UNIVERSITI SAINS MALAYSIA

Peperiksaan Semester Kedua

Sidang 1987/88

CST202 - Kejuruteraan Sofwer

Tarikh: 13 April 1988 Masa: 9.00 pagi - 12.00 tengahari
(3 jam)

Sila pastikan bahawa kertas peperiksaan ini mengandungi 5 muka surat yang bercetak sebelum anda memulakan peperiksaan ini.

Jawab SEMUA soalan.

Semua soalan mesti dijawab dengan menggunakan Bahasa Malaysia.

1. (a) Berikan takrifan tepat untuk fungsi berikut:

pembahagi-sepunya : $N_1 \times N_1 \times N_1 \rightarrow B$

pembahagi-sepunya (x,y,z) bernilai benar jika z adalah pembahagi kepada x dan juga y.

(PERHATIAN: Jangan gunakan operator MOD atau DIV di dalam takrifan anda)

(10/100)

...2/-

(b) Berikan takrifan tersirat (spesifikasi formal) untuk fungsi yang menjalankan tugas berikut:

Apabila diinputkan dua nombor tabii m dan n, yang mana kedua-duanya besar atau sama dengan 3, ia akan mengembalikan satu pasangan nombor tabii p dan q yang mana p ialah nombor genap yang terbesar di kalangan nombor genap yang kurang dari m; manakala q pula ialah nombor perdana yang terbesar di kalangan nombor perdana yang kurang dari n.

(PERHATIAN:

* Nombor perdana ialah nombor integer yang besar atau sama dengan nombor 2 tetapi hanya boleh dibahagi dengan tepat oleh nombor 1 dan nombor itu sendiri sahaja.

* Nombor genap ialah nombor integer yang boleh dibahagi dengan tepat oleh nombor 2.

* Gunakan FUNG sebagai nama fungsi ini dan takrifkan beberapa sub-fungsi untuk membuatkan spesifikasi senang difahami).

(40/100)

(c) Buktikan secara formal:

$$(E_1 \vee E_2) \wedge (E_1 \vee E_3) \vdash E_1 \vee (E_2 \wedge E_3)$$

(50/100)

2. (a) Berikan satu spesifikasi formal untuk satu operasi yang mempunyai kebenaran membaca dan menulis ke atas dua pembolehubah luar, iaitu i dan j. Walaupun kedua-dua pembolehubah tersebut boleh bertukar nilai isinya, adalah diperlukan bahawa hasil tambah nilai i dan j selepas perlaksanaan operasi ini mestilah kurang dari hasil darab nilai kedua-duanya sebelum perlaksanaan operasi. Juga, operasi ini mestilah mengurangkan nilai i setiap kali ia dilaksanakan. Andaikan i dan j mengandungi nombor tabii.

(Gunakan OP1 sebagai nama operasi ini)

(20/100)

(b) Andaikan X ialah satu set terhingga yang mengandungi objek-objek.

Ditakrifkan fungsi berikut:

$$\text{invp} : \text{set of (set of } X) \rightarrow B$$
$$\text{invp}(p) \triangleq \{\ \} \notin p \wedge \forall\, y \in X \cdot \exists!\, s \in p \cdot y \in s$$

Seterusnya kita takrifkan set PETAK (X) seperti berikut:

$$\text{PETAK}(X) = \{ p \in \text{set of (set of } X) \mid \text{invp}(p) \}$$

yang mana p dikatakan sebagai satu **petak** untuk set X.

Andaikan kita berikan takrifan tepat untuk satu fungsi baru seperti berikut:

$$\text{gabung} : \text{PETAK}(X) \rightarrow \text{set of (set of } X)$$
$$\text{gabung}(p) \triangleq \{ s \mid s \in p \wedge \text{fbb}(s) \} \cup \{ \bigcup \{ s \mid s \in p \wedge \neg \text{fbb}(s) \} \}$$

yang mana fungsi fbb mempunyai signatur seperti berikut:

$$\text{fbb} : \text{set of } X \rightarrow B$$

Andaikan X = {a,b,c,d,e,f,g,h,i,j}. Untuk bahagian (i) dan (ii) yang berikut, berikan satu takrifan tepat untuk fbb(s) supaya GABUNG(p) bernilai seperti yang diinginkan apabila p mempunyai nilai seperti yang diberikan. Jika anda fikirkan tiada takrifan untuk fbb wujud untuk tujuan tersebut, beri alasan mengapa.

(i) <u>Diberikan</u>

p = {{a}, {b,c},{d,e,f},{g,h,i,j}}

<u>Inginkan</u>

GABUNG(p) = {{b,c},{a,d,e,f},{g,h,i,j}}

(ii) <u>Diberikan</u>

p = {{a},{b,c},{d,e,f},{g,h,i,j}}

<u>Inginkan</u>

GABUNG(p)={{a,d,e,f},{b,c,g,h,i,j}}

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(iii) Terangkan sifat-sifat takrifan tepat untuk fungsi fbb yang boleh membuatkan GABUNG(p) ε PETAK (X). Buktikan kebenaran jawapan anda dengan secara tak formal.

(50/100)

(c) Buktikan bahawa takrifan tepat untuk fungsi berikut memenuhi takrifan tersirat (spesifikasi) untuknya:

Takrifan tersirat

TUKAR(x : Z) r : Z
pre-TUKAR(x) $\underline{\Delta}$ x = 3 $\vee$ x = 7
post-TUKAR(x,$\overline{r}$) $\underline{\Delta}$ x = 3 $\wedge$ r = 7 $\vee$ x = 7 $\wedge$ r = 3

Takrifan tepat

TUKAR : Z $\rightarrow$ Z
TUKAR(x) $\underline{\Delta}$ 10 - x

Anda perlu buktikan:

$\forall$ m ε Z • pre-TUKAR(m) $\Rightarrow$ TUKAR(m) ε Z $\wedge$ post-TUKAR(m,TUKAR(m))

iaitu:

m ε Z $\vdash$ pre-TUKAR(m) $\Rightarrow$ TUKAR(m) ε Z $\wedge$ post-TUKAR(m,TUKAR(m))

(30/100)

3. (a) Buktikan bahawa takrifan tepat untuk fungsi-fungsi berikut memenuhi takrifan tersirat (spesifikasi) untuknya:

Takrifan tersirat

TIGAAN (x : Z) r : Z
pre-TIGAAN(x) $\underline{\Delta}$ benar
post-TIGAAN(x,$\overline{r}$) $\underline{\Delta}$ 3 * x = r

Takrifan tepat

TIGAAN : Z $\rightarrow$ Z
TIGAAN(x) $\underline{\Delta}$ DUAAN(x) + x

...5/-

Anda perlu buktikan:

$$\forall m \in Z \cdot \text{TIGAAN}(m) \in Z \wedge \text{post-TIGAAN}(m, \text{TIGAAN}(m))$$

iaitu:

$$m \in Z \vdash \text{TIGAAN}(m) \in Z \wedge \text{post-TIGAAN}(m, \text{TIGAAN}(m))$$

Untuk membuktikan ini anda boleh membuat andaian bahawa spesifikasi atau takrifan tersirat untuk fungsi DUAAN adalah seperti berikut:

```
DUAAN(x : Z) r : Z
pre-DUAAN(x) Δ benar
post-DUAAN(x,r̄) Δ r = 2 * x
```

(50/100)

(b) Takrifan tersirat

```
EKS (x : N,y : N) r : N
pre-EKS (x,y) Δ x > 0
post-EKS (x,y,r̄) Δ r = x^y
```

Takrifan tepat

```
EKS : N x N → N
EKS(x,y) Δ if y = 0 then 1
           else EKS(x,y-1) * x
```

Anda perlu buktikan:

$$\forall m, n \in N \cdot \text{pre-EKS}(m,n) \Rightarrow \text{EKS}(m,n) \in N \wedge \text{post-EKS}(m,n, \text{EKS}(m,n))$$

iaitu:

$$m, n \in N \vdash \text{pre-EKS}(m,n) \Rightarrow \text{EKS}(m,n) \in N \wedge \text{post-EKS}(m,n, \text{EKS}(m,n))$$

(50/100)

...ooOoo...

Appendix E

Glossary of Symbols

Numbers

$\mathbf{N}_1 = \{1, 2, \ldots\}$	
$\mathbf{N} = \{0, 1, 2, \ldots\}$	
0, succ	as generators
$\mathbf{Z} = \{\ldots, -1, 0, 1, \ldots\}$	
$\mathbf{R}$ = real numbers	
normal arithmetic operators (e.g. $+, -, <$)	
mod	modulus

Functions

$f: D_1 \times D_2 \rightarrow R$	signature
$f(d)$	application
$\lambda x \in T \cdot t$	abstraction
if ... then ... else ...	conditional
let x = ... in ...	local definition

Logic

$\mathbf{B} = \{\text{true}, \text{false}\}$

E_i are logical expressions, Γ is a list of logical expressions

$\neg E$	negation [1]
$E_1 \wedge E_2$	conjunction
$E_1 \vee E_2$	disjunction
$E_1 \Rightarrow E_2$	implication
$E_1 \Leftrightarrow E_2$	equivalence
$\forall x \in T \cdot E$	universal quantifier [2]
$\exists x \in T \cdot E$	existential quantifier
$\exists ! x \in T \cdot E$	unique existence
$\Gamma \vdash E$	sequent E can be proved from Γ (hypothesis $\vdash$ conclusion)
$\Gamma \models E$	sequent (E is true in all worlds where Γ all true)
$\frac{\Gamma}{E}$	inference rule
$\frac{E_1}{E_2}$ (double line)	bidirectional inference rule

Sets

S, T are sets, t_i are terms

set of T	all finite subsets of T
$\{t_1, t_2, \ldots, t_n\}$	set enumeration
$\{\}$	empty set
$\oplus$	generator
$\{x \in T \mid E\}$	set comprehension
$\{i, \ldots, j\}$	subset of integers
$t \in S$	set membership
$t \notin S$	$\neg(t \in S)$
$S \subseteq T$	set containments (subset of)
$S \subset T$	strict set containment

---

[1] The five propositional operators are given in decreasing order of priority

[2] With all of the quantifiers, the scope extends as far as possible to the right; no parentheses are required but they can be used for extra grouping.

$S \cap T$	set intersection [3]
$S \cup T$	set union
$S - T$	set difference
$S \diamond T$	symmetric set difference
$\bigcup S$	distributed union
card S	cardinality of a set

Maps

M is a map

map D to R	finite maps
dom M	domain
rng M	range
$\{d_1 \mapsto r_1, d_2 \mapsto r_2, \ldots, d_n \mapsto r_n\}$	map enumeration
$\{\}$	empty map
⦶	generator
$\{d \mapsto f(d) \mid E\}$	map comprehension
$m(d)$	application
$S \lhd M$	domain restriction
$S \mathbin{⩤} M$	domain deletion
$M_1 \dagger M_2$	overwriting

Sequences

s, t are sequences

seq of T	finite sequences
len s	length
$[t_1, t_2, \ldots, t_n]$	sequence enumeration
$[\,]$	empty sequence
cons	generator
$s \frown t$	concatenation
hd s	head
tl s	tail
$s(i, \ldots, j)$	sub-sequence

[3] Intersection is higher priority than union.

Composite Objects

o is a composite object

compose N of ... end	
where $inv\text{-}N() \triangleq \ldots$	invariant
::	compose
nil	omitted object
$mk\text{-}N()$	generator
$s_1(o)$	selector
$\mu(o, s_1 \mapsto t)$	modify a component

Function Specification

$f(d{:}D)\ r{:}R$
pre ... d ...
post ... d ... r ...

Operation Specification

$OP\ (p{:}\ Tp)\ r{:}\ Tr$
ext rd $e_1{:}\ T_1$, wr $e_2{:}\ T_2$
pre ... p ... e_1 ... e_2 ...
post ... p ... e_1 ... $\overleftarrow{e_2}$ ... r ... e_2 ...

Appendix A

Rules of Logic

Conventions

1. $E, E_1, \ldots$ denote logical expressions.
2. $x, y, \ldots$ denote variables over proper elements in a universe.
3. $c, c_1, \ldots$ denote constants over proper elements in a universe.
4. $s, s_1, \ldots$ denote terms which may contain partial functions.
5. $E(x)$ denotes a formula in which x occurs free.
6. $E(s/x)$ denotes a formula obtained by substituting all free occurrences of x by s in E. If a clash between free and bound variables would occur, suitable renaming is performed before the substitution.
7. $E[s_2/s_1]$ denotes a formula obtained by substituting some occurrences of s_1 by s_2. If a clash between free and bound variables would occur, then suitable renaming is performed before the substitution.
8. X is a non-empty set.
9. An "arbitrary" variable is one about which no results have been established.

General Properties

inf $\dfrac{E_1 \vdash E_2;\ E_1}{E_2}$

var-I $\dfrac{}{x^1 \in X}$

commutativity ($\vee / \wedge / \Leftrightarrow$-comm)

$$\frac{E_1 \vee E_2}{E_2 \vee E_1} \qquad \frac{E_1 \wedge E_2}{E_2 \wedge E_1} \qquad \frac{E_1 \Leftrightarrow E_2}{E_2 \Leftrightarrow E_1}$$

associativity ($\vee / \wedge / \Leftrightarrow$-ass)

$$\frac{(E_1 \vee E_2) \vee E_3}{\overline{E_1 \vee (E_2 \vee E_3)}} \qquad \frac{(E_1 \wedge E_2) \wedge E_3}{\overline{E_1 \wedge (E_2 \wedge E_3)}} \qquad \frac{(E_1 \Leftrightarrow E_2) \Leftrightarrow E_3}{\overline{E_1 \Leftrightarrow (E_2 \Leftrightarrow E_3)}}$$

transitivity ($\Rightarrow / \Leftrightarrow$-trans)

$$\frac{E_1 \Rightarrow E_2;\ E_2 \Rightarrow E_3}{E_1 \Rightarrow E_3} \qquad \frac{E_1 \Leftrightarrow E_2;\ E_2 \Leftrightarrow E_3}{E_1 \Leftrightarrow E_3}$$

substitution

=t-subs $\dfrac{s_1 = s_2;\ E}{E[s_2/s_1]}$

=v-subs $\dfrac{s \in X;\ x \in X \vdash E(x)}{E(s/x)}$

=-comm $\dfrac{s_1 = s_2}{s_2 = s_1}$

=-trans $\dfrac{s_1 = s_2;\ s_2 = s_3}{s_1 = s_3}$

$f: D \rightarrow R$

$f(d) \triangleq e$

$e_0 = e(d_0/d)$

---

[1] x is arbitrary

$\underline{\Delta}$-subs $\quad \dfrac{d_0 \in D;\ E(e_0)}{E[f(d_0)/e_0]}$

$\underline{\Delta}$-inst $\quad \dfrac{d_0 \in D;\ E(f(d_0))}{E[e_0/f(d_0)]}$

$f(d) \ \underline{\Delta}\ $ if c then et else ef

if-subs $\quad \dfrac{d_0 \in D;\ e_0;\ E(et_0)}{E[f(d_0)/et_0]} \qquad \dfrac{d_0 \in D;\ \neg e_0;\ E(ef_0)}{E[f(d_0)/ef_0]}$

Definitions of Connectives

f-defn $\quad \dfrac{\neg \text{true}}{\text{false}}$

$\wedge$-defn $\quad \dfrac{\neg(\neg E_1 \vee \neg E_2)}{E_1 \wedge E_2}$

$\Rightarrow$-defn $\quad \dfrac{\neg E_1 \vee E_2}{E_1 \Rightarrow E_2}$

$\Leftrightarrow$-defn $\quad \dfrac{(E_1 \Rightarrow E_2) \wedge (E_2 \Rightarrow E_1)}{E_1 \Leftrightarrow E_2}$

$\forall$-defn $\quad \dfrac{\neg \exists x \in X \cdot \neg E(x)}{\forall x \in X \cdot E(x)}$

Relationships between Operators

deM $\quad \dfrac{\neg(E_1 \vee E_2)}{\neg E_1 \wedge \neg E_2} \qquad \dfrac{\neg(E_1 \wedge E_2)}{\neg E_1 \vee \neg E_2}$

$\dfrac{\neg \exists x \in X \cdot E(x)}{\forall x \in X \cdot \neg E(x)} \qquad \dfrac{\neg \forall x \in X \cdot E(x)}{\exists x \in X \cdot \neg E(x)}$

dist $\dfrac{E_1 \vee E_2 \wedge E_3}{(E_1 \vee E_2) \wedge (E_1 \vee E_3)}$ $\dfrac{E_1 \wedge (E_2 \vee E_3)}{E_1 \wedge E_2 \vee E_1 \wedge E3}$

∃∨-dist $\dfrac{\exists x \in X \cdot E_1(x) \vee E_2(x)}{(\exists x \in X \cdot E_1(x)) \vee (\exists x \in X \cdot E_2(x))}$

∃∧-dist $\dfrac{\exists x \in X \cdot E_1(x) \wedge E_2(x)}{(\exists x \in X \cdot E_1(x)) \wedge (\exists x \in X \cdot E_2(x))}$

∀∨-dist $\dfrac{(\forall x \in X \cdot E_1(x)) \vee (\forall x \in X \cdot E_2(x))}{\forall x \in X \cdot E_1(x) \vee E_2(x)}$

∀∧-dist $\dfrac{(\forall x \in X \cdot E_1(x)) \wedge (\forall x \in X \cdot E_2(x))}{\forall x \in X \cdot E_1(x) \wedge E_2(x)}$

Substitution

∧-subs $\dfrac{E_1 \wedge \ldots \wedge E_i \wedge \ldots \wedge E_n;\ E_i \vdash E}{E_1 \wedge \ldots \wedge E \wedge \ldots \wedge E_n}$

∨-subs $\dfrac{E_1 \vee \ldots \vee E_i \vee \ldots \vee E_n;\ E_i \vdash E}{E_1 \vee \ldots \vee E \vee \ldots \vee E_n}$

∃-subs $\dfrac{\exists x \in X \cdot E_1(x);\ E_1(x) \vdash E(x)}{\exists x \in X \cdot E(x)}$

contr $\dfrac{E_1;\ \neg E_1}{E_2}$

⇒-contrp $\dfrac{E_1 \Rightarrow E_2}{\neg E_2 \Rightarrow \neg E_1}$

	INTRODUCTION(*op-I*)	*ELIMINATION*(*op-E*)
$\neg\neg$	$\frac{E}{\neg\neg E}$	$\frac{\neg\neg E}{E}$
$\vee$	$\frac{E_i}{E_1 \vee E_2 \vee \ldots \vee E_n}$	$\frac{E_1 \vee \ldots \vee E_n;\ E_1 \vdash E;\ \ldots;\ E_n \vdash E}{E}$
$\wedge$	$\frac{E_1;\ E_2;\ \ldots;\ E_n}{E_1 \wedge E_2 \wedge \ldots \wedge E_n}$	$\frac{E_1 \wedge E_2 \wedge \ldots \wedge E_n}{E_i}$
$\neg\vee$	$\frac{\neg E_1;\ \neg E_2;\ \ldots;\ \neg E_n}{\neg(E_1 \vee E_2 \vee \ldots \vee E_n)}$	$\frac{\neg(E_1 \vee E_2 \vee \ldots \vee E_n)}{\neg E_i}$
$\neg\wedge$	$\frac{\neg E_i}{\neg(E_1 \wedge \ldots \wedge E_n)}$	$\frac{\neg(E_1 \wedge \ldots \wedge E_n);\ \neg E_1 \vdash E;\ \ldots;\ \neg E_n \vdash E}{E}$
$\Rightarrow$	$\frac{E_1 \vdash E_2;\ E_1 \in \mathbf{B}}{E_1 \Rightarrow E_2}$	
vac $\Rightarrow$	$\frac{E_2}{E_1 \Rightarrow E_2}$	$\frac{E_1 \Rightarrow E_2;\ \neg E_2}{\neg E_1}$
	$\frac{\neg E1}{E_1 \Rightarrow E_2}$	$\frac{E_1 \Rightarrow E_2;\ E_1}{E_2}$
$\Leftrightarrow$	$\frac{E_1 \wedge E_2}{E_1 \Leftrightarrow E_2}$	$\frac{E_1 \Leftrightarrow E_2}{E_1 \wedge E_2 \vee \neg E_1 \wedge \neg E_2}$
	$\frac{\neg E_1 \wedge \neg E_2}{E_1 \Leftrightarrow E_2}$	
$\neg \Leftrightarrow$	$\frac{E_1 \wedge \neg E_2}{\neg(E_1 \Leftrightarrow E_2)}$	$\frac{\neg(E_1 \Leftrightarrow E_2)}{E_1 \wedge \neg E_2 \vee \neg E_1 \wedge E_2}$

$$\frac{\neg E_1 \wedge E_2}{\neg(E_1 \Leftrightarrow E_2)}$$

$\exists$

$$\frac{s \in X;\ E(s/x)}{\exists x \in X \cdot E(x)} \qquad \frac{\exists x \in X \cdot E(x);\quad y^2 \in X, E(y/x) \vdash E_1}{E_1}$$

$\forall$

$$\frac{x^{3} \in X \vdash E(x)}{\forall x \in X \cdot E(x)} \qquad \frac{\forall x \in X \cdot E(x);\ s \in X}{E(s/x)}$$

$\neg\exists$

$$\frac{x \in X \vdash \neg E(x)}{\neg\exists x \in X \cdot E(x)} \qquad \frac{\neg\exists x \in X \cdot E(x);\ s \in X}{\neg E(s/x)}$$

$\neg\forall$

$$\frac{s \in X;\ \neg E(s/x)}{\neg\forall x \in X \cdot E(x)} \qquad \frac{\neg\forall x \in X \cdot E(x);\quad y^4 \in X, \neg E(y/x) \vdash E}{E}$$

Miscellaneous

$\exists$split

$$\frac{\exists x \in X \cdot E(x,x)}{\exists x, y \in X \cdot E(x,y)}$$

$\forall$fix

$$\frac{\forall x, y \in X \cdot E(x,y)}{\forall x \in X \cdot E(x,x)}$$

$\forall \rightarrow \exists$

$$\frac{\forall x \in X^5 \cdot E(x)}{\exists x \in X \cdot E(x)}$$

$$\frac{\exists x \in X \cdot \forall y \in Y \cdot E(x,y)}{\forall y \in Y \cdot \exists x \in X \cdot E(x,y)}$$

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[2] y is arbitrary and not free in E_1

[3] x is arbitrary

[4] y is arbitrary and not free in E

[5] X is non-empty

$$\frac{\forall x \in X \cdot E_1(x) \;\Leftrightarrow\; E_2(x)}{(\forall x \in X \cdot E_1(x)) \;\Leftrightarrow\; (\forall x \in X \cdot E_2(x))}$$

=-contr
$$\frac{\neg(s = s)}{E}$$

=-term
$$\frac{s \in X}{s = s}$$

=-comp
$$\frac{s_1, s_2 \in X}{(s_1 = s_2) \vee \neg(s_1 = s_2)}$$

Δ-I
$$\frac{E}{\Delta E} \qquad\qquad \frac{\neg E}{\Delta E}$$

Δ-E
$$\frac{\Delta E;\; E \vdash E_1;\; \neg E \vdash E_1}{E_1}$$

¬Δ-I
$$\frac{\Delta E \vdash E_1;\; \Delta E \vdash \neg E_1}{\neg \Delta E}$$

¬Δ-E
$$\frac{\neg \Delta E \vdash E_1;\; \neg \Delta E \vdash \neg E_1}{\Delta E}$$

==-refl
$$\frac{}{s == s}$$

==-subs
$$\frac{s_1 == s_2;\; E}{E[s_2/s_1]}$$

==-comm
$$\frac{s_1 == s_2}{s_2 == s_1}$$

==-trans
$$\frac{s_1 == s_2;\; s_2 == s_3}{s_1 == s_3}$$

==-→=
$$\frac{s_1 == s_2;\; s_i \in X}{s_1 = s_2}$$

=-→==
$$\frac{s_1 = s_2}{s_1 == s_2}$$

Appendix B

Properties of Data

Relations

Ordering: Transitive, Reflexive, Antisymmetric.
Equivalence: Transitive, Reflexive, Symmetric.

Natural Numbers (cf. Section 3.2)

$0 : \mathbf{N}$
$succ : \mathbf{N} \rightarrow \mathbf{N}$

N-ind $$\frac{p(0);\ n \in \mathbf{N}, p(n) \vdash p(n+1)}{n \in \mathbf{N} \vdash p(n)}$$

N-indp $$\frac{p(0);\ n \in \mathbf{N}_1, p(n-1) \vdash p(n)}{n \in \mathbf{N} \vdash p(n)}$$

N-cind $$\frac{n, m \in \mathbf{N}, m < n \ \Rightarrow\ p(m) \vdash p(n)}{n \in \mathbf{N} \vdash p(n)}$$

Sets (cf. Section 4.3)

s, s_i are sets

$\{\}$: set of X
$\_ \oplus \_ : X \times$ set of $X \rightarrow$ set of X

Bentuk umum petua Δ-subs dan Δ-inst.

Andaikan:

$$f : D_1 \times D_2 \times \ldots \times D_n \dashrightarrow R$$

$$f\ (m_1,\ m_2,\ \ldots,\ m_n)\ \Delta\ e$$

$$e_0 = e\ (d_1/m_1,\ \ldots,\ d_n/m_n)$$

Δ-subs $$\frac{d_1 \in D_1,\ \ldots.,\ d_n \in D_n\ ;\ E(e_0)}{E\ [f(d_1,\ \ldots.,\ d_n)/e_0]}$$

Δ-inst $$\frac{d_1 \in D_1,\ \ldots.,\ d_n \in D_n;\ E(f(d_1,\ \ldots,\ d_n))}{E[e_0/f\ (d_1,\ \ldots.,\ d_n)]}$$