UNIVERSITI SAINS MALAYSIA

Second Semester Examination
2012/2013 Academic Session

June 2013

MAT 516 – Curve and Surface Methods for CAGD
[Kaedah Lengkung dan Permukaan untuk RGBK]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of SEVEN pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi TUJUH muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions: Answer **all four** [4] questions.

*[**Arahan:** Jawab **semua empat** [4] soalan.]*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai].

1. (a) Let $\binom{n}{r} = \dfrac{n!}{r!(n-r)!}$. Show that

(i) $\dfrac{n}{r}\binom{n}{r} = \binom{n+1}{r+1}$

(ii) $\sum_{j=0}^{n} jB_j^n(t) = nt$ where $B_j^n(t) = \binom{n}{j}(1-t)^{n-j}t^j$, $0 \le t \le 1$.

(b) Given . $B_j^n(t) = \binom{n}{j}(1-t)^{n-j}t^j$, $0 \le t \le 1$

(i) Show that $(1-t)B_j^{n-1}(t) + tB_{j-1}^{n-1}(t) = B_j^n(t)$.

(ii) If $\boldsymbol{p}(t) = \sum_{j=0}^{n+1} B_j^{n+1}(t)\boldsymbol{p}_j$, show that $\boldsymbol{p}'(1) = (n+1)(\boldsymbol{p}_{n+1} - \boldsymbol{p}_n)$.

(iii) Indicate with a diagram the process of evaluating the point $\boldsymbol{p}(0.5)$ of a cubic Bezier curve with control points $\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{p}_2, \boldsymbol{p}_3$ using de Casteljau algorithm.

[25 marks]

1. (a) Biarkan $\binom{n}{r} = \dfrac{n!}{r!(n-r)!}$. *Tunjukkan bahawa*

(i) $\dfrac{n}{r}\binom{n}{r} = \binom{n+1}{r+1}$

(ii) $\sum_{j=0}^{n} jB_j^n(t) = nt$ *dengan* $B_j^n(t) = \binom{n}{j}(1-t)^{n-j}t^j$, $0 \le t \le 1$.

(b) Diberi $B_j^n(t) = \binom{n}{j}(1-t)^{n-j}t^j$, $0 \le t \le 1$

(i) Tunjukkan bahawa $(1-t)B_j^{n-1}(t) + tB_{j-1}^{n-1}(t) = B_j^n(t)$.

(ii) Jika $\boldsymbol{p}(t) = \sum_{j=0}^{n+1} B_j^{n+1}(t)\boldsymbol{p}_j$, *tunjukkan bahawa* $\boldsymbol{p}'(1) = (n+1)(\boldsymbol{p}_{n+1} - \boldsymbol{p}_n)$.

(iii) Nyatakan dengan suatu rajah, proses menilai titik $\boldsymbol{p}(0.5)$ *suatu lengkung Bezier kubik dengan titik kawalan* $\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{p}_2, \boldsymbol{p}_3$ *dengan menggunakan algoritma de Casteljau.*

[25 markah]

2. (a) Given a line with an equation $y = 2x$ and a parabola

$\boldsymbol{r}(t) = (1-t)^2(0,4) + 2t(1-t)(0,0) + t^2(2,0), \quad 0 \le t \le 1.$

(i) Find the value of t where the line intersects the parabola and hence determine the point of intersection.

(ii) Determine the point on the parabola such that the tangent vector at this point is normal to the vector defining the straight line.

(b) A cubic rational Timmer curve with four control points with two internal weights w_1 and w_2 is given by

$$\boldsymbol{r}(t) = \frac{(1-t)^2(1-2t)\boldsymbol{p}_0 + 4(1-t)^2 t w_1 \boldsymbol{p}_1 + 4(1-t)t^2 w_2 \boldsymbol{p}_2 + t^2(2t-1)\boldsymbol{p}_3}{(1-t)^2(1-2t) + 4(1-t)^2 t w_1 + 4(1-t)t^2 w_2 + t^2(2t-1)},$$

$0 \le t \le 1.$

(i) Give an example of the values of the weights w_1 and w_2 such that the point $\boldsymbol{r}(0.5)$ divides the segment $\boldsymbol{p}_1\boldsymbol{p}_2$ in the ratio of $2:3$.

(ii) Show that when the weights are equal and $\boldsymbol{p}_1\boldsymbol{p}_2$ and $\boldsymbol{p}_0\boldsymbol{p}_3$ are parallel then the cubic rational Timmer curve given above will reduce to a conic segment.

Hence, show that the weights w_1 and w_2 of the conic segment is

$$w_1 = w_2 = \frac{1}{2}\frac{(\boldsymbol{p}_0 - \boldsymbol{p}_3).(\boldsymbol{p}_1 - \boldsymbol{p}_2)}{(\boldsymbol{p}_1 - \boldsymbol{p}_2).(\boldsymbol{p}_1 - \boldsymbol{p}_2)}.$$

(c) A cubic polynomial $f(u) = 2u^3 + u^2 - 3u + 1, -1 \le u \le 1$ can be expressed as

$f(t) = (1-t)^3 y_0 + 3(1-t)^2 t y_1 + 3(1-t)t^2 y_2 + t^3 y_3$, with $t = \dfrac{u+1}{2}$

and $0 \le t \le 1.$

Find y_0, y_1, y_2 and y_3..

[25 marks]

2. (a) Diberi suatu garis lurus dengan persamaan $y = 2x$ dan suatu parabola

$\boldsymbol{r}(t) = (1-t)^2(0,4) + 2t(1-t)(0,0) + t^2(2,0), \ 0 \le t \le 1.$

(i) Cari nilai t apabila garis bersilang parabola dan tentukan titik persilangannya.

(ii) Tentukan titik pada parabola supaya vector tangent pada titik ini adalah berserenjang dengan vector yang menghurakan garis lurus.

(b) *Suatu lengkung kubik nisbah Timmer dengan empat titik kawalan dan dua pemberat dalaman* w_1 *dan* w_2 *diberikan sebagai*

$$\boldsymbol{r}(t) = \frac{(1-t)^2(1-2t)\,\boldsymbol{p}_0 + 4(1-t)^2 tw_1\boldsymbol{p}_1 + 4(1-t)t^2 w_2\boldsymbol{p}_2 + t^2(2t-1)\,\boldsymbol{p}_3}{(1-t)^2(1-2t) + 4(1-t)^2 tw_1 + 4(1-t)t^2 w_2 + t^2(2t-1)},$$

$0 \le t \le 1.$

(i) *Berikan contoh nilai pemberat* w_1 *dan* w_2 *supaya titik* $\boldsymbol{r}(0.5)$ *membahagikan segmen* $\boldsymbol{p}_1\boldsymbol{p}_2$ *pada nisbah* $2:3.$

(ii) *Tunjukkan apabila pemberat dalaman adalah sama dan* $\boldsymbol{p}_1\boldsymbol{p}_2$ dan $\boldsymbol{p}_0\boldsymbol{p}_3$ *adalah selari maka lengkung kubik nisbah Timmer yang diberikan diatas akan terturun kepada suatu segmen kon.*
Justeru, tunjukkan bahawa pemberat w_1 *dan* w_2 *suatu segmen kon diberi oleh*

$$w_1 = w_2 = \frac{1}{2}\frac{(\boldsymbol{p}_0 - \boldsymbol{p}_3).(\boldsymbol{p}_1 - \boldsymbol{p}_2)}{(\boldsymbol{p}_1 - \boldsymbol{p}_2).(\boldsymbol{p}_1 - \boldsymbol{p}_2)}.$$

(c) *Suatu polynomial kubik* $f(u) = 2u^3 + u^2 - 3u + 1, \quad -1 \le u \le 1$ *boleh dibentuk oleh*

$$f(t) = (1-t)^3 y_0 + 3(1-t)^2 ty_1 + 3(1-t)t^2 y_2 + t^3 y_3, \text{ dengan } t = \frac{u+1}{2}$$

dan $0 \le t \le 1.$
Cari y_0, y_1, y_2 *dan* y_3. .

[25 markah]

3. (a) A rational quadratic curve is defined as

$$\boldsymbol{r}(t) = \frac{(1-t)^2\,\boldsymbol{p}_0 + 2(1-t)tw\boldsymbol{h} + t^2\,\boldsymbol{p}_1}{(1-t)^2 + 2(1-t)tw + t^2}$$

where $0 \le t \le 1.$ and w is a positive weight.

Let $\boldsymbol{p}_0 = (-2,0),\ \boldsymbol{h} = (0, 2\tan\alpha),\ \boldsymbol{p}_1 = (2,0)$ and the angle $\angle \boldsymbol{h}\boldsymbol{p}_0\boldsymbol{p}_1 = \alpha$.

(i) Determine the value of the curvature $\boldsymbol{r}(t)$ in terms of α.

(ii) If the curve represents a circular arc, then show that $w = \cos\alpha$.

(iii) If tends to $\frac{\pi}{2}$, show that tends to zero and the location of the point tends to infinity.

(b) Let two curves be given as

$\boldsymbol{r}_1(t) = (1-t)\,\boldsymbol{p}_0 + t\boldsymbol{p}_1,\ 0 \le t \le 1,$ and

$\boldsymbol{r}_2(t) = (1-t)^3\,\boldsymbol{p}_0 + 3(1-t)^2 t\boldsymbol{p}_1 + 3(1-t)t^2\,\boldsymbol{p}_2 + t^3\,\boldsymbol{p}_3,\ 0 \le t \le 1,$

.

Find the curvature of $\boldsymbol{r}_2(t)$ at $t = 0$ if the points $\boldsymbol{p}_1, \boldsymbol{p}_2$ and $\boldsymbol{p}_3$ are collinear. Hence determine the locations of $\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{p}_2$ and $\boldsymbol{p}_3$ so that the curves $\boldsymbol{r}_1(t)$ and $\boldsymbol{r}_2(t)$ are joined with curvature continuity at $\boldsymbol{p}_1$.

(c) A cubic function $f(t)$ on the interval $1 \le x \le 3$ can be written as

$$f(t) = \frac{(1-t)^2(1+t)\alpha y_0 + (1-t)^2 t y_1 + (1-t)t^2 y_2 + t^2(2-t)\beta y_3}{(1-t)^2(1+t)\alpha + (1-t)^2 t + (1-t)t^2 + t^2(2-t)\beta}$$

where $t = \dfrac{x-2}{2}$

If the gradient of the function at $x = 1$ is d_0 write y_1 in terms of y_0, d_0

[25 marks]

3. (a) *Suatu lengkung nisbah kuadratik diberi sebagai*

$$\boldsymbol{r}(t) = \frac{(1-t)^2\,\boldsymbol{p}_0 + 2(1-t)tw\boldsymbol{h} + t^2\,\boldsymbol{p}_1}{(1-t)^2 + 2(1-t)tw + t^2}$$

dengan $0 \le t \le 1.$ *dan* w *adalah pemberat positif. Andaikan* $\boldsymbol{p}_0 = (-2,0),\ \boldsymbol{h} = (0, 2\tan\alpha),\ \boldsymbol{p}_1 = (2,0)$ *dan sudut* $\angle \boldsymbol{h}\boldsymbol{p}_0\boldsymbol{p}_1 = \alpha$.

(i) *Tentukan nilai kelengkungan* $\boldsymbol{r}(t)$ *dalam sebutan* α

(ii) *Jika lengkung mewakili suatu lengkuk bulatan, maka tunjukkan bahawa* $w = \cos\alpha$.

(iii) *Jika menuju nilai* $\dfrac{\pi}{2}$, *tunjukkan bahawa kepada nilai sifar dan kedudukan titik menuju kepada infiniti.*

(b) *Biarkan dua lengkung diberi sebagai*

$\boldsymbol{r}_1(t) = (1-t)\,\boldsymbol{p}_0 + t\boldsymbol{p}_1,\ 0 \le t \le 1,$ *dan*

$\boldsymbol{r}_2(t) = (1-t)^3\,\boldsymbol{p}_0 + 3(1-t)^2 t\boldsymbol{p}_1 + 3(1-t)t^2\,\boldsymbol{p}_2 + t^3\,\boldsymbol{p}_3,\ 0 \le t \le 1.$

Cari nilai kelengkungan $\boldsymbol{r}_2(t)$ *pada* $t = 0$ *jika* $\boldsymbol{p}_1, \boldsymbol{p}_2$ *dan* $\boldsymbol{p}_3$ *adalah segaris. Justeru tentukan kedudukkan* $\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{p}_2$ *dan* $\boldsymbol{p}_3$ *supaya lengkung* $\boldsymbol{r}_1(t)$ *dan* $\boldsymbol{r}_2(t)$ *adalah terhubung dengan keselanjaran kelengkungan pada* $\boldsymbol{p}_1$.

(c) *Suatu fungsi nisbah kubik pada selang* $1 \le x \le 3$ *dapat ditulis dalam bentuk*

$$f(t) = \frac{(1-t)^2(1+t)\alpha y_0 + (1-t)^2 t y_1 + (1-t)t^2 y_2 + t^2(2-t)\beta y_3}{(1-t)^2(1+t)\alpha + (1-t)^2 t + (1-t)t^2 + t^2(2-t)\beta}$$

dengan $t = \dfrac{x-2}{2}$

Jika kecerunan lengkung pada $x = 1$ *ialah* d_0 *tuliskan* y_1 *dalam sebutan* y_0, d_0 *dan*

[25 makah]

4. (a) Let a surface be defined parametrically as $S(u,v) = (2u + v^2, u^2 v, u^2 + v - 1),$ where $0 \le u, v \le 2.$
   - (i) Evaluate the point $S(1,1).$
   - (ii) Determine an equation of the tangent plane at $u = 1, v = 1.$
   - (iii) Express $S(u,1)$ as a quadratic Bezier curve with suitable control points.

   (b) A B-spline quadratic curve with knot vector $\{0,0,0,1,2,2,2\}$ and control points $\{\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{p}_2, \boldsymbol{p}_3\}$ is given by

$$\boldsymbol{p}(t) = \sum_{i=0}^{3} N_{i,2}(t)\,\boldsymbol{p}_i$$

Where $N_{i,d}$ t

$$N_{i,0}(t) = \begin{cases} 1 \text{ if } & t \in [t_{i,} t_{i+1}) \\ 0 & \text{elsewhere} \end{cases}$$

$$N_{i,d}(t) = \frac{t - t_i}{t_{i+d} - t_i} N_{i,d-1} \quad t \quad + \frac{t_{i+d+1} - t}{t_{i+d+1} - t_{i+1}} N_{i+1,d-1} \quad t \quad \text{with } 0 \le i \le 3 \text{ and } 1 \le d \le 2.$$

   - (i) Evaluate $N_{3,2}$ 2 and $N_{4,2}$ 2
   - (ii) Express $\boldsymbol{p}(2)$ in terms of the control points $\boldsymbol{p}_1, \boldsymbol{p}_2, \boldsymbol{p}_3.$

[25 marks]

4. *(a)* *Andaikan suatu tampalan permukaan ditakrif secra berparameter* $S(u,v) = (2u + v^2, u^2 v, u^2 + v - 1),$ *dengan* $0 \le u, v \le 2.$
   - *(i)* *Nilaikan titik* $S(1,1)$. .
   - *(ii)* *Tentukan persamaan satah tangen pada* $u = 1, v = 1.$
   - *(iii)* *Ungkapkan* $S(u,1)$ *sebagai suatu lengkung kuadratik dengan menyatakan titk kawalan yang bersesuaian.*

(b) *Suatu splin B kuadratik dengan vector knot* $\{0,0,0,1,2,2,2\}$ *dan titik kawalan* $\{\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{p}_2, \boldsymbol{p}_3\}$ *adalah diberi sebagai*

$$\boldsymbol{p}(t) = \sum_{i=0}^{3} N_{i,2}(t)\,\boldsymbol{p}_i$$

dengan

$$N_{i,0}(t) = \begin{cases} 1 \text{ if } & t \in [t_i, t_{i+1}) \\ 0 & \text{lain tempat} \end{cases}$$

$$N_{i,d}(t) = \frac{t - t_i}{t_{i+d} - t_i} N_{i,d-1}\ t\ + \frac{t_{i+d+1} - t}{t_{i+d+1} - t_{i+1}} N_{i+1,d-1}\ t$$ *dengan* $0 \le i \le 3$ *dan* $1 \le d \le 2.$

(i) *Nilaikan* $N_{3,2}\ 2$ *dan* $N_{4,2}\ 2$

(ii) *Ungkapkan d* $\boldsymbol{p}(2)$ *alam sebutan titik kawalan* $\boldsymbol{p}_1, \boldsymbol{p}_2, \boldsymbol{p}_3$.

[25 markah]

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