UNIVERSITI SAINS MALAYSIA

Peperiksaan Semester Kedua
Sidang Akademik 2002/2003

Februari/Mac 2003

JIM 417/4 – Persamaan Pembezaan Separa

Masa: 3 jam

Sila pastikan bahawa kertas peperiksaan ini mengandungi **SEMBILAN** muka surat yang bercetak sebelum anda memulakan peperiksaan.

Jawab SEMUA soalan.

Baca arahan dengan teliti sebelum anda menjawab soalan.

Setiap soalan diperuntukkan 100 markah.

...2/-

1. Katakan F fungsi sebarangan, manakala $u(x, y, z) = c_1$ dan $v(x, y, z) = c_2$ dengan c_1 dan c_2 pemalar membentuk penyelesaian persamaan

$$\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}.$$

Tunjukkan bahawa penyelesaian am persamaan pembezaan separa linear

$$P\frac{\partial z}{\partial x} + Q\frac{\partial z}{\partial y} = R.$$

dengan P, Q dan R fungsi-fungsi x, y dan z adalah

$$F(u, v) = 0.$$

Seterusnya, selesaikan persamaan pembezaan separa

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = nu, \quad n \text{ adalah pemalar.}$$

(100 markah)

...3/-

2. Dengan menggunakan kaedah pemisahan pembolehubah, selesaikan masalah nilai awal-sempadan berikut:

$$\frac{\partial^2 u}{\partial t^2} - c^2 \frac{\partial^2 u}{\partial x^2} = 0, \qquad 0 < x < \ell,\ t > 0$$

$$u(x, 0) = f(x), \qquad 0 \le x \le \ell$$

$$\frac{\partial u}{\partial t}(x, 0) = g(x), \qquad 0 \le x \le \ell$$

$$u(0, t) = 0, \qquad t \ge 0$$

$$u(\ell, t) = 0, \qquad t \ge 0.$$

(100 markah)

3. (a) Cari siri Fourier, pada selang $-2 \le x \le 2$ bagi fungsi yang ditakrifkan oleh

$$f(x) = \begin{cases} 2, & -2 \le x \le 0 \\ x, & 0 < x \le 2. \end{cases}$$

(50 markah)

(b) Satu batang logam panjangnya 100 cm dan hujungnya pada $x = 0$ dan $x = 100$ ditebat. Pada mulanya, separuh batang logam itu dari hujung $x = 0$ bersuhu 60°C manakala separuh lagi bersuhu 40°C.

(i) Tentukan syarat awal-sempadan bagi masalah di atas.

(ii) Cari suhu batang logam pada masa t jika persamaan pembezaan separa yang terbentuk daripada masalah di atas diberi oleh

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{k}\frac{\partial u}{\partial t}, \quad 0 < x < 100, \quad t > 0.$$

(50 markah)

...4/-

4. Jelmaan Laplace bagi u(x, t) ditakrifkan seperti berikut:

$$\mathscr{L}\{u(x,\ t)\} = \int_0^\infty e^{-st} u(x,\ t)\, dt = F(x,\ s).$$

(a) Tunjukkan bahawa

$$\mathscr{L}\left\{\frac{\partial}{\partial t} u(x,\ t)\right\} = s\,\bar{u}(x,\ s) - u(x,\ 0)$$

dengan $\bar{u}(x, s) = \mathscr{L}\{u(x, t)\}$.

(15 markah)

(b) Seterusnya, dapatkan

$$\mathscr{L}\left\{\frac{\partial^2}{\partial t^2} u(x,\ t)\right\}$$

dalam sebutan $\bar{u}(x, s)$, $u(x, 0)$ dan $\frac{\partial}{\partial t} u(x, 0)$.

(25 markah)

(c) Dengan menggunakan keputusan dalam (a) dan (b), selesaikan persamaan pembezaan separa

$$\frac{\partial}{\partial x} u(x,\ t) + 2\frac{\partial}{\partial t} u(x,\ t) = 0, \quad x > 0,\ t > 0$$

jika $u(0,\ t) = 0$ dan $u(x,\ 0) = 1$.

(60 markah)

...5/-

5. Dengan menggunakan jelmaan Laplace, selesaikan

$$\frac{\partial}{\partial t}u(x, t) = \frac{\partial^2}{\partial x^2}u(x, t), \ \ x > 0, t > 0$$

$$u(x, 0) = 0, \ \ x > 0$$

$$u(0, t) = f(t), t > 0$$

$$\operatorname*{had}_{x \to \infty} u(x, t) = 0, t > 0$$

$$\left[\text{Diberi } \mathscr{L}^{-1}\left\{e^{-a\sqrt{s}}\right\} = \frac{ae^{-a^2/4t}}{2\sqrt{\pi t^3}}\right].$$

(100 markah)

Senarai Rumus

$$u_x = u_\xi \xi_x + u_\eta \eta_x$$

$$u_y = u_\xi \xi_y + u_\eta \eta_y$$

$$u_{xx} = u_{\xi\xi}\xi_x^2 + 2u_{\xi\eta}\xi_x\eta_x + u_{\eta\eta}\eta_x^2 + u_\xi \xi_{xx} + u_n \eta_{xx}$$

$$u_{xy} = u_{\xi\xi}\xi_x\xi_y + u_{\xi\eta}(\xi_x\eta_y + \xi_y\eta_x) + u_{\eta\eta}\eta_x\eta_y + u_\xi \xi_{xy} + u_n \eta_{xy}$$

$$u_{yy} = u_{\xi\xi}\xi_y^2 + 2u_{\xi\eta}\xi_y\eta_y + u_{\eta\eta}\eta_y^2 + u_\xi \xi_{yy} + u_\eta \eta_{yy}$$

$$f(x) = \frac{a_o}{2} + \sum_{n=1}^{\infty}\left[a_n \operatorname{kos}\left(\frac{n\pi x}{L}\right) + b_n \sin\left(\frac{n\pi x}{L}\right)\right]$$

dengan

$$a_o = \frac{1}{L}\int_{-L}^{L} f(x)\,dx$$

$$a_n = \frac{1}{L}\int_{-L}^{L} f(x)\operatorname{kos}\left(\frac{n\pi x}{L}\right)dx, \quad n = 1, 2, 3, \ldots$$

$$b_n = \frac{1}{L}\int_{-L}^{L} f(x)\sin\left(\frac{n\pi x}{L}\right)dx, \quad n = 1, 2, 3, \ldots$$

$$f(x) = \frac{a_o}{2} + \sum_{n=1}^{\infty} a_n \operatorname{kos}\left(\frac{n\pi x}{L}\right)$$

dengan

$$a_n = \frac{2}{L}\int_{0}^{L} f(x)\operatorname{kos}\left(\frac{n\pi x}{L}\right)dx, \quad n = 1, 2, 3, \ldots$$

$$f(x) = \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{L}\right)$$

dengan

$$b_n = \frac{2}{L}\int_{0}^{L} f(x)\sin\left(\frac{n\pi x}{L}\right)dx, \quad n = 1, 2, 3, \ldots$$

$$f(x) = \frac{1}{2} \sum_{-\infty}^{\infty} c_n e^{inx}$$

dengan

$$c_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) e^{-inx} dx, \quad n = 0, \pm 1, \pm 2, \ldots$$

$\dfrac{d^2 y}{dx^2} - \alpha^2 y = 0$ mempunyai penyelesaian

$y = Ae^{\alpha x} + Be^{-\alpha x}$

$y = C \operatorname{kosh} \alpha x + D \sinh \alpha x$

$\dfrac{d^2 y}{dx^2} + \alpha^2 y = 0$ mempunyai penyelesaian

$y = A \operatorname{kos} \alpha x + B \sin \alpha x$

$$r^2 \frac{d^2 R}{dr^2} + r \frac{dR}{dr} - n^2 R = 0$$

$$R_n = C_n r^n + \frac{D_n}{r^n}.$$

$r \dfrac{d^2 R}{dr^2} + \dfrac{dR}{dr} = 0$ mempunyai penyelesaian

$R = A + B \ell \text{n}\, r$

$$\Im[f(t)] = F(\alpha) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t)\, e^{i\alpha t}\, dt$$

$$f(x) = \Im^{-1}[F(\alpha)] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(\alpha)\, e^{-i\alpha x}\, dx$$

$$\mathfrak{F}[f(x)] = F_s(n) = \frac{2}{L}\int_0^L f(x)\sin\frac{n\pi x}{L}\,dx, \quad n=1,2,...$$

$$f(x) = \mathfrak{F}^{-1}[F_s(n)] = \sum_{n=1}^{\infty} F_s(n)\sin\frac{n\pi x}{L}$$

$$\mathfrak{F}[f(x)] = F_c(n) = \frac{2}{L}\int_0^L f(x)\,\mathrm{kos}\frac{n\pi x}{L}\,dx, \quad n = 1, 2, ...$$

$$f(x) = \mathfrak{F}^{-1}[F_c(n)] = \frac{F_c(0)}{2} + \sum_{n=1}^{\infty} F_c(n)\,\mathrm{kos}\,\frac{n\pi x}{L}$$

$$\mathfrak{F}[f''(x)] = \frac{2n}{\pi}[f(0)-(-1)^n f(\pi)] - n^2 F_s(n)$$

$$\mathfrak{F}[f''(x)] = \frac{2}{\pi}[(-1)^n f'(\pi) - f'(0)] - n^2 F_c(n)$$

$$\mathfrak{L}[f(t)] = F(s) = \int_0^{\infty} e^{-st} f(t)\,dt$$

$$\mathfrak{L}[e^{\alpha t} f(t)] = F(s-\alpha)$$

Jika $g(t) = \begin{cases} 0 & , t < \alpha \\ f(t-\alpha), & t > 0 \end{cases}$

maka

$$\mathfrak{L}[g(t)] = e^{-\alpha s} F(s)$$

$$\mathfrak{L}[f^{(n)}(t)] = s^n F(s) - s^{n-1} f(0) - s^{n-2} f'(0) - ... - sf^{(n-2)}(0) - f^{(n-1)}(0)$$

$$\mathfrak{L}[tf(t)] = -F'(s) = -\frac{d}{ds}\mathfrak{L}[f(t)]$$

$$\mathfrak{L}\left[\int_0^t f(u)\,du\right] = \frac{F(s)}{s}$$

$$\mathfrak{L}^{-1}[F(s)G(s)] = \int_0^t f(u)\,g(t-u)\,du = f * g$$

Jadual Jelmaan Laplace

$f(t)$	$\mathcal{L}\{f(t)\} = F(s)$
1	$\frac{1}{s}$
t_n, $n = 1, 2, 3, \ldots$	$\frac{n!}{s^{n+1}}$
e^{at}	$\frac{1}{s-a}$
kos at	$\frac{s}{s^2+a^2}$
sin at	$\frac{a}{s^2+a^2}$
kosh at	$\frac{s}{s^2-a^2}$
sinh at	$\frac{a}{s^2-a^2}$
t kos bt	$\frac{s^2-a^2}{(s^2+b^2)^2}$
t sin bt	$\frac{2bs}{(s^2+b^2)^2}$
e^{at} kos bt	$\frac{s-a}{(s-a)^2+b^2}$
e^{at} sin bt	$\frac{b}{(s-a)^2+b^2}$

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