# UNIVERSITI SAINS MALAYSIA 

$2^{\text {nd }}$. Semester Examination
2003/2004 Academic Session

February / March 2003

## EUM 213/3 - Operational Research

Duration : 3 hours

## Instructions to candidates:

1. Ensure that this paper contains FOUR (4) printed pages before you start your examination.
2. This paper contains FIVE (5) questions. Answer FOUR (4) questions only. Marks will be given to the FIRST FOUR (4) questions put in order on the answer script and NOT the BEST FOUR (4).
3. All questions carry equal marks.
4. All questions MUST BE answered in Bahasa Malaysia.
5. Write answered question numbers on the cover sheet of the answer script.
6. A company produces two products (Product A and Product B) requiring resources P and Q . Management wants to determine how many units of each product to produce so as to maximize profit. For each unit of Product $A$, 1 unit of resource $P$ and 2 units of resource $Q$ are required. For each unit of Product $B, 3$ units of resource $P$ and 2 units of resource $Q$ are required. The company has 200 units of resource $P$ and 300 units of resource Q. Each unit of Product A gives a profit of RM1. Each unit of Product B, up to 60 units, gives a profit of RM2 that is it will not bring any profit to the company if it produces in excess of 60 units.

Formulate a linear programming model for this problem and solve it using any appropriate simplex method.
(25 marks)
2. (a) Explain clearly the meaning of the following terms:
I. Optimization rule for the M method.
II. The two phase technique.
III. Degeneracy.
IV. North-west corner rule.
V. Hungarian method.
(10 marks)
(b) Solve the following linear programming problem using the M method: Maximize $z=2 x_{1}+4 x_{2}+3 x_{3}$ subject to

$$
\begin{array}{ll}
x_{1}+3 x_{2}+2 x_{3} & =20 \\
x_{1}+5 x_{2} & \geq 10 \\
x_{1}, x_{2}, x_{3} \geq 0 &
\end{array}
$$

3. Assume that a project consists of nine activities. The following table provides the activities of this project:

| Activity | Immediate <br> Predecessor | Optimistic <br> Time | Pessimistic <br> Time | Most Likely <br> Time |
| :---: | :---: | :---: | :---: | :---: |
| A | - | 2 | 5 | 8 |
| B | A | 6 | 9 | 12 |
| C | A | 6 | 7 | 8 |
| D | B,C | 1 | 4 | 7 |
| E | A | 8 | 8 | 8 |
| F | D,E | 5 | 14 | 17 |
| G | C | 3 | 12 | 21 |
| H | F,G | 3 | 6 | 9 |
| I | H | 5 | 8 | 11 |

I. Draw the project network.
II. Find the expected duration and variance of this project.
(8 marks)
III. What is the probability that the project can be completed in 42 days?
4. (a) Explain clearly the FOUR (4) cost components in an inventory model.
(b) State clearly the TWO (2) types of inventory model.
4. (c) A university must decide between two new programs which will be introduced. Management wants to know the interest in the programs will be high, medium or low. Projected increases in student population and their probabilities are shown below:

| Interest | Probability | Increase |  |
| :---: | :---: | :---: | :---: |
|  |  | Program 1 | Program 2 |
| High | 0.6 | 220 | 200 |
| Medium | 0.3 | 170 | 180 |
| Low | 0.1 | 110 | 150 |

What is the optimal action under the minimax criterion, maximax criterion and the Bayes decision rule?
(12 marks)
5. (a) Explain clearly the meaning of the Poisson process.
(b) If $p_{n}$ is the probability that there are n customers in the system at steady state, show that

$$
p_{n}=\frac{\lambda_{n-1} \lambda_{n-2} \cdots \lambda_{1} \lambda_{0}}{\mu_{n} \mu_{n-1} \cdots \mu_{2} \mu_{1}}
$$

(10 marks)
(c) A service station has one gasoline pump. Cars wanting gasoline arrive according to the Poisson process at a mean rate of 15 per hour. However, if the pump is already being used, these potential customers may balk (they will drive to another service station) that is, if there are n cars already at the service station, the probability that the arriving potential customer will balk is $\frac{n}{3}$ with $n=1,2,3$. The time required to service a car has an exponential distribution with a mean of 4 minutes.
I. Construct a rate diagram for this queueing system.
II. Find the steady state distribution for the number of cars at the station.
III. Find the expected waiting time in the system

