UNIVERSITI SAINS MALAYSIA

2nd. Semester Examination
2003/2004 Academic Session
Peperiksaan Semester Kedua
Sidang Akademik 2003/2004

Februari / Mac 2004

EAS 567/4 – Plate and Shell Structures

EAS 567/4 – Struktur Plat dan Kelompang

Duration: 3 hours
Masa : 3 jam

Instructions to candidates:

1. Ensure that this paper contains **FIVE (5)** printed pages before you start your examination.
   *Sila pastikan kertas peperiksaan ini mengandungi **LIMA (5)** muka surat bercetak sebelum anda memulakan peperiksaan ini.*

2. This paper contains **FOUR (4)** questions. Answer **ALL (4)** questions.
   *Kertas ini mengandungi **EMPAT (4)** soalan. Jawab **SEMUA (4)** soalan.*

3. All questions **CAN BE** answered in English or Bahasa Malaysia or combination of both languages.
   Semua soalan boleh dijawab dalam Bahasa Inggeris atau Bahasa Malaysia ataupun kombinasi kedua-dua bahasa.

4. Each question **MUST BE** answered on a new page.
   *Tiap-tiap jawapan **MESTILAH** dimulakan pada muka surat yang baru.*

5. Write the answered question numbers on the cover sheet of the answer script.
   Tuliskan nombor soalan yang dijawab di luar kulit buku jawapan anda.

1. (a) Shells are structures with curved surfaces. Explain the advantages of a curved surface from the point of view of load transfer when compared to a flat surface.
(6 marks)

Kelompang adalah struktur dengan permukaan melengkung. Terangkan kebaikan permukaan melengkung berbanding dengan permukaan plat yang rata dari segi pemindahan beban.

(b) State the **THREE (3)** basic equations which are necessary in the analysis of shell structures. Explain clearly all symbols used in the corresponding equations.
(9 marks)

*Nyatakan **TIGA (3)** persamaan asas yang diperlukan dalam analisis kelompang. Terangkan dengan jelas simbol-simbol yang dipakai sekiranya persamaan diguna dalam penerangan.*

(c) Kirchhoff-Love's assumptions are used in the derivation of basic equations for shells from the corresponding equations from the theory of 3D elasticity. State all the assumptions and explain how basic equations in the theory of 3D elasticity are simplified as a consequence of each of the assumption.
(10 marks)

Anggapan Kirchhoff-Love digunakan di dalam penerbitan persamaan asas untuk kelompang dari persamaan yang berkaitan dalam teori keanjalan 3D. Nyatakan anggapan-anggapan tersebut dan terangkan cara bagaimana persamaan dalam teori keanjalan 3D diringkaskan akibat daripada setiap anggapan.

2. (a) By using separate diagrams of (i) a stressed shell element and (ii) a shell element neutral surface, show clearly the stress resultants and the corresponding components of stress which are considered in the theory of membrane.
(6 marks)

Tunjukkan dengan jelas paduan tegasan dan juga komponen tegasan yang berkaitan yang dipertimbangkan di dalam teori membran dengan menggunakan gambarajah elemen kelompang tertegas dan permukaan neutral yang berkaitan.

(b) Making use of suitable diagrams, explain the main difference between theory of membrane and theory of bending from the point of view of stress and moment resultants which are considered in each theory.
(5 marks)

Dengan menggunakan gambarajah-gambarajah yang sesuai, terangkan perbezaan di antara teori membran dan teori lenturan dari segi paduan tegasan dan momen yang dipertimbangkan dalam setiap teori.

2. (c) Table 1 shows the set of equations for membrane stress resultants N_φ and N_θ for an axisymmetrically loaded shell of revolution. Derive the equations of N_φ and N_θ for the case of a spherical shell.

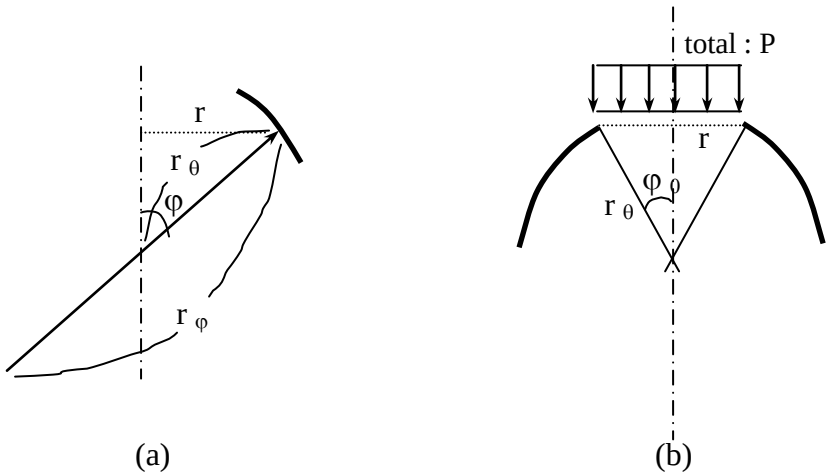
Jadual 1 menunjukkan set persamaan untuk paduan tegasan membran N_φ dan N_θ untuk kelompang putaran yang dibebani secara simetri paksi. Terbitkan persamaan yang berkaitan untuk kes kelompang sfera.

Table 1

$$N_\theta = r_\theta q_n - \frac{r_\theta}{r_\varphi} N_\varphi$$

$$N_\varphi = -\frac{1}{r \sin\varphi}\left[C_1 + \int_{\varphi_0}^{\varphi} r\, r_\varphi\, (q_\varphi \sin\varphi - q_n \cos\varphi)\, d\varphi\right] \quad , P = 2\pi C_1 \, , C_1 = \text{constant}$$

Meaning of symbols used in the equations are as shown in Figure 1.



(a) (b)

Figure 1

By using the derived equations, show that the equations for membrane stress resultants for a spherical shell which is loaded with a varying liquid pressure as shown in Figure 2 could be written as follows:

Seterusnya, dengan menggunakan persamaan yang diperolehi, tunjukkan bahawa paduan tegasan membran untuk satu kelompang sfera yang dibebani oleh tekanan cecair yang berubah-ubah seperti yang ditunjukkan dalam Rajah 2 adalah seperti berikut:

$$N_\varphi = -\frac{\gamma a^2}{6}\left[1 + \frac{3H}{a} - \frac{2\cos^2\varphi}{1+\cos\varphi}\right]$$

$$N_\theta = -\frac{\gamma a^2}{6}\left[-1 + \frac{3H}{a} - \frac{4\cos^2\varphi - 6}{1+\cos\varphi}\right]$$

where γ : unit weight of liquid, a : radius of dome and H : height of liquid to the apex of dome.

di mana γ : berat unit cecair, a : jejari dom dan H : kedalaman cecair kepada puncak kubah.

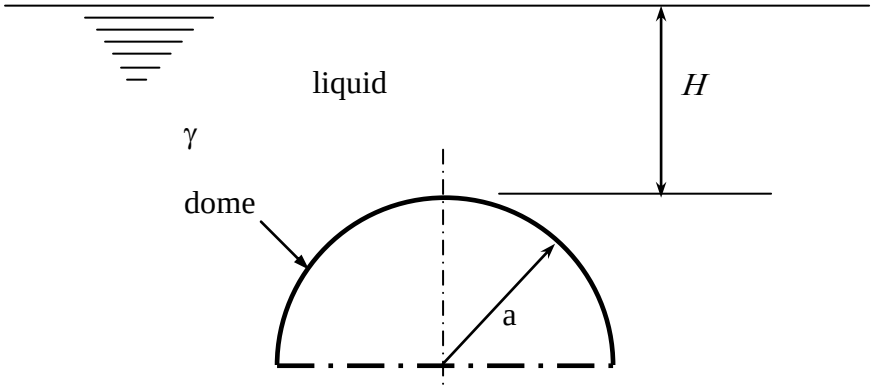


Figure 2

(14 marks)

3. Figure 3 shows the bending stress resultants M_x and M_y acting on sections of a thin plate. Show that the bending moment M_x and M_y are given by the following equations:

Rajah 3 menunjukkan paduan tegasan lenturan M_x dan M_y yang bertindak ke atas keratan satu plat nipis. Tunjukkan bahawa persamaan untuk M_x dan M_y boleh ditulis dalam bentuk seperti berikut:

$$M_x = -D\left[\frac{\partial^2 w}{\partial x^2} + \mu\frac{\partial^2 w}{\partial y^2}\right]; \quad \text{with } D = \frac{Eh^3}{12(1-\mu^2)}$$

$$M_y = -D\left[\frac{\partial^2 w}{\partial y^2} + \mu\frac{\partial^2 w}{\partial x^2}\right]$$

where E : elastic modulus of plate, h : thickness of plate, μ : Poisson's ratio of plate and w : deflection of plate in z-direction.

(25 marks)

di mana E : modulus keanjalan plat, h : ketebalan plat, μ : nisbah Poisson plat dan w : pesongan plat dalam arah z .

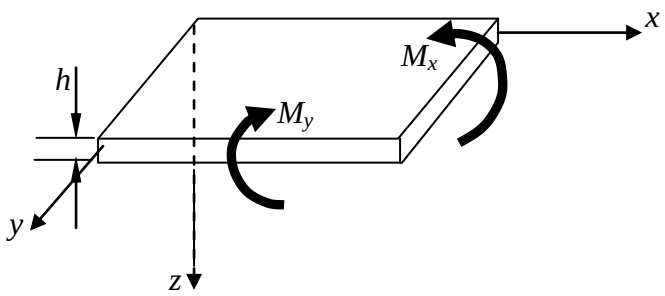


Figure 3

4. (a) Plate is often considered as a 2-dimensional structure as opposed to beam which is considered as a 1-dimensional structure for the purpose of analysis. Explain briefly the main difference between a plate and a beam in terms of the way transverse load is being carried.

(4 marks)

Untuk tujuan analisis, plat biasanya dianggap sebagai struktur 2-dimensi manakala rasuk dianggap sebagai struktur 1-dimensi. Terangkan secara ringkas perbezaan utama antara satu plat dengan satu rasuk dari segi cara pemindahan beban melintang.

(b) A 5m x 5m square simply supported slab with thickness of 275mm carries a distributed load with intensity of $3kN/m^2$ at the center of slab. The load varies sinusoidally to zero at the supporting edges. Calculate the maximum bending moment. Take $E = 25kN/mm^2$ and $\mu = 0.3$.

(21 marks)

Satu papak disokong mudah dengan ukuran 5m x 5m dan ketebalan 275mm menanggung satu beban teragih dengan keamatan $3kN/m^2$ pada titik tengah papak. Beban teragih ini berubah secara sinus sehingga nilai sifar di sepanjang sisi penyokong papak. Kira nilai maksima momen lentur. Anggap $E=25kN/mm^2$ dan $\mu=0.3$.

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Soalan asal

EAS 567 (Theory of Plates and Shells)

Q1.
(a) Shells are structures with curved surfaces. Explain the advantages of a curved surface from the point of view of load transfer when compared to a flat surface.

(6 marks)

(b) State the three basic equations which are necessary in the analysis of shell structures. Explain clearly all symbols used in the corresponding equations .

(9 marks)

(c) Kirchhoff-Love's assumptions are used in the derivation of basic equations for shells from the corresponding equations from the theory of 3D elasticity. State all the assumptions and explain how basic equations in the theory of 3D elasticity are simplified as a consequence of each of the assumption.

(10 marks)

Q2.
(a) By using separate diagrams of (i) a stressed shell element and (ii) a shell element neutral surface, show clearly the stress resultants and the corresponding components of stress which are considered in the theory of membrane.

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(b) Making use of suitable diagrams, explain the main difference between theory of membrane and theory of bending from the point of view of stress and moment resultants which are considered in each theory.

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(c) Table 1 shows the set of equations for membrane stress resultants N_φ and N_θ for an axisymmetrically loaded shell of revolution. Derive the equations of N_φ and N_θ for the case of a spherical shell.

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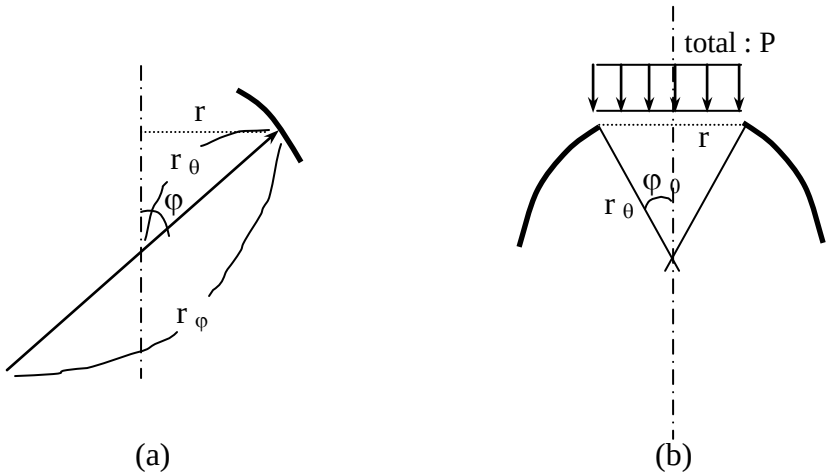

Fig. 1

By using the derived equations, show that the equations for membrane stress resultants for a spherical shell which is loaded with a varying liquid pressure as shown in Fig.2 could be written as follows :

$$N_{\varphi} = -\frac{\gamma a^2}{6}\left[1+\frac{3H}{a}-\frac{2\cos^2\varphi}{1+\cos\varphi}\right]$$

$$N_{\theta} = -\frac{\gamma a^2}{6}\left[-1+\frac{3H}{a}-\frac{4\cos^2\varphi-6}{1+\cos\varphi}\right]$$

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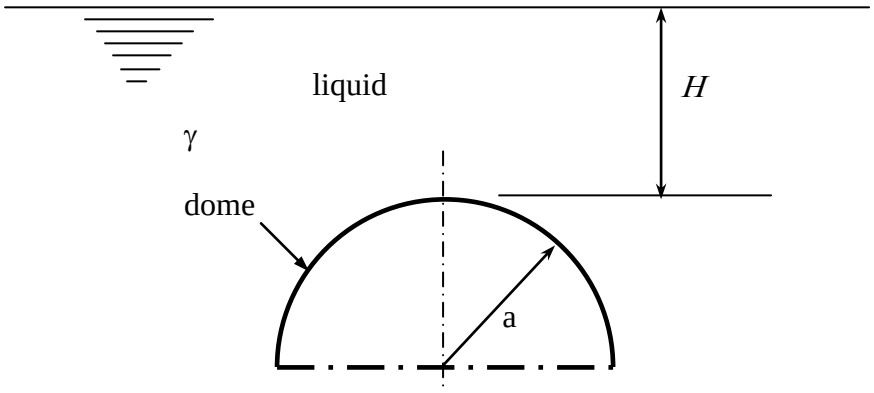

Fig. 2

(14 marks)

Q3.

Fig.3 shows the bending stress resultants M_x and M_y acting on sections of a thin plate. Show that the bending moment M_x and M_y are given by the following equations :

$$M_x = -D\left[\frac{\partial^2 w}{\partial x^2} + \mu\frac{\partial^2 w}{\partial y^2}\right]; \quad \text{with } D = \frac{Eh^3}{12(1-\mu^2)}$$

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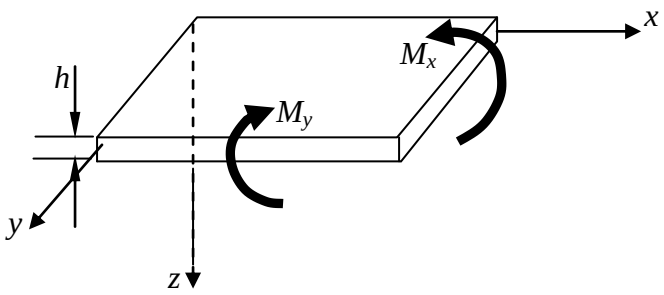

Fig.3

Q4.

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(b) A 5m x 5m square simply supported slab with thickness 275mm carries a distributed load of intensity 3kN/m^2 at the center of slab. The load varies sinusoidally to zero at the supporting edges. Calculate the maximum bending moment. Take E=25kN/mm^2 and μ=0.3.

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