UNIVERSITI SAINS MALAYSIA

Peperiksaan Semester Pertama
Sidang Akademik 2005/2006
First Semester Examination
2005/2006 Academic Session

November 2005
November 2005

ESA 473/3 – Aero-Anjalan
Aero-Elasticity

Masa : 3 jam
Duration : 3 hours

Sila pastikan bahawa kertas peperiksaan ini mengandungi LIMA BELAS mukasurat dan EMPAT soalan sebelum anda memulakan peperiksaan ini.

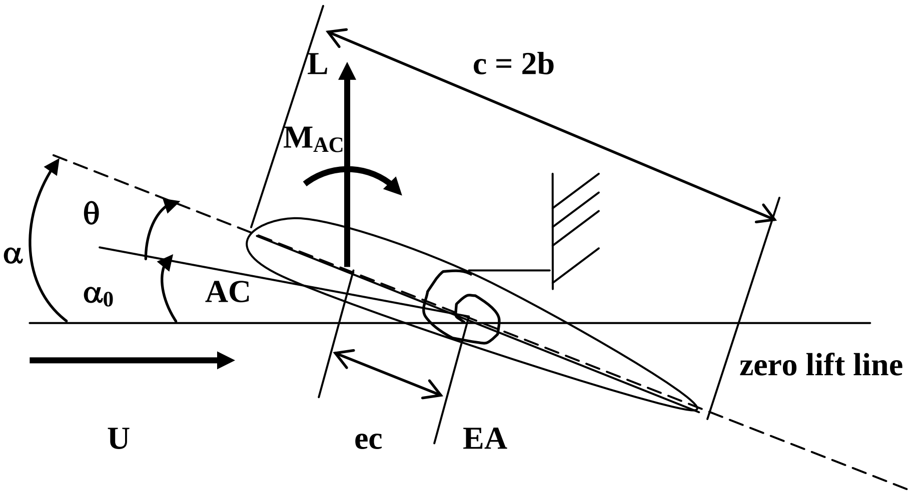
Please check that this examination paper consists of FIFTEEN printed pages and FOUR questions before you begin the examination.

Arahan : Jawab **TIGA(3)** soalan sahaja.

Intructions : *Answers* ***THREE (3)*** *questions only.*

1. Pertimbangkan sebuah sayap yang digambarkan dalam gambarajah di bawah, ia ditahan daripada terpiuh di EA

Consider a wing represented by a typical section, which is restrained in twisting at EA.



Semua petunjuk adalah positif dalam Gambarajah
All signs are positive in the Figure

Dianggap pesongan lentur adalah tidak penting (untuk sayap lurus)
Di sini:

α_0 - sudut serangan awal
θ - sudut piuh disebabkan aliran udara
$L = C_L q S$, q = tekanan dinamik
S - luas permukaan sayap
$q = ½ \rho U^2$

Bending deflection is assumed to be unimportant (for straight wings).
Here:

α_0 - *initial angle of attack*
θ - *angle of twist due to airflow*
$L = C_L q S$, *q = dynamic pressure*
S - wing surface area
$q = ½ \rho U^2$

Tork penyimpanan semula yang disebabkan oleh keanjalan sayap (diwakilkan oleh pegas kilasan K_α):

$$T = K_\alpha \theta$$

Restoring torque due to elasticity of wing (represented by torsional spring K_α):

$$T = K_\alpha \theta$$

(a) Terbitkan sebutan untuk sudut piuh,θ yang disebabkan oleh daya aerodinamik dan momen.

Derive the expression for the angle of twist θ due to aerodynamic force and moments.

(20 markah/*marks*)

(b) Apakah tekanan dinamik kecapahan dan halaju kecapahan kritikal, merangkapi kekukuhan kilasan sayap, parameter sayap e, c, S, $C_{L\alpha}$ dan tekanan dynamic jejurus bebas?

What is the divergence dynamic pressure and critical divergence speed, as a function of wing torsional stiffness, wing parameters e, c, S, $C_{L\alpha}$ and the dynamic pressure of free stream?

(20 markah/*marks*)

(c) Pertimbangkan sayap serupa-Isogai dengan :

$C_{L\alpha} = 2\pi$ s = luas permukaan separuh sayap
$\alpha 0 = 0$ S = kawasan permukaan separuh sayap
$CMAC = 0$ $\cong$ bs
$e = 0.25$
$\mu = 100 = m / (\pi \rho b_2 s)$
$r_\alpha^2 = 3.48$

dan :

$K_\alpha = \omega_\alpha^2 I_\alpha$
$I_\alpha = r_\alpha^2 mb^2$

Consider an Isogai-like wing with:

$C_{L\alpha} = 2\pi$ s = *half wing span*

$\alpha 0 = 0$ S = *half wing surface area*

CMAC = 0 $\cong$ *bs*

e = 0.25

μ = 100 = $m / (\pi\rho b_2 s)$

r_α^2 = 3.48

and :

$K_\alpha = \omega_\alpha^2 I_\alpha$

$I_\alpha = r_\alpha^2 mb^2$

Kirakan halaju kecapahan merangkapi frekuensi kilasanω_α

Calculate the divergence speed as a function of torsional frequency ω_α

(20 markah/*marks*)

(d) Plotkan U_D melawan ω_α (dalam rad/saat atau cps), dan kirakan ω_α jika ia dikehendaki untuk mencapai $U_D >$ 400 m/saat

Plot U_D *versus* ω_α *(in rad/sec or cps), and calculate* ω_α *if it is desired to have* $U_D >$ *400 m/sec.*

(20 markah/*marks*)

(e) Tuliskan kelajuan kecapahan U_D dengan menggunakan definisi K_α dalam sebutan I_α dan I_α dalam sebutan mb^2 .

Write the divergence speed U_D *using the definition of* K_α *in terms of* I_α *and* I_α *in terms of* mb^2 .

(10 markah/*marks*)

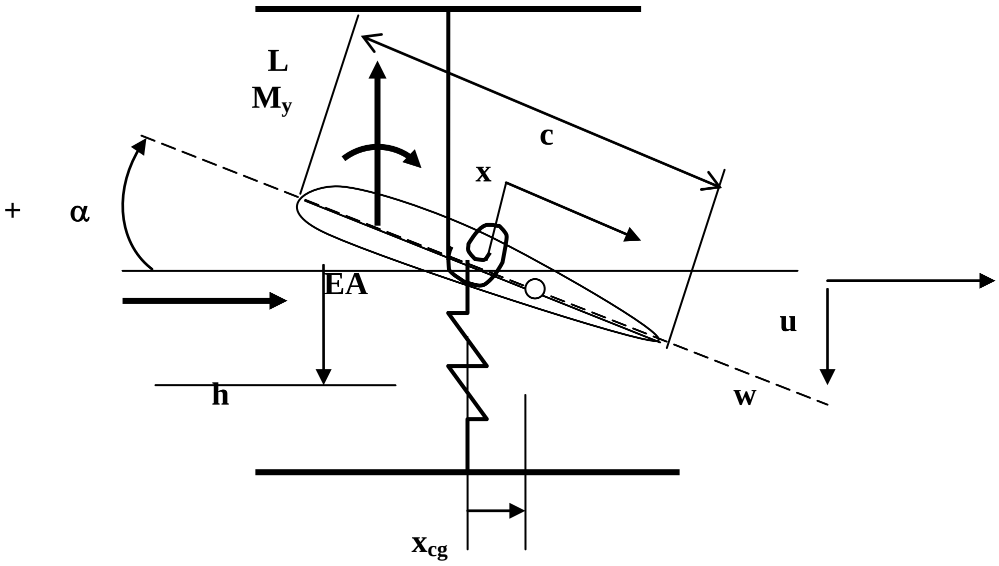
(f) Daripada sebutan yang diterbitkan dalam (e), apakah pilihan lain yang ditunjukkan di dalamnya untuk meningkatkan U_D jika ω_α tidak boleh diubah?:

From expression derived in (e), what are other options indicated there to increase U_D *, if* ω_α *cannot be modified at will ?:*

(10 markah/*marks*)

2. Pertimbangkan bahagian sayap di bawah:

Consider a typical wing section below:



x diukur di sepanjang perentas dari EA
x is measured along chord from EA.

Pertimbangkan koordinat umum berikut:

$$q_1 = h$$
$$q_2 = \alpha$$

Consider the following generalized coordinate:

$$q_1 = h$$
$$q_2 = \alpha$$

Anjakan mana-mana titik pada airfoil ialah:

Displacement of any point on the airfoil is:

Daripada geometri

$r = u\,\mathbf{i} + w\,\mathbf{k}$

$u = 0$

$w = -h - x\alpha$

From geometry

$r = u\,\boldsymbol{i} + w\,\boldsymbol{k}$

$u = 0$

$w = -h - x\alpha$

Biarkan:

$$m = \int \rho dx$$

$$S_\alpha = \int \rho x dx = x_{cg} m$$

$$I_\alpha = \int \rho x^2 dx$$

Let:

$$m = \int \rho dx$$

$$S_\alpha = \int \rho x dx = x_{cg} m$$

$$I_\alpha = \int \rho x^2 dx$$

Dan tenaga upaya:

$$U = \frac{1}{2} K_h h^2 + \frac{1}{2} K_\alpha \alpha^2$$

And the potential energy:

$$U = \frac{1}{2} K_h h^2 + \frac{1}{2} K_\alpha \alpha^2$$

Kerja tak mengabadi:

$$\delta W_{NC} = \delta h(-L) + \delta\alpha(M_y)$$

The non-conservative work:

$$\delta W_{NC} = \delta h(-L) + \delta\alpha(M_y)$$

Dimana :

$$L \equiv \int p dx = -Q_h$$
$$M \equiv \int pxdx = Q_\alpha$$

Where:

$$L \equiv \int p dx = -Q_h$$
$$M \equiv \int pxdx = Q_\alpha$$

Terbitkan persamaan gerakan untuk bahagian tersebut merangkapi sistem binar dengan menggunkan prinsip Hamilton dan persamaan Langrange.

Derive the equation of motion for the typical section as a binary system using Hamilton Principle and Lagrange equations.

(100 markah/*marks*)

3. Pertimbangkan dinamik sebuah sayap biasa yang diwakilkan dengan sistem binar:

$$\mathrm{m}\ddot{\mathrm{h}} + \mathrm{K}_{\mathrm{h}}\mathrm{h} + \mathrm{S}_{\alpha}\ddot{\alpha} = -\mathrm{L} \qquad \textbf{(1)}$$

$$\mathrm{S}_{\alpha}\ddot{\mathrm{h}} + \mathrm{I}_{\alpha}\ddot{\alpha} + \mathrm{K}_{\alpha}\alpha = \mathrm{M}_{\mathrm{y}} \qquad \textbf{(2)}$$

Consider the dynamics of a typical section represented by the binary system:

$$m\ddot{h} + K_h h + S_{\alpha}\ddot{\alpha} = -L \qquad \textbf{(1)}$$

$$S_{\alpha}\ddot{h} + I_{\alpha}\ddot{\alpha} + K_{\alpha}\alpha = M_y \qquad \textbf{(2)}$$

Seperti biasa, ia ditakrifkan sebagai:

$$L \equiv \int pdx$$

$$M \equiv \int pxdx$$

as defined in the usual manner:

$$L \equiv \int pdx$$

$$M \equiv \int pxdx$$

Frekuensi tabii yang tidak terganding ditakrifkan sebagai:

$$\omega_h{}^2 = \frac{K_h}{m}$$

$$\omega_{\alpha}{}^2 = \frac{K_{\alpha}}{I_{\alpha}}$$

The uncoupled natural frequencies are defined as:

$$\omega_h{}^2 = \frac{K_h}{m}$$

$$\omega_{\alpha}{}^2 = \frac{K_{\alpha}}{I_{\alpha}}$$

Anggap pergerakan berbentuk sinus sebagai:

$$L = \overline{L}e^{i\omega t}$$
$$M_y = \overline{M}_y e^{i\omega t}$$
$$h = \overline{h}e^{i\omega t}$$
$$\alpha = \overline{\alpha}e^{i\omega t}$$

Assume sinusoidal motion:

$$L = \overline{L}e^{i\omega t}$$
$$M_y = \overline{M}_y e^{i\omega t}$$
$$h = \overline{h}e^{i\omega t}$$
$$\alpha = \overline{\alpha}e^{i\omega t}$$

Di mana $\overline{L}$, $\overline{M}_y$, $\overline{h}$ dan $\overline{\alpha}$ ialah amplitud.

where $\overline{L}$, $\overline{M}_y$, $\overline{h}$ *and* $\overline{\alpha}$ *are the amplitudes.*

(a) Dengan menggunakan pergerakan berbentuk sinus, tuliskan persamaan matrik daripada sistem (1) dan sistem (2).

Using sinusoidal motion, write the matrix equation from system (1) and system (2).

(50 markah/*marks*)

(b) Tindakbalas sistem tersebut yang disebabkan oleh daya-daya aerodinamik L dan M_y adalah ampilitud-amplitud untuk anjakan linear $\overline{h}$ dan anjakan sudut $\overline{\alpha}$ yang merangkapi amplitud-amplitud ujaan (aerodinamik) $\overline{L}$, $\overline{M}_y$. Untuk meringkaskan lagi (supaya hanya pembolehubah $\overline{L}$ sahaja digunakan), takrifkan

$$d \equiv \frac{\overline{M}_y}{\overline{L}}$$

Cari sebutan $\frac{\bar{h}}{\bar{L}}$ merangkapi parameter-parameter bahagian sayap biasa tersebut.

The response of the system due to aerodynamic forces L and M_y *are the amplitudes of the linear displacement* $\bar{h}$ *and angular displacement* $\bar{\alpha}$ *as a function of the excitation amplitudes (aerodynamic)* $\bar{L}$ *,* $\bar{M}_y$*. For further simplification (so that we have to deal with single variable* $\bar{L}$ *only),define*

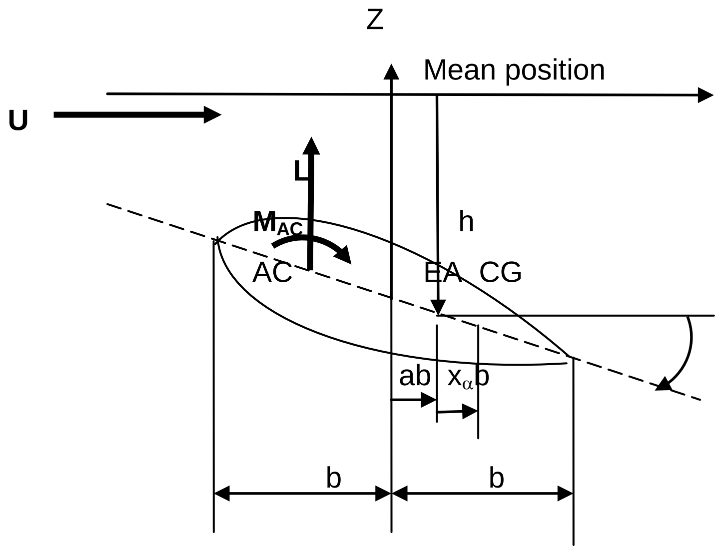
$$d \equiv \frac{\bar{M}_y}{\bar{L}}$$

Find the expression of $\frac{\bar{h}}{\bar{L}}$ *as the function of the typical section parameters.*

(50 markah/*marks*)

4. Untuk mendapatkan gambaran debaran yang baik, kita akan pertimbangkan model aerodinamik pegun yang diaplikasikan kepada aeroanjalan sistem bahagian biasa dengan dua darjah kebebasan: lenturan dan kilasan (sistem penduaan). Hanya pergerakan kilasan yang akan mengaruhkan daya aerodinamik:

   To obtain a good impression of flutter, we will consider steady aerodynamic model applied to aeroelasticity of a two-degree-of-freedom typical section system.: bending and torsion (binary system). Only torsional motion induces aerodynamic forces:



$L(t) = q\,S\,C_{L\alpha}\;\alpha(t)$ 1

$M_{AC}(t) = 0$ 2

$M_{EA} = 2\,L\,e\,b + M_{AC} = 2\,q\,S\,e\,b\,C_{L\alpha}\;\alpha(t)$ 3

Persamaan Debaran

Flutter equations :

$$m\ddot{h} + S_\alpha\ddot{\alpha} + K_h h + qSC_{L_\alpha}\alpha = 0 \qquad 4$$

$$S_\alpha\ddot{h} + I_\alpha\ddot{\alpha} + K_\alpha\alpha - 2qSebC_{L_\alpha}\alpha = 0 \qquad 5$$

(a) Tuliskan persamaan 4 dan 5 dalam bentuk matrik

Write equation 4 and 5 in matrix notation

(20 markah/*marks*)

Analisis kestabilan debaran boleh dibuat dengan menganggap ianya adalah gerakan harmoni:

Analysis of flutter stability can be carried out by assuming harmonic motion:

$$h = \hat{h} \; e^{pt} \tag{6a}$$

$$\alpha = \hat{\alpha} \; e^{pt} \tag{6b}$$

atau

or

$$\{x\} = \{\hat{x}\} e^{pt} \tag{6c}$$

Penggantian akan menghasilkan

Substitution leads to

$$(mp^2 + K_h)\hat{h} + (S_\alpha p^2 + qSC_{L_\alpha})\hat{\alpha} = 0 \tag{7}$$

$$S_\alpha p^2 \hat{h} + (I_\alpha p^2 + K_\alpha - 2qSebC_{L_\alpha})\hat{\alpha} = 0 \tag{8}$$

Penentu persamaan 6.11 dan 6.12 ialah:

The determinant of the set of equations 6.11 and 6.12 is:

$$I_\alpha mp^4 - S_\alpha^2 p^4 + p^2\left[mK_\alpha 2qSebC_{L_\alpha} m + I_\alpha K_h\right] - p^2 S_\alpha qSC_{L_\alpha} + K_\alpha K_h - 2qSebC_{L_\alpha} K_h = 0$$

iaitu Persamaan Ciri dalam bentuk

which is a Characteristic Equation of the form

$$a_4\, p^4 + a_2 p^2 + a_0 = 0 \tag{9}$$

dengan

with:

$a_4 = \quad mI_\alpha - S_\alpha^2$ 10

$a_2 = \quad m(K_\alpha - 2qSebC_{L\alpha}) + I_\alpha K_h - qSS_\alpha C_{L\alpha}$ 11

$\quad = \quad mK_\alpha + I_\alpha K_h - (2meb + S_\alpha)qSC_{L\alpha}$

$a_0 = \quad K_h(K_\alpha - 2qSebC_{L\alpha})$ 12

Persamaan 9 adalah polinomial turutan ke-4 (atau turutan ke-2 dlm p) dengan 4 punca:

Equation 9 is 4th order polynomial (or 2nd order in p^2) with 4 roots:

$$p_{1,2,3,4} = (\sigma + i\omega)_{1,2,3,4} = \pm\sqrt{\frac{1}{2a_4}\left(-a_2 \pm \sqrt{a_2^2 - 4a_4a_0}\right)} \qquad 13$$

Jenis penyelesaian adalah:

Solutions are of the type:

$$\{x\} = \{\hat{x}\}e^{\sigma t}e^{i\omega t} \qquad 14$$

Anggap persamaan-persamaan berikut benar di sini:

Let us assume further, that the following relations hold:

$K_h = m\,\omega_h^2$, $K_\theta = I_\theta\,\omega_\theta^2$

$S_\theta = m\,x_\alpha\,b$ $I_\theta = m\,r_\alpha^2\,b^2$

x_α adalah momen lengan pegun dan r_α adalah jejari legaran, dan ragam tabii memuaskan persamaan ciri.

x_α is the static moment arm, and r_α is the radius of gyration, and that the natural modes satisfy the characteristic equation.

Anggap bahagian biasa sayap boleh digambarkan sebagai yang dicadangkan oleh Isogai, 1979, untuk sayap pesawat pengangkut moden:

Let us assume further, that the typical section of the wing can be represented as proposed by Isogai, 1979, for a modern transport wing:

$a = -2.0$
$x_\alpha = 1.8$
$r_\alpha^2 = 3.48$
$\frac{\omega_h}{\omega_\varepsilon} = 1.0$

$C_{L_\alpha} = 2.0\pi$
e = -0.75
$\mu = 100$

(b) Carikan nilai a_4, a_2 dan a_0

Find a_4, a_2 and a_0

(40 markah/*marks*)

Untuk pertimbangkan jenis ketidakstabilan yang akan berlaku, jadual di bawah harus dirujuk

To consider the type of instability that will occur, one may refer to the following table

$a_2^2 - 4a_4a_0$	>0			<0
a_0	> 0		< 0	>< 0
a_2	> 0	< 0	>< 0	>< 0
p^2	$-\omega_1^2$, $-\omega_2^2$	σ_1^2 , σ_2^2	σ^2 , $-\omega^2$	-g ± ih
p	$\pm\ i\omega_1$, $\pm\ i\omega_2$	$\pm\sigma_1$, $\pm\sigma_2$	$\pm\sigma$, $\pm\ i\omega$	$\pm\sigma \pm\ i\omega$
Jenis pergerakan Type of motion	Harmonik: 2 frek. positif 2 frek. negatif *Harmonic:* *2 pos. freq.* *2 neg. freq*	Tak berkala: 2 mencapah 2 menumpu *Aperiodic:* *2 diverging* *2 converging*	*Aperiodic:* *1 diverging* *1 converging* *Harmonic:* *1 pos.freq.* *1 neg.freq.*	*Oscillatory:* *1 div.pos.freq* *1 conv.pos.freq* *1 div.neg.freq* *1 conv.neg.freq*
Jenis Ketidakstabilan *Type of instab.*	Neutral *Neutral*	Kecapahan *Divergence*	Kecapahan *Divergence*	Debaran *Flutter*
Category	I	III	IV	II

(c) Untuk kes yang dipertimbangkan, ketidakstabilan jenis apakah yang akan berlaku, anggap $\omega_\theta = 1$? Huraikan.

What type of instability takes place in the case considered, assume $\omega_\theta = 1$? *Elaborate. (40 marks)*

(40 markah/*marks*)

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