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UNIVERSITI SAINS MALAYSIA

Second Semester Examination  
2012/2013 Academic Session

June 2013

**MAA 101 - Calculus for Science Students I**  
***[Kalkulus untuk Pelajar Sains I]***

Duration : 3 hours  
*[Masa : 3 jam]*

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Please check that this examination paper consists of FIVE pages of printed material before you begin the examination.

*[Sila pastikan bahawa kertas peperiksaan ini mengandungi LIMA muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]*

**Instructions:** Answer **all six** [6] questions.

**Arahan:** Jawab **semua enam** [6] soalan].

In the event of any discrepancies, the English version shall be used.

*[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai.]*

1. (a) Given the functions  $f(x) = \sqrt{x+1}$  and  $g(x) = \frac{1}{x}$ , find the domain of  $f$ ,  $g$ ,  $f \circ g$  and  $g \circ f$ .
- (b) Show that  $f(x) = \frac{3x}{x-1}$ ,  $x > 1$  is a one-to-one function. Find its inverse and hence show that  $f(f^{-1}(x)) = f^{-1}(f(x))$ .

[15 marks]

1. (a) Diberi fungsi  $f(x) = \sqrt{x+1}$  dan  $g(x) = \frac{1}{x}$ , dapatkan domain  $f$ ,  $g$ ,  $f \circ g$  dan  $g \circ f$ .
- (b) Tunjukkan bahawa  $f(x) = \frac{3x}{x-1}$ ,  $x > 1$  ialah fungsi satu ke satu. Dapatkan songsangannya dan seterusnya tunjukkan  $f(f^{-1}(x)) = f^{-1}(f(x))$ .

[15 markah]

2. (a) Evaluate the following limits. The L'Hospital rule can be applied whenever applicable.

(i)  $\lim_{x \rightarrow 1} \frac{x^2 - 4x + 3}{x^2 - 6x + 9}$

(ii)  $\lim_{x \rightarrow 1} \left( \frac{x}{x-1} - \frac{1}{\ln x} \right)$

(iii)  $\lim_{x \rightarrow 0^+} \cos x^{\frac{1}{x}}$

- (b) Find the value of  $c$  that makes  $f$  continuous at  $x = 3$ .

$$f(x) = \begin{cases} cx^2 - \frac{x}{2} + 3 & , x < 3 \\ 2x - c + \frac{1}{2} & , x \geq 3 \end{cases}$$

[16 marks]

2. (a) Nilaiikan had-had berikut, Petua L'Hospital boleh digunakan jika memadai.

(i) had  $\lim_{x \rightarrow 1} \frac{x^2 - 4x + 3}{x^2 - 6x + 9}$

(ii) had  $\lim_{x \rightarrow 1} \left( \frac{x}{x-1} - \frac{1}{\ln x} \right)$

(iii) had  $\lim_{x \rightarrow 0^+} \cos x^{\frac{1}{x}}$

(b) Dapatkan nilai  $c$  yang menjadikan  $f$  selanjar pada  $x=3$ .

$$f(x) = \begin{cases} cx^2 - \frac{x}{2} + 3 & , \quad x < 3 \\ 2x - c + \frac{1}{2} & , \quad x \geq 3 \end{cases}$$

[16 markah]

3. Find the derivative of the following functions.

(a)  $f(x) = \frac{5x^3 + 2}{x + \sin x}$

(b)  $y = \ln \left( \frac{x^2 + 2^5}{x^3 + 1} \right)^{2/3}$

(c)  $y = (\cos x)^{x^3}$

[14 marks]

3. Dapatkan pembezaan fungsi-fungsi berikut.

(a)  $f(x) = \frac{5x^3 + 2}{x + \sin x}$

(b)  $y = \ln \left( \frac{x^2 + 2^5}{x^3 + 1} \right)^{2/3}$

(c)  $y = (\cos x)^{x^3}$

[14 markah]

4. (a) The function  $f(x) = 2x^2 + 3x + 5$  satisfies the Mean-Value Theorem at  $x = 2$  on the interval  $[1, a]$ . Find all the values of  $a$ .

(b) Suppose that the graph of a function  $y = f(x)$  satisfies the following properties:

$$f(2) = 1; \quad f(4) = 4; \quad f(6) = 7.$$

$$f'(2) = 0 = f'(6) \quad \text{and} \quad f''(4) = 0.$$

$$f'(x) < 0 \quad \text{on} \quad (-\infty, 2) \quad \text{and} \quad (6, \infty); \quad f'(x) > 0 \quad \text{on} \quad (2, 6).$$

$$f''(x) > 0 \quad \text{on} \quad (-\infty, 4) \quad \text{and} \quad f''(x) < 0 \quad \text{on} \quad (4, \infty).$$

$$\lim_{x \rightarrow -\infty} f(x) = \infty \quad \text{and} \quad \lim_{x \rightarrow \infty} f(x) = -\infty.$$

State

- (i) the critical numbers of  $f$ .
- (ii) the interval on which  $f$  is increasing or decreasing.
- (iii) the local maximum and minimum points, if any.
- (iv) the interval of concavity and the inflection points, if exist.

Sketch the graph of  $f$  with the above properties.

[18 marks]

4. (a) Fungsi  $f(x) = 2x^2 + 3x + 5$  memenuhi Teorem Nilai Min pada  $x = 2$  dalam selang  $[1, a]$ . Cari semua nilai  $a$ .

(b) Andaikan graf bagi suatu fungsi  $y = f(x)$  memenuhi sifat-sifat berikut.

$$f(2) = 1; \quad f(4) = 4; \quad f(6) = 7.$$

$$f'(2) = 0 = f'(6) \text{ dan } f''(4) = 0.$$

$$f'(x) < 0 \text{ pada } (-\infty, 2) \text{ dan } (6, \infty); \quad f'(x) > 0 \text{ pada } (2, 6).$$

$$f''(x) > 0 \text{ pada } (-\infty, 4) \text{ dan } f''(x) < 0 \text{ pada } (4, \infty).$$

$$\lim_{x \rightarrow -\infty} f(x) = \infty \text{ dan } \lim_{x \rightarrow \infty} f(x) = -\infty.$$

Nyatakan

- (i) nombor genting.
- (ii) selang yang mana  $f$  menokok atau menyusut.
- (iii) titik maksimum and minimum tempatan, jika ada.
- (iv) selang kecekungan dan titik lengkung balas, jika wujud.

Lakarkan graf  $f$  dengan sifat-sifat di atas.

[18 markah]

5. (a) Evaluate the integral,  $\int_{-1}^4 x|x-2| dx$ .

(b) Find the following integrals.

(i)  $\int e^x \sqrt{1+e^{2x}} dx$

(ii)  $\int \frac{x+1}{x^2-x-6} dx$

[15 marks]

5. (a) Nilai kamiran,  $\int_{-1}^4 x|x-2| dx$ .

(b) Dapatkan kamiran-kamiran berikut.

(i)  $\int e^x \sqrt{1+e^{2x}} dx$

(ii)  $\int \frac{x+1}{x^2-x-6} dx$

[15 markah]

6. (a) Find the equation of the tangent to the curve  $y = F(x)$  at  $x = 1$ , where

$$F(x) = \int_1^{\sqrt{x}} \frac{2t+1}{t+2} dt.$$

- (b) Find the area of the region bounded by the curves  $y = \sin x$ ,  $y = \cos x$ ,  $x = 0$  and  $x = 1$ .
- (c) Find the volume of the solid obtained when the region bounded by the curve  $y = \sqrt{x+1}$ ,  $x$ -axis, line  $x = 2$  and  $x = 4$  is rotated about the  $x$ -axis.

[22 marks]

6. (a) Dapatkan persamaan garis tangen kepada lengkung  $y = F(x)$  pada  $x = 1$ ,

dengan  $F(x) = \int_1^{\sqrt{x}} \frac{2t+1}{t+2} dt.$

- (b) Dapatkan luas rantau yang dibatasi oleh lengkung-lengkung  $y = \sin x$ ,  $y = \cos x$ ,  $x = 0$  dan  $x = 1$ .
- (c) Dapatkan isipadu bongkah kisanan yang terhasil apabila rantau yang dibatasi oleh lengkung-lengkung  $y = \sqrt{x+1}$ , paksi- $x$ , garis  $x = 2$  dan  $x = 2$  diputarkan sekitar paksi- $x$ .

[22 markah]