UNIVERSITI SAINS MALAYSIA

1st. Semester Examination
2004/2005 Academic Session
Peperiksaan Semester 1
Sidang Akademik 2004/2005

October 2004

EAS 663/4 – Dynamics and Stability of Structures
EAS 663/4 – Dinamik dan Kestabilan Struktur

Duration: 3 hours
Masa : 3 jam

Instructions to candidates:

1. Ensure that this paper contains **TEN (10)** printed pages included appendices.
   Sila pastikan kertas peperiksaan ini mengandungi SEPULUH (10) muka surat bercetak termasuk lampiran sebelum anda memulakan peperiksaan ini.

2. This paper contains **FIVE (5)** questions. Answer **ALL (5)** questions.
   Kertas ini mengandungi FIVE (5) soalan. Jawab KESEMUA LIMA (5) soalan..

3. All questions **CAN BE** answered in English or Bahasa Malaysia or combination of both languages.
   Semua soalan boleh dijawab dalam Bahasa Inggeris atau Bahasa Malaysia ataupun kombinasi kedua-dua bahasa.

4. Each question carry equal marks.
   Tiap-tiap soalan mempunyai markah yang sama.

5. All question **MUST BE** answered on a new sheet.
   Semua jawapan MESTILAH dijawab pada muka surat yang baru.

6. Write the answered question numbers on the cover sheet of the answer script.
   Tuliskan nombor soalan yang dijawab di luar kulit buku jawapan anda.

1. (a) List two characteristics that distinguish structural dynamic problems from static ones.

(*a*) *Senaraikan dua ciri yang membezakan masalah struktur dinamik daripada masalah statik.*

(4 marks)

(b) Define viscous damping. Sketch the displacement response, (v) versus (t) of undamped and damped SDOF systems for free vibration. Does the natural period of vibration, T, change with the present of damping?

(b) Takrifkan redaman likat. Lakarkan sambutan anjakan, v, melawan masa (t) untuk sistem Kebebasan Satu Darjah yang tanpa redaman dan dengan redaman. Adakah kala getaran, T, berubah dengan kehadiran redaman?

(6 marks)

(c) Figure 1.0 shows a model of spring-mass SDOF system that is subjected to a harmonic excitation, p(t) = 50 cos 10t N. The weight of the mass block is 150 kN and the spring stiffness, k = 7000 N/m. Assume the damping of the system is equal to 5% of the critical damping. Determine the total displacement response of the system which is given by the following equation:

$$v(t) = V\cos(\Omega t - \alpha) + e^{-\zeta\omega t}(A_1 \cos\omega_d t + A_2 \sin\omega_d t)$$

$$V = \frac{v_r}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}}$$

where ω : natural circular frequency of the system and v_0 : static displacement due to p_o.

(c) Rajah 1.0 menunjukkan satu sistem jisim- pegas dengan Kebebasan Satu Darjah yang dikenakan satu getaran harmonik , p(t) = 50 cos 10t N. Berat jisim blok ialah 150 kN dan kekukuhan pegas, k = 7000 N/m. Anggap bahawa redaman dalam sistem bersamaan 5% daripada redaman kritikal. Kira nilai sambutan anjakan sistem tersebut yang diberi dalam persamaan berikut :

$$v(t) = V\cos(\Omega t - \alpha) + e^{-\zeta\omega t}(A_1 \cos\omega_d t + A_2 \sin\omega_d t)$$

$$V = \frac{v_r}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}}$$

iaitu ω ialah frekuensi bulatan tabii sistem dan v_0 ialah anjakan statik disebabkan oleh p_o.

(10 marks)

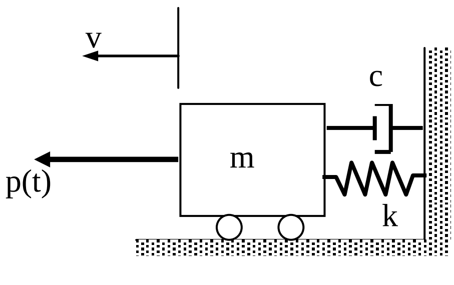


Figure 1.0

2. (a) Duhamel Integral is normally used for the evaluation of a linear SDOF system subjected to <u>arbitrary time varying force</u>. Define the underlined term with the help of a graph Force, (P) versus (t).

(a) Kamilan Duhamel biasanya digunakan untuk penilaian sistem Kebebasan Satu Darjah yang dikenakan <u>daya ubah waktu sembarangan</u>. Takrifkan perkataan yang bergaris dengan bantuan rajah daya, (P)v, melawan masa (t).

(5 marks)

(b) Figure 2.0 shows a spring-mass model for 2DOF system under free vibration. Derive the equations of motion for the system.

(b) Rajah 2.0 menunjukkan satu model jisim-pegas dengan Kebebasan Dua Darjah yang dikenakan getaran bebas. Terbitkan persamaan gerakan untuk sistem tersebut.

(5 marks)

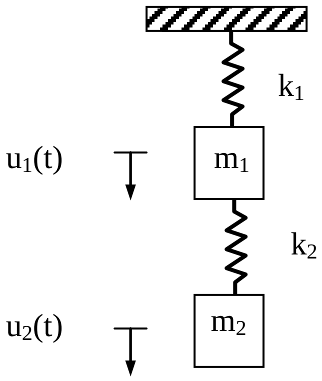


Figure 2.0 (a)

(c) The water tower as shown in Figure 2.0 (b) weighs 700kN when filled with water is subjected to step force with rise time [Figure 2.0 (c)]. It is observed that a horizontal jack force of 30kN is required to displace the tower top by a distance of 20mm. Estimate the maximum lateral displacement response due to dynamic forces. The constant phase is given by the following equation:

$$v(t) = v_0 \left\{ 1 + \frac{1}{\omega t_r} \left[A \sin \langle \omega (t - t_r) + \alpha \rangle \right] \right\}$$

$$A = \sqrt{(1 - \cos \omega t_r)^2 + (\sin \omega t_r)^2} \quad , \quad \tan \alpha = -\frac{\sin \omega t_r}{(1 - \cos \omega t_r)}$$

where ω : natural circular frequency of the system , v_0 : static displacement due to p_o , v_{max} : maximum response and T_n : natural period of vibration. A plot of R_d (= v_{max} / v_0) versus t_r / T_n is shown in Figure 2.0 (d). Comment on the effect of ratio t_r / T_n on R_d , without carrying out any "exact" dynamic analysis.

(c) *Menara air yang ditunjukkan dalam Rajah 2.0 (b) mempunyai berat 700kN apabila penuh dengan air yang dikenakan daya langkah dengan masa kenaikan yang terhingga [Rajah 2.0 (c)]. Diberi bahawa satu daya ufuk sebesar 30kN diperlukan untuk menganjakkan bahagian atas menara air sebanyak 20mm. Anggarkan sambutan anajakan maksima yang disebabkan oleh satu daya dinamik. Persamaan sambutan anjakan semasa fasa malar adalah seperti berikut:*

$$v(t) = v_0 \left\{ 1 + \frac{1}{\omega\, t_r} \left[A \sin\langle \omega\,(t - t_r) + \alpha \rangle \right] \right\}$$

$$A = \sqrt{(1 - \cos\omega\, t_r)^2 + (\sin\omega\, t_r)^2} \quad , \quad \tan\alpha = -\frac{\sin\omega\, t_r}{(1 - \cos\omega\, t_r)}$$

iaitu ω *: frekuensi bulatan tabii sistem ,* v_0 *: anjakan statik disebabkan oleh* p_o *,* v_{max} *: sambutan maksimum dan* T_n *: kala tabii getaran. Satu plot* R_d *(=* v_{max} / v_0*) lawan nisbah* t_r / T_n *ditunjukkan dalam Rajah 2.0 (d). Beri ulasan tentang kesan nisbah* t_r / T_n *ke atas* R_d *tanpa menjalankan sebarang analisis dinamik "sebenar".*

(10 marks)

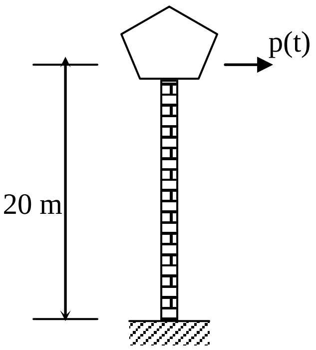


Figure 2.0 (b)

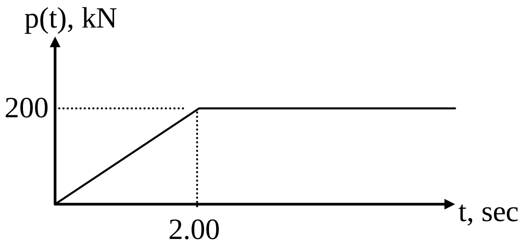


Figure 2.0 (c)

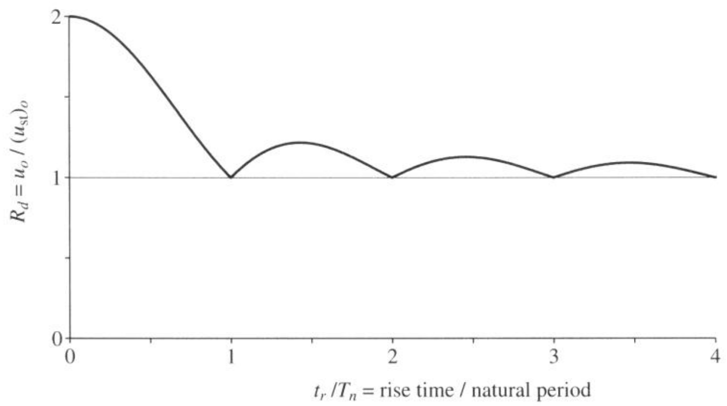


Figure 2.0 (d)

3. (a) By using an axially loaded and perfectly straight column with both ends pinned, explain the concepts of stable, unstable and neutral equilibrium.

(6 marks)

(b) Figure 3.0 shows an initially straight column subjected to an axial load P which acts at an eccentricity e from the centroidal axis of the column. Obtain the following relation between mid-height deflection δ and ratio P/P_E where P_E : Euler buckling load $=\pi^2 EI/L^2$:

$$\delta = e\left[\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_E}}\right)-1\right]$$

Sketch a plot of P/P_E versus δ for three different values of e.

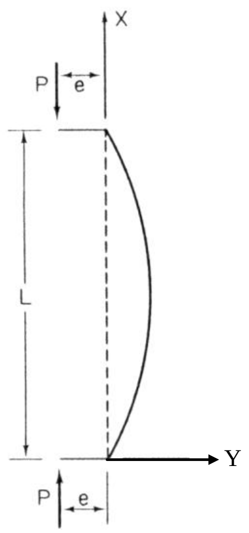


Figure 3.0

Sketch also on the same plot the graph representing the behaviour of an initially straight column with e=0. Based on the graph, discuss the effect of imperfection of load on the behaviour of an axially loaded column.

(14 marks)

4. (a) Derive the following fourth order differential equation for beam-column :

$$y^{iv} + k^2 y'' = 0 \ , \ k^2 = \frac{P}{EI}$$

where y : lateral displacement of beam-column, P : axial force acting at both ends of beam-column, EI : flexural rigidity and $(\ldots)' = d(\ldots)/dx$. Next, explain how the above fourth order differential equation is used to determine the critical load of beam-column with different end conditions. You are required to specifically point out in your explanation how starting from the fourth order differential equation, one can arrive at the eigenvalue problem which can be used to solve for the critical load of beam-column with different end conditions.

(10 marks)

(b) A simple two-bar frame is shown in Figure 3.0. A load P acts at end B of vertical member AB. Both supports A and C are fixed. Obtain the effective length L_e for the two-bar frame by using the following equation for an elastically restrained column:

$$(1-\lambda_1-\lambda_2-\lambda_1\lambda_2\Phi^2)\Phi\sin\Phi+(2+\lambda_1\Phi^2+\lambda_2\Phi^2)\cos\Phi-2=0$$

where $\lambda_1=EI/(\alpha_1 L)$, $\lambda_2=EI/(\alpha_2 L)$, $\Phi=kL$, $k^2=P/EI$, EI : flexural rigidity , L : length of column, α_1, α_2 : rotational stiffness of end 1 and 2 of column being studied, respectively. Justify your solution for the effective length obtained by using information provided in Table 1.0 (see Appendix 1).

(10 marks)

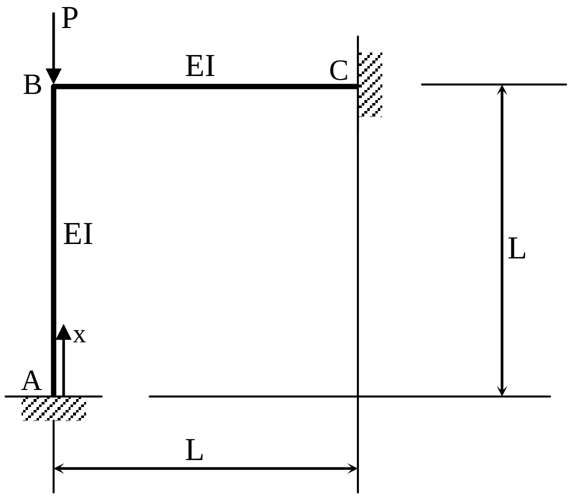


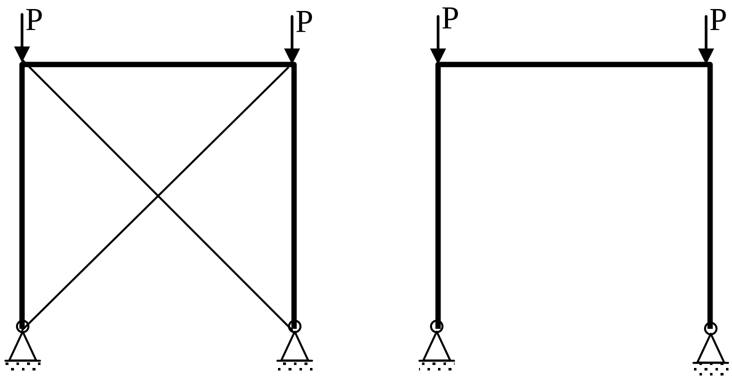
Figure 3.0

5. (a) Figure 4.0 (a) and (b) show braced and unbraced frames, respectively. For each frame, sketch the buckling mode corresponding to the lowest critical load. Using suitable eigenvalue analysis, it can be shown that effective length factor K for

   i. column in braced frame is K < 1.0 and
   ii. column in unbraced frame is K > 1.0

   Justify the above conclusions by referring to behavior of columns with other standard end conditions.

(6 marks)



(a) Braced frame (b) Unbraced frame

Figure 4.0

(b) Slope deflection equations for a beam-column are given as follows :

$$M_A = \frac{EI}{L}\left(s_{ii}\theta_A + s_{ij}\theta_B\right)$$
$$M_B = \frac{EI}{L}\left(s_{ji}\theta_A + s_{jj}\theta_B\right)$$

where s_{ii}, $s_{ij}(=s_{ji})$, s_{jj} are stability functions and M_A, M_B, θ_A and θ_B are as shown in Figure 5.0.

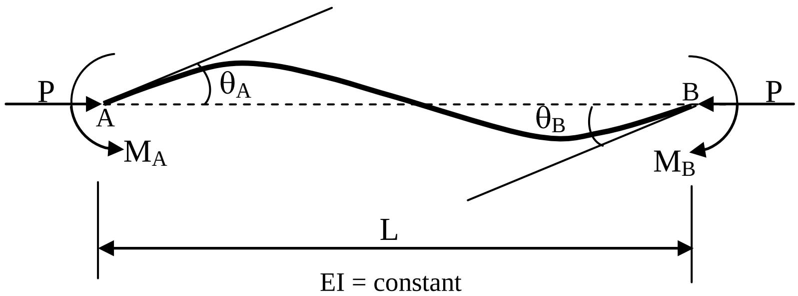


Figure 5.0

Making use of the previous set of slope-deflection equations and the following assumptions for a member in a braced frame:

i. All members are prismatic and behave elastically
ii. The axial forces in the beam are negligible
iii. All columns in a storey buckle simultaneously
iv. At a joint, the restraining moment provided by the beams is distributed among the columns in proportion to their stiffness
v. At buckling, the rotations at the near and far ends of the beams are equal and opposite

show the process of deriving the following eigenvalue problem for the determination of effective length of a column in a braced frame :

$$\begin{bmatrix} s_{ii} + \dfrac{2}{G_A} & s_{ij} \\ s_{ij} & s_{jj} + \dfrac{2}{G_B} \end{bmatrix} \begin{Bmatrix} \theta_A \\ \theta_B \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

where G_A , G_B are defined as follows :

$$G_A = \frac{\sum_A (I/L)_{column}}{\sum_A (I/L)_{beam}} = \frac{\sum \text{of column stiffness meeting at end A}}{\sum \text{of beam stiffness meeting at end A}}$$

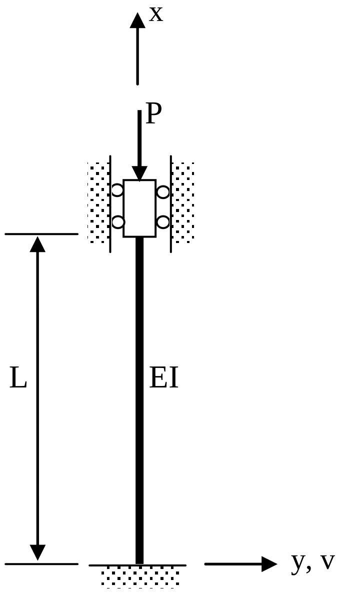
$$G_B = \frac{\sum_B (I/L)_{column}}{\sum_B (I/L)_{beam}} = \frac{\sum \text{of column stiffness meeting at end B}}{\sum \text{of beam stiffness meeting at end B}}$$

You are required:

i. to show the equations involved in arriving at the eigenvalue problem
ii. to show a suitable sketch showing the column in a braced frame
iii. to state how the assumptions listed above are used in the derivation

(14 marks)

Supplements :



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