UNIVERSITI SAINS MALAYSIA

Stamford College

First Semester Examination
2002/2003 Academic Session
September 2002

External Degree Programme
Bachelor of Computer Science (Hons.)

CPT104 - Introduction to Logic and Abstraction

Duration : 3 hours

INSTRUCTIONS TO CANDIDATE:

- Please ensure that this examination paper contains **FOUR** questions in **SIX** printed pages before you start the examination.
- Answer **ALL** questions.
- This is an "Open Book" Examination.
- On each page, write *only your **Student ID***.

1. (a) Consider the following argument:

If Ali has money, then he must have robbed a bank or won the lottery. Ali doesn't have money and didn't rob any bank. So, Ali won the lottery.

(i) Identify the atomic proposition in the above argument and write them in symbolic form.

(15/100)

(ii) Construct a truth table for the expression given in 1(a)(i) above.

(15/100)

(iii) Identify a situation that makes the above argument invalid.

(10/100)

(b) Consider the following atomic sentences:

B : I go to town.
H : It is raining.
P : I have umbrella.

Convert the following expressions into simple English:

(i) $H \Rightarrow (B \Leftrightarrow P)$

(10/100)

(ii) $((\neg H) \Rightarrow P) \vee B$

(10/100)

(iii) $(H \wedge B) \Rightarrow P$

(10/100)

(c) Simplify the following expression using equivalent rules:

$$(P \wedge Q) \vee \neg(P \Rightarrow Q) \vee (R \vee (R \wedge Q)) \wedge \neg R)$$

(15/100)

(d) Prove the following argument using formal proof method:

$$Q \Rightarrow P,\ P \rightarrow (R \vee S),\ Q \wedge \neg S \vdash R \vee S$$

(15/100)

2. (a) Let P(x), Q(x), R(x) and S(x) be open statements that are defined for a universe of discourse that consists of 5 elements, A, B, C, D and E. The assignments are:

x	P(x)	Q(x)	R(x)	S(x)
A	T	T	T	T
B	F	T	F	T
C	T	F	T	F
D	F	T	T	T
E	T	T	F	T

Determine the truth value of the following predicate expressions. Give an example or situation to support your answer.

(i) $\forall x\,[P(x) \Rightarrow Q(x)]$

(ii) $\forall x\,[Q(x) \Leftrightarrow S(x)]$

(iii) $\forall x\,[\neg R(x) \vee S(x)]$

(iv) $\exists x\,[P(x) \wedge Q(x)]$

(v) $\exists x\,[P(x) \vee R(x)) \Rightarrow S(x)]$

(35/100)

(b) For each variable in the following expression, determine whether it is free or bounded:

(i) $(\exists x\,(\exists y\,P(x, y)) \Rightarrow \forall z\,\neg R(z))$

(ii) $\forall x\,P(x) \vee \neg\exists x\,Q(x, y)$

(iii) $\exists y\,P(x, y) \Rightarrow R(y)$

(20/100)

(c) Write the negation of the following expression without using the connective '$\neg$':

$$A = \forall x\,\exists y\,(x < y \Rightarrow \exists k\,(kx = y))$$

Determine the truth value of A if the universe of discourse is N_1.

(20/100)

(d) Write the following statements in predicate calculus:

(i) At least one student is hardworking.

(ii) No hardworking students fail in the examination.

(iii) There is a student who has not failed in the examination.

(iv) Some students are neither hardworking nor fail in the examination.

(25/100)

3. (a) One function, *check* receives two different natural numbers as input and determines whether the result of subtraction between these two numbers is a negative value.

(i) Give the implicit specification and direct specification of the *check* function.

(10/100)

(ii) By using the *check* function, give the implicit specification of a function, *absolute* which calculates the absolute value for the subtraction of two natural numbers.

(5/100)

(iii) Give the direct specification of a function, *card* which calculates the number of elements in a given set.

(5/100)

(iv) Given a set of natural numbers as input, a function, *total* yields the sum of all elements in that set. Give the implicit specification of that function.

(5/100)

(v) By using the functions above, give the implicit specification of a function, *tsubtraction* which receives two sets of natural numbers as input and calculates absolute value for the difference of total amount between two set of natural numbers.

(5/100)

(b) (i) Write an implicit specification for a function CONVERT that will convert a raw mark to its equivalent letter grade based on the scale given below:

Letter Grade	Marks
A	Above 80
B	75 - 80
C	60 - 75
D	40 - 60
F	Below 40

(15/100)

(ii) Given a list of marks, write an implicit specification of a function BEST_GRADE to determine the best grade and then convert it into its equivalent letter grade. Use the function CONVERT in 3(b)(i).

(15/100)

(c) An electric company charges for each residential unit 0.318 sen for the first 150 units consumed and 0.365 sen for the following units. Write an implicit specification that calculates the amount to be charged given the number of units consumed per residential units as input.

(25/100)

(d) Given S = [3, 4, 5, 3, 7] and
R = [2, 3, 6, 3]

Determine:

(i) S (len R)

(ii) Card (elems R)

(iii) hd (tl (tl R))

(15/100)

4. (a) A supplier gives a rebate of RM10 if the quantity is 100 or more and a rebate of RM50 if the quantity is 500 or more. Given the price and quantity as inputs, write the implicit specification and the direct definition of the function 'payment' to calculate the payment amount.

(25/100)

(b) The management of a construction company would like to keep information on its projects. The information to be kept are name of project, owner of project, cost of project, project start date, project end date and project manager's name.

(i) Write a suitable specification to model the data for the above problem (Note that the start date must be less than the end date).

(10/100)

(ii) Given the information of a project as input, write an implicit specification of a function that will return the cost of the project.

(10/100)

(iii) Write an implicit specification of an operation to calculate the total cost of all the projects handled by the company. Use the function in 4(b)(ii).

(20/100)

(iv) Given a project manager's name, write a specification for an operation to determine the total cost of all the projects managed by him/her.

(20/100)

(v) Write a specification for an operation that will return the name of project with highest cost.

(15/100)

- oooOooo -