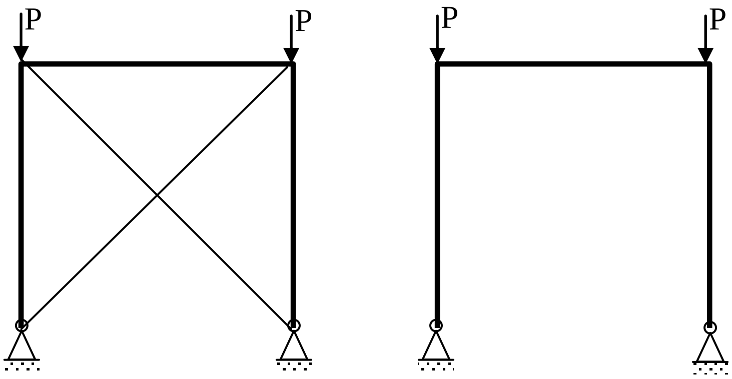


Figure 4

5. (a) Explain the meaning of effective length of a compression member without using any equation.

Figures 5(a) and (b) show a braced and unbraced frame, respectively. For each frame, sketch the buckling mode corresponding to the lowest critical load. Without carrying out any calculation, justify that effective length factor *K* for column: in braced frame is $K > 1$ and in unbraced frame is $K < 1$.

(8 marks)



(a) Braced frame (b) Unbraced frame

Figure 5

(b) Slope deflection equations for a beam-column are given as follows:

$$M_A = \frac{EI}{L}\left(s_{ii}\theta_A + s_{ij}\theta_B\right)$$
$$M_B = \frac{EI}{L}\left(s_{ji}\theta_A + s_{jj}\theta_B\right)$$

where s_{ii}, $s_{ij}(=s_{ji})$, s_{jj} are stability functions and M_A, M_B, θ_A and θ_B are as shown in Figure 6.

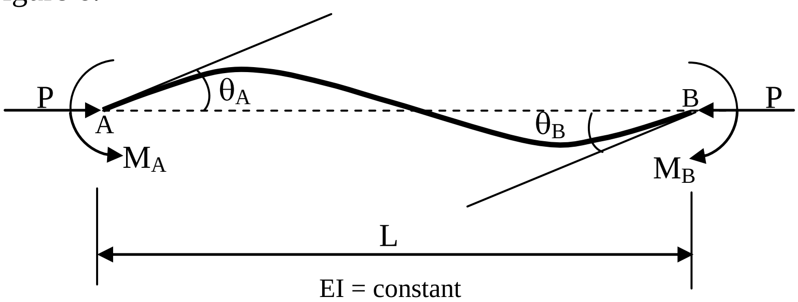


Figure 6

Making use of the above set of slope-deflection equations and the following assumptions for a member in a braced frame:

(a) All members are prismatic and behave elastically
(b) The axial forces in the beam are negligible
(c) All columns in a storey buckle simultaneously
(d) At a joint, the restraining moment provided by the beams is distributed among the columns in proportion to their stiffness
(e) At buckling, the rotations at the near and far ends of the beams are equal and opposite

show the process of derivation of the following eigenvalue problem for the determination of effective length of a column in a braced frame :

$$\begin{bmatrix} s_{ii} + \dfrac{\mathbf{2}}{G_A} & s_{ij} \\ s_{ij} & s_{jj} + \dfrac{\mathbf{2}}{G_B} \end{bmatrix} \begin{Bmatrix} \theta_A \\ \theta_B \end{Bmatrix} = \begin{Bmatrix} \mathbf{0} \\ \mathbf{0} \end{Bmatrix}$$

where G_A , G_B are defined as follows :

$$G_A = \frac{\sum_A (I/L)_{column}}{\sum_A (I/L)_{beam}} = \frac{\sum \text{of column stiffness meeting at end A}}{\sum \text{of beam stiffness meeting at end A}}$$

$$G_B = \frac{\sum_B (I/L)_{column}}{\sum_B (I/L)_{beam}} = \frac{\sum \text{of column stiffness meeting at end B}}{\sum \text{of beam stiffness meeting at end B}}$$

You are required:
(i) to show the equations used in arriving at the eigenvalue problem
(ii) to include a suitable sketch showing the column in a braced frame
(iii) to state how the assumptions listed above are used in the derivation

(12 marks)

- ooo O ooo -