UNIVERSITI SAINS MALAYSIA

Second Semester Examination
2012/2013 Academic Session

June 2013

MAT 102 – Advanced Calculus
[Kalkulus Lanjutan]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of FIVE pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi LIMA muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions : Answer **all ten** [10] questions.

*[**Arahan** : Jawab **semua sepuluh** [10] soalan.]*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai].

1. For a sequence a_n , if $\lim_{n\to\infty} |a_n| = 0,$ prove that $\lim_{n\to\infty} a_n = 0.$ [5 marks]

1. Bagi jujukan a_n , jika $\lim_{n\to\infty} |a_n| = 0,$ buktikan bahawa $\lim_{n\to\infty} a_n = 0.$ [5 markah]

2. Discuss the continuity of f at $0,0$.

$$f\ x,y\ = \begin{cases} \dfrac{x^2 - y^2}{x^2 + y^2} & , \quad x,y\ \neq\ 0,0 \\ 0 & , \quad x,y\ =\ 0,0 \end{cases}$$

[6 marks]

2. Bincangkan keselanjaran bagi f pada $0,0$.

$$f\ x,y\ = \begin{cases} \dfrac{x^4 - y^4}{x^4 + y^4} & , \quad x,y\ \neq\ 0,0 \\ 0 & , \quad x,y\ =\ 0,0 \end{cases} .$$

[6 markah]

3. Find the radius of convergence of power series $\sum_{n=0}^{\infty} -1^{\ n} \sqrt{n+1}\ 1+x\ ^{2n+1}.$

[7 marks]

3. Dapatkan jejari penumpuan siri kuasa $\sum_{n=0}^{\infty} -1^{\ n} \sqrt{n+1}\ 1+x\ ^{2n+1}.$

[7 markah]

4. A function $f\ x,y$ has a directional derivative 8 in the direction $\mathbf{v}_1 = 3\mathbf{i} - 4\mathbf{j}$ and 1 in the direction $\mathbf{v}_2 = 12\mathbf{i} + 5\mathbf{j}$ at the point $P_0\ -1,2$. Find the directional derivative of f at P_0 in the direction $\mathbf{v} = -6\mathbf{i} + 8\mathbf{j}.$

[7 marks]

4. Satu fungsi $f\ x,y$ mempunyai terbitan berarah 8 dalam arah $\mathbf{v}_1 = 3\mathbf{i} - 4\mathbf{j}$ dan 1 dalam arah $\mathbf{v}_2 = 12\mathbf{i} + 5\mathbf{j}$ pada titik $P_0\ -1,2$. Dapatkan terbitan berarah bagi f pada P_0 dalam arah $\mathbf{v} = -6\mathbf{i} + 8\mathbf{j}.$

[7 markah]

5. Using the power series representation for the function $f(x) = \frac{1}{1-x}$ for $|x| < 1$, write $f'(x)$ and $f''(x)$ in terms of power series.

Hence, show that $\frac{2x^2}{(x-1)^3} = \sum_{n=1}^{\infty} \frac{(n+1)\,n}{x^n}$ for $|x| > 1$.

[8 marks]

5. *Menggunakan perwakilan siri kuasa bagi fungsi* $f(x) = \frac{1}{1-x}$ *untuk* $|x| < 1$, *tuliskan* $f'(x)$ *dan* $f''(x)$ *dalam sebutan siri kuasa.*

Seterusnya, tunjukkan bahawa $\frac{2x^2}{(x-1)^3} = \sum_{n=1}^{\infty} \frac{(n+1)\,n}{x^n}$ *untuk* $|x| > 1$.

[8 markah]

6. Evaluate the integral $\int_0^2 \frac{dx}{\sqrt{|x-1|}}$. [10 marks]

6. *Nilaikan kamiran* $\int_0^2 \frac{dx}{\sqrt{|x-1|}}$. *[10 markah]*

7. A rectangular box has a volume 1000 cm^3. Find the base area of the box that has minimal surface area.

[12 marks]

7. *Satu kotak segi empat tepat mempunyai isipadu* $1000\,cm^3$. *Dapatkan keluasan tapak kotak yang mempunyai keluasan permukaan yang minimum.*

[12 markah]

8. Determine whether the following series converges absolutely, converges conditionally or diverges?

(a) $\sum_{k=1}^{\infty} \left(\frac{8^k}{k^2} \right)$

(b) $\frac{1}{1\cdot 2} - \frac{1}{3\cdot 2} + \frac{1}{5\cdot 2} - \frac{1}{7\cdot 2} + \cdots$

[15 marks]

8. *Tentukan samada siri berikut menumpu secara mutlak, menumpu secara bersyarat atau mencapah.*

(a) $\sum_{k=1}^{\infty}\left(\frac{8^k}{k^2}\right)$

(b) $\frac{1}{1\cdot 2}-\frac{1}{3\cdot 2}+\frac{1}{5\cdot 2}-\frac{1}{7\cdot 2}+\cdots$

[15 markah]

9. Let $w=f\ x,y$, where $x=r\cos\theta$ and $y=r\sin\theta$.

(a) Show that $\frac{\partial w}{\partial r}=f_x\cos\theta+f_y\sin\theta$ and $\frac{1}{r}\frac{\partial w}{\partial\theta}=-f_x\sin\theta+f_y\cos\theta.$

(b) Write f_x and f_y in terms of $\frac{\partial w}{\partial r}$ and $\frac{\partial w}{\partial\theta}$.

Hence, show that $f_x{}^2+f_y{}^2=\left(\frac{\partial w}{\partial r}\right)^2+\frac{1}{r^2}\left(\frac{\partial w}{\partial\theta}\right)^2$.

[15 marks]

9. *Biar* $w=f\ x,y$, $x=r\cos\theta$ *dan* $y=r\sin\theta$.

(a) Tunjukkan bahawa $\frac{\partial w}{\partial r}=f_x\cos\theta+f_y\sin\theta$ *dan*

$\frac{1}{r}\frac{\partial w}{\partial\theta}=-f_x\sin\theta+f_y\cos\theta.$

(b) Tuliskan f_x *and* f_y *dalam sebutan* $\frac{\partial w}{\partial r}$ *dan* $\frac{\partial w}{\partial\theta}$.

Seterusnya, tunjukkan bahawa $f_x{}^2+f_y{}^2=\left(\frac{\partial w}{\partial r}\right)^2+\frac{1}{r^2}\left(\frac{\partial w}{\partial\theta}\right)^2$.

[15 markah]

10. Evaluate the following integrals.

(a) $\int_0^2\int_{\sqrt[3]{x}}^2\frac{1}{y^4+1}\,dydx$

(b) $\iiint_E\frac{\sqrt{z}}{x^2+y^2}\,dV$, where E is a solid region above the xy-plane, below the cone $z=\sqrt{x^2+y^2}$ and inside the cylinder $x^2+y^2=9.$

[15 marks]

10. Nilaikan kamiran berikut.

(a) $\int_0^2 \int_{\sqrt[3]{x}}^2 \frac{1}{y^4+1}\, dydx$

(b) $\iiint_E \frac{\sqrt{z}}{x^2+y^2} dV$ *, E merupakan rantau pepejal di atas satah- xy, di bawah kon* $z=\sqrt{x^2+y^2}$ *dan di dalam silinder* $x^2+y^2=9.$

[15 markah]

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