UNIVERSITI SAINS MALAYSIA

1st. Semester Examination
2005/2006 Academic Session

November 2005

EAS 661/4 – Advanced Structural Mechanics

Duration: 3 hours

Instructions to Candidates:

1. Ensure that this paper contains **SIX (6)** printed pages before you start your examination.

2. This paper contains **FIVE (5)** questions. Answer **ALL (5)** questions.

3. Each question carry equal marks.

4. All questions **CAN BE** answered in English or Bahasa Malaysia or combination of both languages.

5. Each question **MUST BE** answered on a new sheet.

6. Write the answered question numbers on the cover sheet of the answer script.

1. (a) The state of stress at a point within an elastic continuum is fully described by six Cartesian stress components. Show clearly the six stress components using a sketch of an infinitesimal volume in an elastic continuum. Explain in details the meaning of subscripts used in the representation of the stress components.

(6 marks)

(b) The six stress components as mentioned in (a) above must be in equilibrium with the Cartesian components of body force at all points in the interior of a three dimensional continuum. By choosing anyone of the three Cartesian co-ordinate direction, derive the corresponding equilibrium equation between the stress and body force components. Use suitable sketches in your derivation. Explain in details all meaning of symbols used in the derivation.

(6 marks)

(c) Plane stress problem is a specialization of three dimensional continua problem to that of a two dimensional one. Explain in details, with the help of suitable sketches, what plane stress problem is.

(8 marks)

2. (a) Define the Principle of Virtual Displacement(PvD). Show that the statement of PvD when specialized to the case of conservative problem can be expressed as follows:

$\delta W_e = \delta U_p$

where δW_e : variation in external work and δU_p : variation in strain energy, during virtual displacement.

Using the above statement of PvD for conservative problem, derive the equation of equilibrium for the linearly elastic spring as shown in Fig.1 in terms of *F*, *k* and *u*, where *k* : spring constant, *u* : elongation of spring due to *F* and *F* : force acting on the spring.

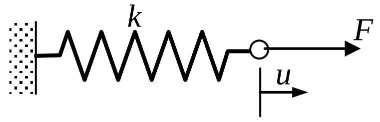

Figure 1

(8 marks)

2. (b) Figure 2 shows a simply supported beam with an elastic spring prop at point C. The beam is subjected to a point load P acting at the mid-span. The following expression for lateral displacement field v has been suggested :

$$v = A \sin(\pi x / L)$$

where A is a constant. Show that the above displacement field is kinematically admissible. Next, solve for the constant A by applying the principle of minimum potential energy. Flexural rigidity of beam is EI and spring constant for elastic spring is k.

(12 marks)

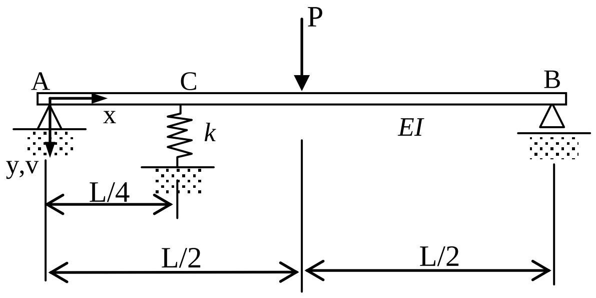

Figure 2

3. (a) Using the three basic relations necessary for the analysis of structural mechanics problems, derive the following stiffness equation for a simple 1D linearly elastic prismatic bar subjected to an end force P as shown in Figure 3.

$$P = \frac{EA}{L}\Delta$$

where Δ : elongation of bar due to force P , E : elastic modulus of material of bar, A : cross-sectional area and L : original length of bar.

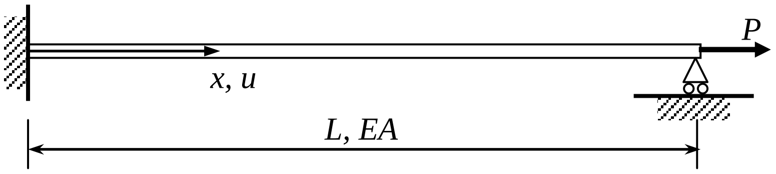

Figure 3

(6 marks)

3. (b) The stepped beam with both end fixed as shown in Figure 4 is to be analysed using piece-wise Rayleigh-Ritz method where the beam will be divided artificially into two portions <A-B> and <B-C>. The beam is subjected to a uniformly distributed load, *w* and a point load, *P* at B.

Explain the process involved in solving the beam problem in order to obtain the lateral displacement *v*, distributions of shear force, *S* and bending moment, *M* along the beam. In your explanation, you are required to show clearly the steps and the equations/relations used in each step in the solution process. Detailed solutions of *v*, *S* and *M* are not required.

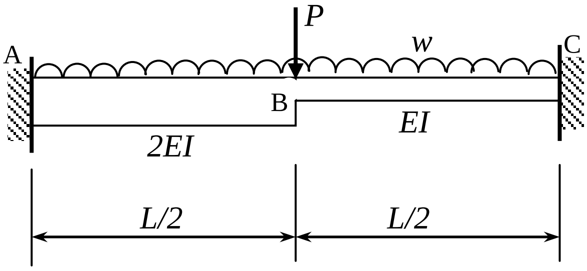

Figure 4

(14 marks)

4. (a) Write down the element stiffness matrices and global matrix for the three bars assembly which is loaded with force P, and constrained at the two ends in terms of E, A and L as shown in Figure 5(a).

(5 marks)

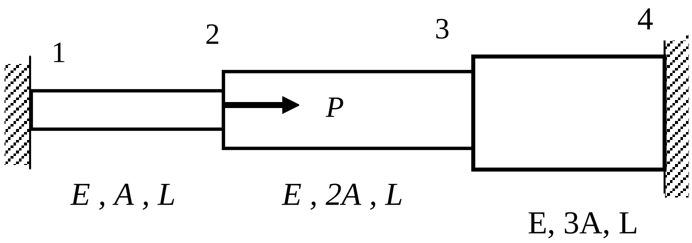

Figure 5(a)

(b) Clearly define the difference between a bar and beam in the analysis using Finite Element Method.

(5 marks)

4. (c) Figure 5(b) shows a system of two beams labeled as node 1, 2 and 3 and a spring labeled as node 3 and 4 subjected to a nodal forces, P = 50 kN at node 3. The beam is fixed at node 1, simply supported at node 2 and spring support at node 3. The spring system can only displace in axial direction and is supported at node 4. Given the value of k = 200 kN/m, $L_1 = L_2$ = 3m, E = 210GPa and I = $2\text{x}10^{-4}$ m^4,

i. Obtain the element stiffness matrix for the beam and spring.
ii. Derive the global stiffness matrix for the system.
iii. Evaluate the deflection v_3, θ_2 and θ_3 in unit metre and rad, respectively.

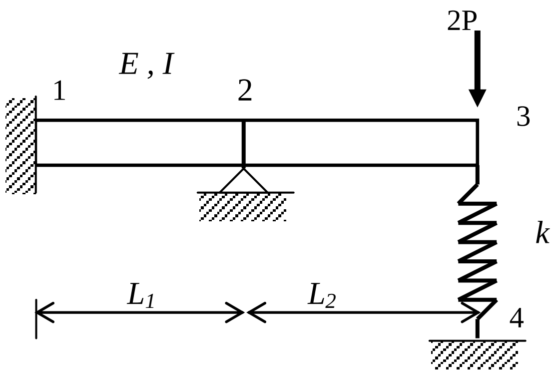

Figure 5(b)

(10 marks)

Given the stiffness of the beam element in dimensional space:

$$k = \frac{EI}{L^3}\begin{array}{c} \begin{array}{cccc} v_i & \theta_i & v_j & \theta_j \end{array} \\ \begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix} \end{array} \text{ for the beam element}$$

$$k = \begin{array}{c} \begin{array}{cc} u_i & u_j \end{array} \\ \begin{bmatrix} k & -k \\ -k & k \end{bmatrix} \end{array} \text{ for the spring element}$$

5. (a) Explain the assumptions made in the modeling procedures for material properties and loading conditions in Finite Element Method.

(5 marks)

(b) Figure 6(a) shows a cantilever beam carrying a concentrated load of 1000kN at point B. The beam is modeled with linear four-noded rectangular elements (□) and three nodded triangular elements (△). Given the Elastic Modulus E = 200 GPa, thickness, t = 10 mm and the Poisson ratio, v = 0.3. Sketch the results for the maximum stress in xx and yy direction for the beam especially at point A and B.

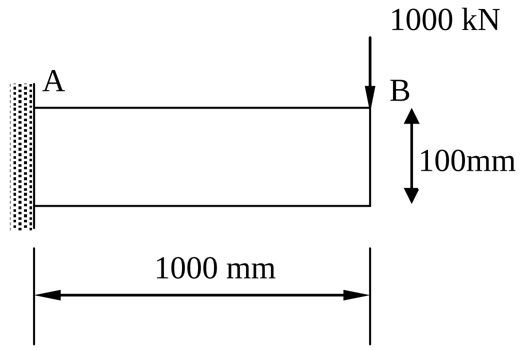

Figure 6(a)

(10 marks)

(c) Derive the stiffness matrix for an element shown in Figure 6(b) in term of applied axial loads F1, F2, displacements u1, u2, axial rigidity EA and initial length, L.

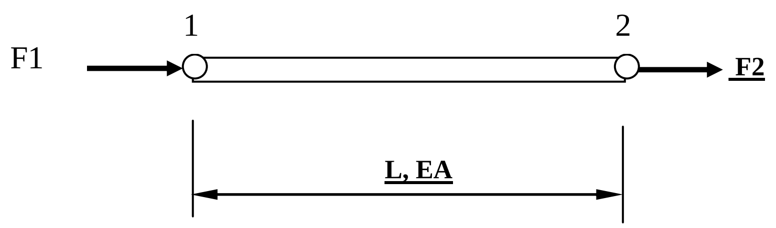

Figure 6(b)

(5 marks)

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