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UNIVERSITI SAINS MALAYSIA

2<sup>ND</sup>. Semester Examination  
2003/2004 Academic Session

February / March 2004

**JUM 221/3 - Probability And Applied Statistics**

Duration: 3 hours

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**Instructions to candidates:**

1. Ensure that this paper contains **FIVE (5)** printed pages before you start your examination.
2. This paper contains **SIX (6)** questions. Answer **FOUR (4)** questions only. Marks will be given to the **FIRST FOUR (4)** questions put in order on the answer script and **NOT** the **BEST FOUR (4)**.
3. All questions **MUST BE** answered in Bahasa Malaysia.
4. Each question **MUST BE** answered on a new page.
5. Write the answered question numbers on the cover sheet of the answer script.

1. (a) Explain clearly the meaning of the following terms:

- I. The properties of the joint probability distribution for two continuous variables.
- II. Independent random variables  $X_1$  and  $X_2$ .
- III. Covariance.
- IV. Correlation.
- V. Point estimator.

(10 marks)

(b) Let  $X_1$  and  $X_2$  be the scores for two intelligence tests. The density function for  $[X_1, X_2]$  is given as follows:

probability

$$f(x_1, x_2) = \begin{cases} 4x_1x_2e^{-(x_1^2+x_2^2)} & 0 \leq x_1 \leq \infty, 0 \leq x_2 \leq \infty \\ 0 & \text{otherwise} \end{cases}$$

Find

- I. the marginal distribution for  $X_1$  and  $X_2$ .
- II. the conditional probability distribution for  $X_1$  and  $X_2$ .

(15 marks)

2. (a) Two new processes are being tested. The following data is given:

Process A		Process B	
2748	3149	3027	2910
2700	3257	3356	2889
2655	3213	3359	2902
2822	3220	3297	
2511	2753	3125	

Under the assumption that the population variance for the two processes are equal but unknown, find the 90% confidence interval for the differences between the means of the two processes. What assumptions are required?

(15 marks)

2. (a) Assume that a random sample of size  $2n$  was obtained (and denoted by  $X_1, X_2, \dots, X_{2n}$ ) from a population with  $E(X) = \mu$  and  $V(X) = \sigma^2$ .

Assume that

$$\bar{X}_1 = \frac{1}{2n} \sum_{i=1}^{2n} X_i \quad \text{and} \quad \bar{X}_2 = \frac{1}{n} \sum_{i=1}^n X_i$$

are two estimators for  $\mu$ .

Which is the best estimator?

(10 marks)

3. (a) Explain clearly the meaning of minimum variance unbiased estimator.

(3 marks)

- (b) The following result gives the mean and standard deviation of the breaking strength of two types of steel. A random sample of 100 specimens of each steel were used.

Steel type	Sample mean	Sample standard deviation
1	60.1	1.0
2	59.9	1.0

By using an appropriate hypothesis test, is there any significant difference between the true mean breaking strength using the two different steels. Assume that the samples were collected from the normal distribution.

(10 marks)

- (c) A modification was done towards a process which produces a type of film. The new process will only be used if the modification can decrease the true mean of developing the film by more than 1 second. The following data is given:

Original process	8.6	5.1	4.5	5.4	6.3	6.6	5.7	8.5
New process	5.5	4.0	3.8	6.0	5.8	4.9	7.0	5.7

By using the Wilcoxon rank sum test, should the new process be used?

(12 marks)

4. An electrical engineer is investigating the effective conductivity of five different tube coatings for cathode ray tubes which is used in visual displays of a telecommunication system. The conductivity data was obtained as follows:

Type of coatings	Conductivity			
1	143	141	150	146
2	152	149	137	143
3	134	133	132	127
4	129	127	132	129
5	147	148	144	142

- I. Is there any difference in the conductivity based on different coatings? (15 marks)
  - II. Assume that coating type 4 is being used, what is the engineer's recommendation to the management? Assume that we want to minimize conductivity. (10 marks)
5. (a) A random sample of 200 married and retired men are categorized according to their education background and the number of siblings they have. The following relationship was obtained:

Education	Number of siblings		
	0-1	2-3	>3
School	14	37	32
College	19	42	17
University	12	17	10

- Is family size related to education background? (10 marks)

5. (b) Two different devices for measuring sulphur monoxide in atmosphere was compared in a test for measuring air pollution. The following data was obtained:

Sulphur monoxide			
Device A		Device B	
0.96	0.68	0.87	0.57
0.82	0.65	0.74	0.53
0.75	0.84	0.63	0.88
0.61	0.59	0.55	0.51
0.89	0.94	0.76	0.79
0.64	0.91	0.70	0.84
0.81	0.77	0.69	0.63

By using the sign test, determine whether the measurement for sulphur monoxide is different using the two different devices.

(15 marks)

6. The following data gives the compressive strength of an alloy at different thickness level for an additive material.

Thickness, $x$	Compressive strength, $y$		
10	25.2	27.3	28.7
15	29.8	31.1	27.8
20	31.2	32.6	29.7
25	31.7	30.1	32.3
30	29.4	30.8	32.8

- I. Fit a linear regression model of the form  $y = \beta_0 + \beta_1x + \beta_2x^2$ .

(10 marks)

- II. Test for the adequacy of the regression model.

(15 marks)

$$\text{Note : } (X^T X)^{-1} = \begin{pmatrix} 5.266667 & -0.5600 & 0.013333 \\ -0.5600 & 0.062286 & -0.001524 \\ 0.013333 & -0.001524 & 0.000038 \end{pmatrix}$$

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