UNIVERSITI SAINS MALAYSIA

Peperiksaan Semester Kedua
Sidang Akademik 1993/94

April 1994

CSI501 - Logic and Inference Systems

Masa: [3 jam]

ARAHAN KEPADA CALON:

- Sila pastikan bahawa kertas peperiksaan ini mengandungi **LIMA** muka surat yang bercetak sebelum anda memulakan peperiksaan ini.

Answer **ALL** questions.

1. (a) For each of the following formula, argue informally whether it is valid or not. If it is not, show whether it is consistent(satisfiable) or inconsistent(unsatisfiable).

   (i) $\forall x\, P(x) \Rightarrow \exists x\, P(x)$

   (ii) $\exists y\, Q(y) \Rightarrow \forall x\, Q(x)$

   (iii) $\neg[\, Q(a) \Rightarrow \exists x\, Q(x)\,]$

   (7/25)

   (b) Let $\mathbf{I_1}$ be an interpretation as follows :

   Domain $\mathbf{D} = \{ a, b \}$.
   $P(a,a) \leftarrow T$
   $P(a,b) \leftarrow F$
   $P(b,a) \leftarrow T$
   $P(b,b) \leftarrow F$

   Determine the truth value of the following formulas under $\mathbf{I_1}$:

   (i) $\forall x\, \exists y\, P(x,y)$

   (ii) $\exists y\, \forall x\, P(x,y)$

   (6/25)

   (c) Put the following predicate-calculus formula into clause form :

   $$\exists x\, \exists y\, \forall z\, \{\, (\, P(x,y) \Rightarrow [\, P(y,z) \wedge P(z,z)\,]\,) \wedge (\,[\, P(x,y) \wedge Q(x,y)\,] \Rightarrow [\, Q(x,z) \wedge Q(z,z)\,]\,)\,\}$$

   (7/25)

   (d) Given the following substitutions α and β,

   $\alpha = \{\, x \leftarrow g(y),\ y \leftarrow g(b),\ z \leftarrow u\,\}$

   $\beta = \{\, y \leftarrow g(b),\ u \leftarrow z,\ v \leftarrow g(g(b))\,\}$

   $E = p(u,v,x,y,z)$

   Find the value of $E\alpha$, $(E\alpha)\beta$, $\alpha\beta$, $E(\alpha\beta)$.

   (5/25)

82

2. (a) Let S : P(x) ∨ Q(x, f(y)) be a clause with two literals. Describe the Herbrand universe of S, Herbrand base of S and give one Herbrand interpretation which a model of S.

(5/25)

(b) Consider the following interpretation $\mathbf{I_2}$:

Domain : $D = \{ 1, 2 \}$

Assignment of constants a and b :

$a \leftarrow 1$
$b \leftarrow 2$

Assignment for function f :

$f(1) = 2$
$f(2) = 1$

Assignment for predicate P :

$P(1,1) \leftarrow T$
$P(1,2) \leftarrow T$
$P(2,1) \leftarrow F$
$P(2,2) \leftarrow F$

(i) Let **G** : $\forall x \ \forall y \ [P(x,y) \Rightarrow P(f(x), f(y))]$. Evaluate the truth value of **G** under $\mathbf{I_2}$. Is $\mathbf{I_2}$ a model of **G** ? If not, find another interpretation $\mathbf{I_3}$ which is a model of **G**.

(ii) Find a Herbrand interpretation which satisfies **G**. Justify your answer.

(8/25)

(c) What is meant by a ground instance of a clause ? Show informally that the following observations are true :

(i) A clause C is falsified by an interpretation **I** if and only if there is at least one ground instance C' of C such that C' is not satisfied by **I**.

(ii) A set S of clauses is unsatisfiable if and only if for every interpretation **I**, there is at least one ground instance C' of some clause C in S such that C' is not satisfied by **I**.

(7/25)

(d) State and discuss the significance of the Herbrand's Theorem in mechanical theorem proving.

(5/25)

83

3. (a) Use the ground resolution method to prove that the following well-formed formula is valid :

$$[\ \forall x\ \exists y\ P(x,y)\ \wedge \forall y\ \exists z\ Q(y,z)\ ] \Rightarrow \forall x\ \exists y\ \exists z\ (P(x,y)\ \wedge\ Q(y,z))$$

(5/25)

(b) Consider the following set **S** of clauses :

1. $D(x,x)$
2. $\neg D(x,y)\quad \neg D(y,z)\quad D(x,z)$
3. $P(x)\quad D(g(x), x)$
4. $P(x)\quad L(w, g(x))$
5. $P(x)\quad L(g(x), x)$
6. $L(w,a)$
7. $\neg P(x)\quad \neg D(x,a)$
8. $\neg L(w,x)\quad \neg L(x,a)\quad P(f(x))$
9. $\neg L(w,x)\quad \neg L(x,a)\quad D(f(x),x)$

Use either input resolution or set of support strategy (SOS) to find a refutation for S. If you choose to use SOS, you are free to find an appropriate initial support set.

Note : Only **a** is a skolem constant in **S**.

(10/25)

(c) Consider the following statements :

S1 : Anyone who likes Ali will choose Ahmad for his team.
S2 : Ahmad is not a friend of anyone who is a friend of Abdul.
S3 : Kassim will choose no one but a friend of Karim for his team.

Use the resolution method to prove that if Karim is a friend of Abdul, then Kassim does not like Ali.

(6/25)

(d) Describe the main differences between paramodulation and demodulation rules of inference.

(4/25)

4. (a) Briefly discuss the relationship between mechanical theorem proving, logic programming and PROLOG. (5/25)

(b) Find an SLD-refutation to show that **p(n,b)** is a logical consequence of the following logic program :

$p(x,y) \leftarrow q(x), f(y).$
$q(x) \leftarrow r(x).$
$f(x) \leftarrow t(x).$
$t(a) \leftarrow$
$t(b) \leftarrow$
$r(n) \leftarrow$

<u>Note</u> : Only a,b, and n are Skolem constants. (6/25)

(c) Consider the following PROLOG program :

```
rel(a,b).
rel(c,b).
rel(X,Z) :- rel(X,Y), rel(Y,Z).
rel(X,Y) :- rel(Y,X).
```

Describe the behavior of the typical PROLOG interpreter in finding the answer to the query **rel(a,c)**. Is the behavior or answer given desireable ? If not, show how the desireable behavior and answer can be obtained by any other logic programming method. What conclusion can you say about PROLOG ? (7/25)

(d) Write a PROLOG program that can find an intersection of two sets which are given in the form of lists.

<u>Example</u>

?- intersect([a,b,c,d], [b,c,e,g], Y) gives Y = [b, c]
?- intersect([a, b], [c,d], Y) gives Y = []
?- intersect([], [a,b], Y) gives Y = []
?- intersect([a,b]; [], Y) gives Y = []

(7/25)

- oooOooo -