UNIVERSITI SAINS MALAYSIA

1st. Semester Examination
2004/2005 Academic Session

October 2004

EAS 661/4 – Advanced Structural Mechanics

Time : 3 hours

<u>Instruction to candidates:</u>

1. Ensure that this paper contains **<u>EIGHT</u> (8)** printed pages.

2. This paper contains **<u>FIVE</u> (5)** questions. Answer **<u>ALL</u> (5)** questions.

3. All questions carry the same mark..

4. All questions **<u>MUST BE</u>** answered in English.

5. Write answered question numbers on the cover sheet of the answer script.

1. (a) Three dimensional continua problem could be specialized to two well-known cases of plane stress and plane strain problems. Explain clearly with the help of suitable sketches the difference between plane stress and plane strain problems.

(6 marks)

(b) Figure 1.0 shows the stress components acting on an infinitesimal volume in a three dimensional body. Using the notation of :

$\boldsymbol{\sigma} = [\ \sigma_x\ \ \sigma_y\ \ \sigma_z\ \ \tau_{xy}\ \ \tau_{yz}\ \ \tau_{zx}\ ]^T$

and

$\boldsymbol{\varepsilon} = [\ \varepsilon_x\ \ \varepsilon_y\ \ \varepsilon_z\ \ \gamma_{xy}\ \ \gamma_{yz}\ \gamma_{zx}\ ]^T$

for the Cartesian components of stress and the corresponding strain, respectively, derive the constitutive equation $\boldsymbol{\varepsilon}=\mathbf{D}\boldsymbol{\sigma}$ for the case of a homogeneous isotropic body. Specialize it to the case of a plane stress problem.

(8 marks)

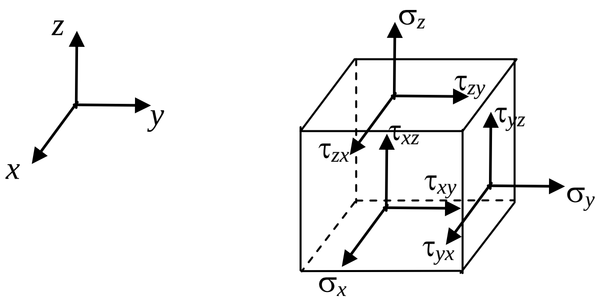


Figure 1.0

(c) The sets of equilibrium equations and strain-displacement equations for an infinitesimal volume in a three dimensional body as shown in Figure 1.0 are given as follows, respectively :

Equilibrium equations :

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} + R_x = 0$$

$$\frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{yz}}{\partial z} + \frac{\partial \tau_{xy}}{\partial x} + R_y = 0$$

$$\frac{\partial \sigma_z}{\partial z} + \frac{\partial \tau_{zx}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + R_z = 0$$

where R_x, R_y and R_z are body forces per unit volume in x, y and z-directions, respectively;

Strain-displacement equations :

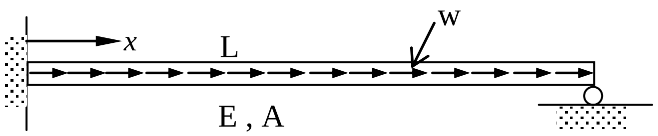
$$\varepsilon_x = \frac{\partial u}{\partial x} \quad , \quad \varepsilon_y = \frac{\partial v}{\partial y} \quad , \quad \varepsilon_z = \frac{\partial w}{\partial z}$$

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \quad , \quad \gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \quad , \quad \gamma_{zx} = \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}$$

where u, v and w are components of displacement in x, y and z-directions, respectively, of a point within the three dimensional body.

Using the above sets of equations together with the general constitutive equations for a homogeneous isotropic body derived in (b) earlier, specialize them to the case of a 1D bar loaded with uniformly distributed body force as shown in Figure 2.0. Explain clearly all assumptions made in the process of specialization. State also the boundary condition for the problem.

(6 marks)



E, A : constant

Figure 2.0

2. (a) Prove that identical equilibrium equation will be obtained for the elastic spring subjected to a load f as shown in Figure 3.0 by using both :

    i. principle of virtual displacement and
    ii. principle of minimum potential energy.

    Express the equation of equilibrium in terms of k and f, where k is the spring constant.

    (6 marks)

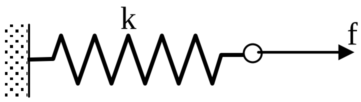


Figure 3.0

(b) By specializing the equation of principle of virtual displacement to the case of 1D-problem, derive the governing equation of equilibrium for the problem of a tapered bar subjected to concentrated load as shown in Figure 4.0, where A, L and P are the cross-sectional area, length of the bar and concentrated load, respectively. State also the force and displacement boundary conditions. Young modulus of the bar is E.

(6 marks)

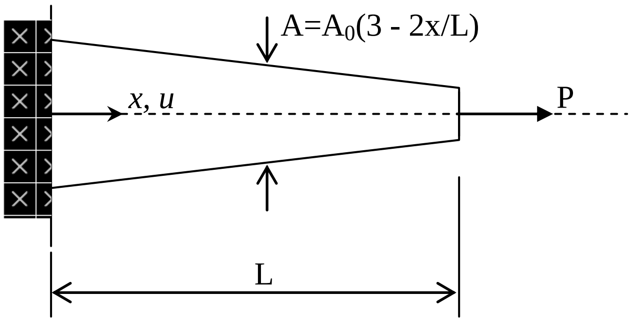


Figure 4.0

(c) Figure 5.0 shows a simply supported beam with an elastic spring prop at point C. The beam is subjected to a point load P acting at the mid-span. The following expression for lateral displacement field *v* has been suggested :

$$v = A \sin(\pi x / L)$$

where *A* is a constant. Show that the above displacement field is admissible. Next, solve for the constant *A* by applying the principle of minimum potential energy. Flexural rigidity of beam is EI and spring constant for elastic spring is k.

(8 marks)

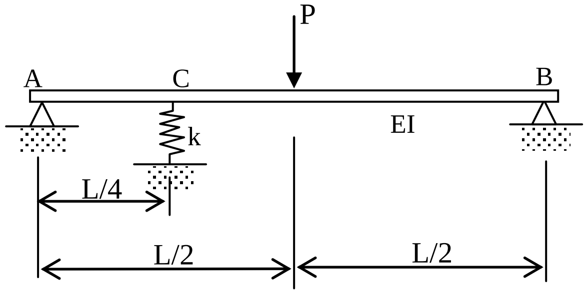


Figure 5.0

3. (a) Write down the element stiffness matrices and global matrix for the two bar assembly which is loaded with force P, and constrained at the two ends in terms of E, A and L as shown in Figure 6.0 (a).

(5 marks)

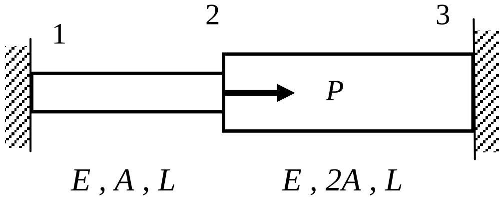


Figure 6.0 (a)

(b) Show clearly in a step by step manner the development process of a stiffness matrix, $[K]^e$, for a triangular element in a state of plane stress as shown in Figure 2.0. Given E = 200 GN/m^2, ν = 0.3 and t = 2 cm.

(15 marks)

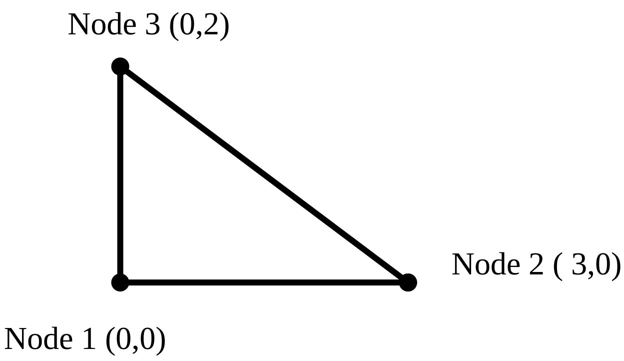


Figure 6.0 (b)

4. (a) Clearly define the difference between a bar and beam in the analysis using Finite Element Method.

(5 marks)

(b) Figure 7.0 shows a system of two beams labeled as node 1, 2 and 3 and a spring labeled as node 3 and 4 subjected to a nodal forces of P = 50 kN at node 3. The beam is fixed at node 1, simply supported at node 2 and spring support at node 3. The spring system can only displace in axial direction and is supported at node 4. Given the value of k = 200 kN/m, $L_1 = L_2$ = 3m, E = 210GPa and I = $2x10^{-4}$ m^4 .

i. Obtain the element stiffness matrix for the beam and the spring.
ii. Derive the global stiffness matrix for the system.
iii. Evaluate the deflection v_3, θ_2 and θ_3 in unit metre and radian respectively.

(15 marks)

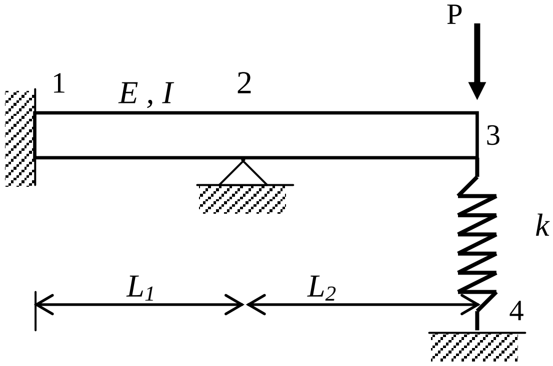


Figure 7.0

Given the stiffness of the beam element in dimensional space:

$$k = \frac{EI}{L^3} \begin{array}{c} \begin{array}{cccc} \quad v_i & \quad \theta_i & \quad v_j & \quad \theta_j \end{array} \\ \begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix} \end{array} \text{ for the beam element}$$

$$k = \begin{array}{c} \begin{array}{cc} u_i & \quad u_j \end{array} \\ \begin{bmatrix} k & -k \\ -k & k \end{bmatrix} \end{array} \text{ for the spring element}$$

5. (a) Describe the plan stresses and plane strain structures in the analysis of finite element method.

(5 marks)

(b) Explain what is the Variational Method (Rayleigh-Ritz) and the Weighted-Residual Method (Galerkin)

(5 marks)

(c) Figure 8.0 (b) shows a cantilever beam carrying a concentrated load of 100N at point B.The beam is modeled with linear four-noded rectangular elem□ts ( ) and three nodded triangular △ments ( ). Given Elastic Modulus E = 200 GPa, thickness, t = 10 mm and the Poisson ratio, v = 0.3. The results for the percentage of deflection error is shown in Figure 8.0 (c). Give 3 suggestions that can be done to improve the result of the deflection. If the deflection at B has converged, discuss about the convergence of stresses at A.

(5 marks)

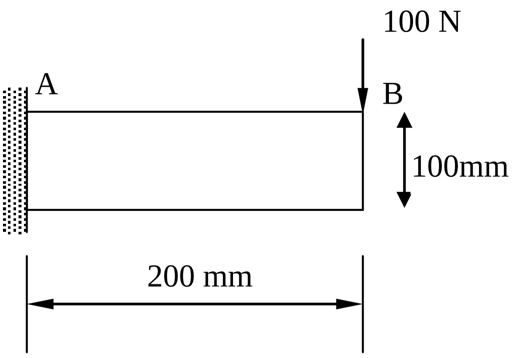


Figure 8.0 (b)

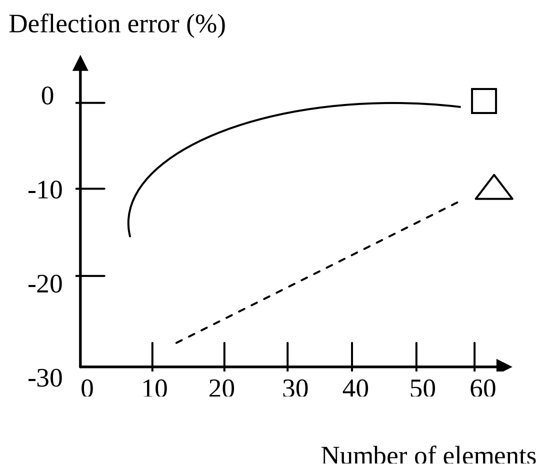


Figure 8.0 (c)

(d) Derive the stiffness matrix for an element shown in Figure 8.0 (d) in terms of applied axial loads F1, F2, displacements u1, u2 , axial rigidity EA and initial length, L.

(5 marks)

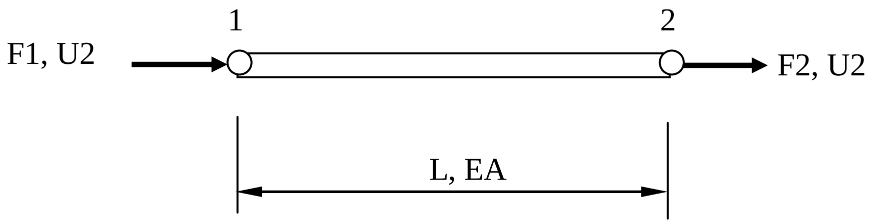


Figure 8.0 (d)

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