UNIVERSITI SAINS MALAYSIA

Second Semester Examination
2012/2013 Academic Session

June 2013

MST 565 - Linear Models
[Model Linear]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of EIGHT pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi LAPAN muka surat yang bercetak sebelum anda memulakan peperiksaan ini].

Instructions: Answer **all nine** [9] questions.

*[**Arahan:*** *Jawab **semua sembilan** [9] soalan.]*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai].

1. Let $\mathbf{A} = \begin{pmatrix} 6 & 2 & 3 \\ 4 & -1 & 6 \\ 2 & 0 & -4 \end{pmatrix}$ and $\mathbf{x} = \begin{pmatrix} 7 \\ 3 \\ 4 \end{pmatrix}$

(a) Find a matrix $\mathbf{B} \neq \mathbf{A}$ such that $\mathbf{Ax} = \mathbf{Bx}$. Why is this possible? Can **A** and **B** be nonsingular? Can **A-B** be nonsingular?

(b) Find a matrix $\mathbf{C} \neq 0$ such that $\mathbf{Cx} = 0$. Can **C** be nonsingular?

[10 marks]

1. *Biarkan* $\mathbf{A} = \begin{pmatrix} 6 & 2 & 3 \\ 4 & -1 & 6 \\ 2 & 0 & -4 \end{pmatrix}$ *dan* $\mathbf{x} = \begin{pmatrix} 7 \\ 3 \\ 4 \end{pmatrix}$

(a) *Cari matriks* $\mathbf{B} \neq \mathbf{A}$ *supaya* $\mathbf{Ax} = \mathbf{Bx}$. *Kenapa perkara ini boleh berlaku? Bolehkah **A** dan **B** adalah tak singular? Bolehkah* **A-B** *tak singular?*

(b) *Cari matriks* $\mathbf{C} \neq 0$ *supaya* $\mathbf{Cx} = 0$. *Bolehkah **C** tak singular?*

[10 markah]

2. Let $\mathbf{T} = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix}$

(a) Find $\mathbf{T}^{\frac{1}{2}}$.

(b) Show that $\left(\mathbf{T}^{\frac{1}{2}}\right)^2 = \mathbf{T}$.

[10 marks]

2. *Biarkan* $\mathbf{T} = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix}$

(a) *Cari* $\mathbf{T}^{\frac{1}{2}}$.

(b) *Tunjukkan* $\left(\mathbf{T}^{\frac{1}{2}}\right)^2 = \mathbf{T}$.

[10 markah]

3. Assume $\mathbf{X}=\begin{pmatrix} X_1 \\ \vdots \\ X_n \end{pmatrix}$ where $X_1, X_2, \ldots, X_n$ are $N\ \mu, \sigma^2$ and independent. Let $\boldsymbol{\mu}=\begin{pmatrix} \mu \\ \vdots \\ \mu \end{pmatrix}$ and defined vector $\bar{\mathbf{X}}=\begin{pmatrix} \bar{X} \\ \vdots \\ \bar{X} \end{pmatrix}$.

(a) Show that $\bar{\mathbf{X}}$ and $\mathbf{X}\text{-}\bar{\mathbf{X}}$ are independent vectors.

(b) Find the distribution of $\dfrac{\mathbf{X}\text{-}\bar{\mathbf{X}}\ '\ \mathbf{X}\text{-}\bar{\mathbf{X}}}{\sigma^2}$.

[10 marks]

3. Andaikan $\mathbf{X}=\begin{pmatrix} X_1 \\ \vdots \\ X_n \end{pmatrix}$ *di mana* $X_1, X_2, \ldots, X_n$ *adalah* $N\ \mu, \sigma^2$ *dan tak bersandar. Biarkan* $\boldsymbol{\mu}=\begin{pmatrix} \mu \\ \vdots \\ \mu \end{pmatrix}$ *dan definisikan vektor* $\bar{\mathbf{X}}=\begin{pmatrix} \bar{X} \\ \vdots \\ \bar{X} \end{pmatrix}$.

(a) Tunjukkan $\bar{\mathbf{X}}$ *dan* $\mathbf{X}\text{-}\bar{\mathbf{X}}$ *adalah vektor tak bersandar.*

(b) Cari taburan $\dfrac{\mathbf{X}\text{-}\bar{\mathbf{X}}\ '\ \mathbf{X}\text{-}\bar{\mathbf{X}}}{\sigma^2}$.

[10 markah]

4. Given that $\begin{pmatrix} U \\ V \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{21} & \sigma_2^2 \end{pmatrix}\right)$, show that $Cov\ U^2, V^2\ = 2\left[Cov\ U, V\ \right]^2$.

[8 marks]

4. Diberikan $\begin{pmatrix} U \\ V \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{21} & \sigma_2^2 \end{pmatrix}\right)$, *tunjukkan* $Cov\ U^2, V^2\ = 2\left[Cov\ U, V\ \right]^2$.

[8 markah]

5. Let **y** be a normal random vector with mean $\boldsymbol{\mu} = \begin{pmatrix} 0 \\ -4 \\ 1 \end{pmatrix}$ and variance **I**. Let $\mathbf{S} = \frac{1}{3}\begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix}$.

(a) Find E **y'Sy** .

(b) Find the distribution of **y'Sy** .

(c) Let **z** be a normal random vector that is independent of **y,** with mean $\begin{pmatrix} 2 \\ 0 \\ 2 \end{pmatrix}$ and variance **I**. Find the distribution of **y'y+z'z** .

[12 marks]

5. Biarkan **y** *vektor rawak normal dengan min* $\boldsymbol{\mu} = \begin{pmatrix} 0 \\ -4 \\ 1 \end{pmatrix}$ *dan varians* **I**. *Jika* $\mathbf{S} = \frac{1}{3}\begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix}$.

(a) Cari E **y'Sy** .

(b) Cari taburan bagi **y'Sy** .

(c) Biarkan **z** *adalah vektor rawak normal yang tak bersandar kepada* **y,** *dengan min* $\begin{pmatrix} 2 \\ 0 \\ 2 \end{pmatrix}$ *dan varians* **I**. *Cari taburan bagi* **y'y+z'z** .

[12 markah]

6. The number of pounds of steam used per month at a plant is thought to be related to the average monthly ambient temperature. The past year's usages and temperatures are in Table 1.

Table 1

Month	Temperature	Usage/100	Month	Temperature	Usage/100
Jan.	21	185.79	Jul.	68	621.55
Feb.	24	214.47	Aug.	74	675.06
Mar.	32	288.03	Sept.	62	562.03
Apr.	47	424.84	Oct.	50	452.93
May	50	454.68	Nov.	41	369.95
Jun.	59	539.03	Dec.	30	273.98

(a) Write down the linear model in matrix form.
(b) Write down the normal equations for this model.
(c) Solve the normal equations to estimate the parameters.

[12 markah]

6. Bilangan paun stim digunakan sebulan di kilang dianggap berkait rapat dengan purata suhu persekitaran bulanan. Penggunaan dan suhu pada tahun lalu adalah seperti dalam Jadual 1.

Jadual 1

Bulan	*Suhu*	*Penggunaan/100*	*Bulan*	*Suhu*	*Penggunaan/100*
Jan.	*21*	*185.79*	*Jul.*	*68*	*621.55*
Feb.	*24*	*214.47*	*Ogs.*	*74*	*675.06*
Mac	*32*	*288.03*	*Sept.*	*62*	*562.03*
Apr.	*47*	*424.84*	*Okt.*	*50*	*452.93*
Mei	*50*	*454.68*	*Nov.*	*41*	*369.95*
Jun	*59*	*539.03*	*Dis.*	*30*	*273.98*

(a) Tuliskan model linear dalam bentuk matriks.
(b) Tuliskan persamaan normal untuk model ini.
(c) Selesaikan persamaan normal untuk menganggarkan parameter.

[12 marks]

7. An engineer interested in predicting the delivery time required by a driver to service a vending machine in an outlet. He has suggested that the two most important variables affecting the delivery time are the number of cases of product stocked and the distance walked by the driver. The engineer has collected 25 observations. Based on SAS output given in Figure 1, discuss the suggested fitted model.

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	2	5550.81092	2775.40546	261.24	<.0001
Error	22	233.73168	10.62417		
Corrected Total	24	5784.54260			

Root MSE	3.25947	R-Square	0.9596
Dependent Mean	22.38400	Adj R-Sq	0.9559
Coeff Var	14.56162		

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|
Intercept	1	2.34123	1.09673	2.13	0.0442
cases	1	1.61591	0.17073	9.46	<.0001
distance	1	0.01438	0.00361	3.98	0.0006

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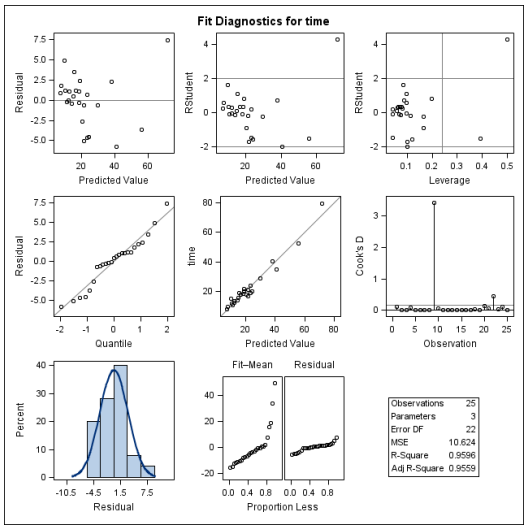

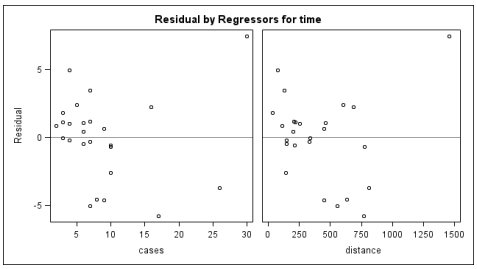

Figure 1

[14 marks]

7. Seorang jurutera berminat untuk meramalkan masa penghantaran yang diperlukan oleh pemandu untuk servis mesin layan diri di sebuah kedai. Beliau telah mencadangkan bahawa dua pembolehubah paling penting yang mempengaruhi masa penghantaran adalah bilangan kes produk dipenuhi dan jarak berjalan oleh pemandu. Jurutera tersebut telah mengumpul 25 cerapan. Berdasarkan output SAS yang diberikan dalam Rajah 1, bincangkan penyuaian model yang disyorkan.

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	2	5550.81092	2775.40546	261.24	<.0001
Error	22	233.73168	10.62417		
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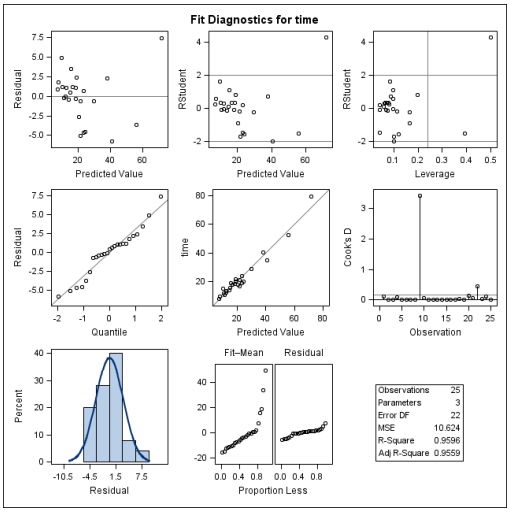

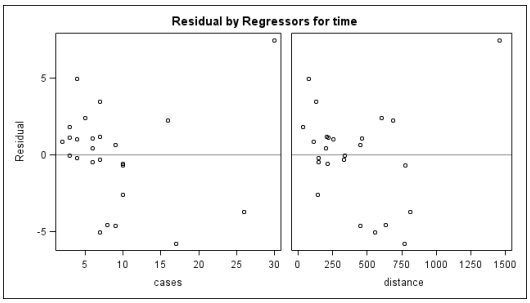

Rajah 1

[14 markah]

8. Refer to the previous question on delivery time required by a driver to service a vending machine. Based on SAS outputs given in Figure 1 and Figure 2, answer the following questions.
   (a) Construct and interpret the 99% confidence interval for the parameters.
   (b) Construct and interpret the 95% confidence interval on the mean delivery time for an outlet requiring 15 cases and where the distance is 425 feet.
   (c) Construct and interpret the 95% prediction interval on the mean delivery time for an outlet requiring 15 cases and where the distance is 425 feet.

Model Crossproducts X'X X'Y Y'Y

Variable	*Intercept*	*cases*	*distance*	*time*
Intercept	25	219	10232	559.6
cases	219	3055	133899	7375.44
distance	10232	133899	6725688	337071.69
time	559.6	7375.44	337071.69	18310.629

X'X Inverse, Parameter Estimates, and SSE

Variable	*Intercept*	*cases*	*distance*	*time*
Intercept	0.1132151861	-0.004448593	-0.000083673	2.3412311452
cases	-0.004448593	0.0027437833	-0.000047857	1.6159072106
distance	-0.000083673	-0.000047857	1.2287447E-6	0.0143848263
time	2.3412311452	1.6159072106	0.0143848263	233.73167742

Figure 2

[14 marks]

8. Merujuk kepada soalan sebelumnya berkaitan masa penghantaran yang dikehendaki oleh pemandu untuk servis mesin layan diri. Berdasarkan output SAS yang diberikan dalam Rajah 1 dan Rajah 2, jawab soalan-soalan berikut.
   (a) Bina dan tafsir selang keyakinan 99% bagi parameter-parameter tersebut.
   (b) Bina dan tafsir selang keyakinan 95% bagi min masa penghantaran ke sebuah kedai yang terdapat 15 kes dan di mana jarak 425 kaki.
   (c) Bina dan tafsir selang ramalan 95% bagi min masa penghantaran ke sebuah kedai yang terdapat 15 kes dan di mana jarak 425 kaki.

Model Crossproducts X'X X'Y Y'Y

Variable	*Intercept*	*cases*	*distance*	*time*
Intercept	25	219	10232	559.6
cases	219	3055	133899	7375.44
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Intercept	0.1132151861	-0.004448593	-0.000083673	2.3412311452
cases	-0.004448593	0.0027437833	-0.000047857	1.6159072106
distance	-0.000083673	-0.000047857	1.2287447E-6	0.0143848263
time	2.3412311452	1.6159072106	0.0143848263	233.73167742

Rajah 2

[14 markah]

9. Three varieties of potato are planted on three plots at each of four locations. The yields in bushels are given in Table 1. Produce the ANOVA table for these data. Does the interaction term appear necessary? Describe your conclusions.

Table 1

Variety	Location 1	Location 2	Location 3	Location 4
A	15, 19, 22	17, 10, 13	9, 12, 6	14, 8, 11
B	20, 24, 18	24, 18, 22	12, 15, 10	21, 16, 14
C	22, 17, 14	26, 19, 21	10, 5, 8	19, 15, 12

[10 marks]

9. Tiga jenis kentang ditanam dalam tiga plot di setiap empat lokasi. Hasil dalam busyel diberikan dalam Jadual 1. Hasilkan jadual ANOVA bagi data ini. Adakah kesan diperlukan? Huraikan kesimpulan anda.

Jadual 1

Variasi	*Lokasi 1*	*Lokasi 2*	*Lokasi 3*	*Lokasi 4*
A	*15, 19, 22*	*17, 10, 13*	*9, 12, 6*	*14, 8, 11*
B	*20, 24, 18*	*24, 18, 22*	*12, 15, 10*	*21, 16, 14*
C	*22, 17, 14*	*26, 19, 21*	*10, 5, 8*	*19, 15, 12*

[10 markah]

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