UNIVERSITI SAINS MALAYSIA

First Semester Examination
2012/2013 Academic Session

January 2013

MST 564 – Statistical Reliability
[Kebolehpercayaan Statistik]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of SIXTEEN pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi ENAM BELAS muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions: Answer **all four** [4] questions.

*[**Arahan:** Jawab **semua empat** [4] soalan.]*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai].

1. (a) 'The reliability of an item is the probability that it will adequately perform its specified purpose for a specified period of time under specified environmental conditions.' Give two examples and reasoning for utilizing such a definition.

(b) In reliability data analysis, once the definition of a failure event is in place, you have to determine what data is important to fully describe the failure event. Provide a list and description of critical information that is important to collect for any failure event.

(c) When we examine reliability as a function of time, this leads us to the definition of failure rate.
  (i) How would you define failure rate?
  (ii) What sort of curve would represent a time-dependent failure rate?
  (iii) How does the curve in (ii) reveal the behaviour of failure rates with time?
  (iv) Is the curve in (ii) still valid in our digital age? If no, explain why not? Is there another curve which will better represent the failure rate?

[100 marks]

1. (a) 'Kebolehpercayaan sesuatu benda adalah kebarangkalian bahawa ia akan melaksanakan tujuan tertentu dengan sempurna bagi tempoh masa yang dinyatakan di bawah syarat-syarat alam sekitar tertentu.'
Berikan dua contoh dan penjelasan untuk menggunakan definisi yang dinyatakan.

(b) Dalam analisis data kebolehpercayaan, setelah definisi peristiwa kegagalan ditentukan, anda perlu menentukan data apa adalah penting untuk menggambarkan sepenuhnya peristiwa kegagalan. Sediakan suatu senarai dan perihalan maklumat kritikal yang penting untuk mengumpul sebarang peristiwa kegagalan.

(c) Apabila kita memeriksa kebolehpercayaan sebagai fungsi masa, ianya membawa kita kepada takrif kadar kegagalan.
  (i) Bagaimana anda akan mendefinisikan kadar kegagalan?
  (ii) Apakah jenis lengkungan yang akan mewakili kadar kegagalan yang bergantung kepada masa?
  (iii) Bagaimanakah lengkungan dalam (ii) mendedahkan tingkah laku kadar kegagalan dengan masa?
  (iv) Adakah lengkungan dalam (ii) masih sah dalam zaman digital kita? Jika tidak, jelaskan mengapa tidak? Adakah terdapat satu lagi lengkungan yang akan mewakili kadar kegagalan dengan lebih baik?

[100 markah]

2. (a) A component has a constant failure rate of $\lambda = 0.02$ per hour.
   - (i) What is the probability that it will fail during the first 10 hours of operation?
   - (ii) Suppose that the component has successfully operated for 100 hours. What is the probability that it will fail during the next 10 hours of operation?

   (b) Given a Weibull failure distribution with a shape parameter of 1/3 and a scale parameter of 16,000, completely characterize the failure process.
   - (i) What is the reliability function of this process?
   - (ii) How does the shape parameter provide insight into the behaviour of this failure process?
   - (iii) Compute the MTTF. If the distribution is highly skewed, how would you represent a better average time to failure? Then compute the better alternative.
   - (iv) If a 90% reliability is desired, determine the design life.

[100 marks]

2. (a) Suatu komponen mempunyai kadar kegagalan malar $\lambda = 0.02$ setiap jam.
   - *(i) Apakah kebarangkalian bahawa ia akan gagal dalam operasi pada 10 jam pertama?*
   - *(ii) Katakan bahawa komponen telah berjaya dikendalikan selama 100 jam. Apakah kebarangkalian bahawa ia akan gagal dalam operasi 10 jam seterusnya?*

   (b) Diberi taburan kegagalan Weibull dengan parameter bentuk 1/3 dan parameter skala 16,000, mencirikan proses kegagalan dengan sepenuhnya.
   - *(i) Apakah fungsi kebolehpercayaan proses ini?*
   - *(ii) Bagaimanakah parameter bentuk memberi penjelasan ke atas tingkah laku proses kegagalan ini?*
   - *(iii) Kirakan MTTF. Jika taburan adalah sangat pencong, bagaimana anda akan mewakili suatu purata masa kegagalan yang lebih baik? Seterusnya, kira alternatif yang lebih baik.*
   - *(iv) Jika kebolehpercayaan 90% diingini, tentukan masa hidup reka bentuk.*

[100 markah]

3. (a) Suppose that you follow thirteen patients for survival and observe the following times (in months) :

5, 12, 25, 26, 27, 28, 32+, 33+, 34+, 37, 39, 40+, 42+

(The notation '+' denotes 'censoring'.)

(i) Calculate the Kaplan-Meier survival estimates for the thirteen patients.
(ii) Interpret the Kaplan-Meier survival function for the thirteen patients.
(iii) Find S(36) where S is the survival function.
(iv) Obtain the actuarial life table taking intervals of width six months.
(v) Find the estimated hazard at the midpoint of each interval.
(vi) Discuss the strengths and weaknesses of using the product limit method and the actuarial method.

(b) The data in **Table 1** gives the remission times (in weeks) for patients subjected to two different treatments. Twenty patients were assigned to each of the two treatments; '+' denotes 'right censoring'.

Table 1. Remission times (in weeks) for twenty patients

Treatment A	**Treatment B**
1 3 3 6 7 7 10 12 14 15 18 19 22 26 28+ 29 34 40 48+ 49+	1 1 2 2 3 4 5 8 8 9 11 12 14 16 18 21 27+ 31 38+ 44

(i) Compare the survival prospects of the two groups by interpreting the graphs of their survival functions.
(ii) Obtain estimates of the probability of survival beyond 12 weeks.
(iii) What is your conclusion based on the insight from parts (i) and (ii)?

[100 marks]

3. (a) Andaikan bahawa anda mengikuti hayat hidup tiga belas orang pesakit dan memerhatikan masa berikut (dalam bulan):

5, 12, 25, 26, 27, 28, 32 +, 33 +, 34 +, 37, 39, 40 +, 42 +

(Catatan '+' menandakan 'censoring'.)

(i) Kirakan anggaran hayat Kaplan-Meier bagi 13 orang pesakit.
(ii) Tafsirkan fungsi hayat Kaplan-Meier bagi 13 orang pesakit.
(iii) Cari S (36) di mana S adalah fungsi hayat.
(iv) Dapatkan jadual jangka hayat aktuari dengan selang lebar enam bulan.

(v) *Cari anggaran bahaya di titik tengah setiap selang.*

(vi) *Bincangkan kekuatan dan kelemahan menggunakan kaedah had produk dan kaedah aktuari.*

(b) *Data dalam **Jadual 1** memberikan masa remitan (dalam minggu) untuk pesakit yang diberi dua rawatan yang berbeza. Dua puluh pesakit telah ditugaskan untuk setiap dua rawatan; '+' menandakan 'right censoring'.*

***Jadual 1**. Masa remitan (dalam minggu) untuk dua puluh pesakit*

Rawatan A	***Rawatan B***
1 3 3 6 7 7 10 12 14 15 18 19 22 26 28+ 29 34 40 48+ 49+	*1 1 2 2 3 4 5 8 8 9 11 12 14 16 18 21 27+ 31 38+ 44*

(i) *Bandingkan prospek survival kedua-dua kumpulan dengan mentafsir graf fungsi hayat mereka.*

(ii) *Dapatkan anggaran kebarangkalian hayat melebihi 12 minggu.*

(iii) *Apakah kesimpulan anda berdasarkan pandangan dari bahagian (i) dan (ii)?*

[100 markah]

4. (a) The data set given in **Table 2** was obtained by Nelson & Hahn (1972). Hours to failure of components are given as a function of operating temperatures (classified as Low and High). There is severe censoring (0 = censored, 1 = uncensored) with only 17 out of 40 components failing. The experiment is conducted at higher temperatures to speed up failure time.

(i) Find the distribution which provides the best fit to the failure data in **Table 2**.

(ii) Apply a suitable model which takes into account the accelerated process. Interpret the application of this model.

(iii) Use your favourite statistical package to obtain the acceleration factor for the components to fail.

(iv) What is your overall conclusion?

Table 2. Hours to failure of forty components.

Component	Time (hours)	Status	Temperature
1	1764	0	Low
2	2772	0	Low
3	3444	0	Low
4	3542	0	Low
5	3780	0	Low
6	4860	0	Low
7	5196	0	Low
8	5448	0	Low
9	5448	0	Low
10	5448	0	Low
11	8064	0	Low
12	8064	0	Low
13	8064	0	Low
14	8064	0	Low
15	8064	0	Low
16	8064	0	Low
17	8064	0	Low
18	8064	0	Low
19	8064	0	Low
20	8064	0	Low
21	408	1	High
22	408	1	High
23	1344	1	High
24	1344	1	High
25	1440	1	High
26	1680	0	High
27	1680	0	High
28	1680	0	High
29	1680	0	High
30	1680	0	High
31	408	1	High
32	408	1	High
33	504	1	High
34	504	1	High
35	504	1	High
36	528	0	High
37	528	0	High
38	528	0	High
39	528	0	High
40	528	0	High

(b) One hundred and forty-nine diabetic patients were followed for 17 years (a subset of data from Lee et al., 1988). **Table 3** (only 5 cases are displayed) gives the survival time from baseline examination, survival status, and several potential prognostic factors at baseline: age (years), body mass index (BMI), age at diagnosis of diabetes (years), smoking status (0=No, 1=Ex-smoker, 2=Current), systolic blood pressure (SBP), diastolic blood pressure (DBP), electrocardiogram reading (ECG) (1=Normal, 2=Borderline, 3=Abnormal), and whether the patient had any coronary heart disease (CHD) (0=No, 1=Yes).

(i) Determine the questions you ought to ask in the analysis of the data in Table 3.

(ii) Perform an appropriate analysis of the data in **Table 3** (use the data file provided).

(iii) Interpret the results obtained in (ii). What can you conclude?

Table 3. Data of 149 diabetic patients (first 5 cases displayed)

Patient	Status	Sur-vival Time (yrs)	Age (yrs)	BMI	Age at Diagnosis (yrs)	Smoking Status	SBP (mmHg)	DPB (mmHg)	ECG	CHD
1	0	12.4	44	34.2	41	0	132	96	1	0
2	0	12.4	49	32.6	48	2	130	72	1	0
3	0	9.6	49	22.0	35	2	108	58	1	1
4	0	7.2	47	37.9	45	0	128	76	2	1
5	0	14.1	43	42.2	42	2	142	80	1	0

Note that:

Status (0 = alive, 1 = dead)
Smoking status (0 = no, 1 = ex-smoker, 2 = current)
ECG (1 = normal, 2 = borderline, 3 = abnormal)
CHD (0 = no, 1 = yes)

[100 marks]

4. (a) *Set data yang diberikan dalam **Jadual 2** telah diperoleh oleh Nelson & Hahn (1972). Waktu kegagalan komponen diberikan sebagai fungsi suhu operasi (diklasifikasikan sebagai Rendah dan Tinggi). Terdapat 'censoring' teruk (0 = 'censored', 1 = 'uncensored') dengan hanya 17 daripada 40 komponen gagal. Eksperimen dijalankan pada suhu yang lebih tinggi untuk mempercepatkan masa kegagalan.*

(i) *Cari taburan yang menyediakan penyuaian terbaik untuk data kegagalan dalam **Jadual 2**.*

(ii) *Aplikasikan model yang sesuai yang mengambil kira proses dipercepatkan. Tafsirkan aplikasi model ini.*

(iii) *Gunakan pakej statistik kegemaran anda untuk mendapatkan faktor pecutan bagi komponen gagal.*

(iv) *Apakah kesimpulan anda secara keseluruhan?*

***Jadual 2**. Jam untuk kegagalan empat puluh komponen.*

Komponen	Masa (jam)	Status	Suhu
1	1764	0	Rendah
2	2772	0	Rendah
3	3444	0	Rendah
4	3542	0	Rendah
5	3780	0	Rendah
6	4860	0	Rendah
7	5196	0	Rendah
8	5448	0	Rendah
9	5448	0	Rendah
10	5448	0	Rendah
11	8064	0	Rendah
12	8064	0	Rendah
13	8064	0	Rendah
14	8064	0	Rendah
15	8064	0	Rendah
16	8064	0	Rendah
17	8064	0	Rendah
18	8064	0	Rendah
19	8064	0	Rendah
20	8064	0	Rendah
21	408	1	Tinggi
22	408	1	Tinggi
23	1344	1	Tinggi
24	1344	1	Tinggi
25	1440	1	Tinggi
26	1680	0	Tinggi
27	1680	0	Tinggi
28	1680	0	Tinggi
29	1680	0	Tinggi

30	*1680*	*0*	*Tinggi*
31	*408*	*1*	*Tinggi*
32	*408*	*1*	*Tinggi*
33	*504*	*1*	*Tinggi*
34	*504*	*1*	*Tinggi*
35	*504*	*1*	*Tinggi*
36	*528*	*0*	*Tinggi*
37	*528*	*0*	*Tinggi*
38	*528*	*0*	*Tinggi*
39	*528*	*0*	*Tinggi*
40	*528*	*0*	*Tinggi*

(b) *Seratus dan empat puluh sembilan pesakit kencing manis telah diikuti selama 17 tahun (subset data daripada Lee et al., 1988).* ***Jadual 3*** *(hanya 5 kes dipaparkan) memberikan masa survival daripada pemeriksaan asas, status hidup, dan beberapa faktor ramalan potensi pada asas: umur (tahun), indeks jisim badan (BMI), umur semasa diagnosis diabetes (tahun), status merokok (0 = Tidak, 1 = Bekas-perokok, 2 = Semasa), tekanan darah sistolik (SBP), tekanan darah diastolik (DBP), bacaan elektrokardiogram (ECG) (1 = Normal, 2 = Sempadan, 3 = Tidak Normal), dan sama ada pesakit mempunyai sebarang penyakit jantung koronari (CHD) (0 = Tiada, 1 = Ya).*

(i) *Tentukan soalan yang anda harus bertanya dalam analisis data dalam* ***Jadual 3****.*

(ii) *Lakukan analisis yang sesuai pada data dalam* ***Jadual 3*** *(menggunakan fail data yang disediakan).*

(iii) *Tafsirkan keputusan yang diperoleh dalam (ii). Apa yang anda boleh simpulkan?*

Jadual 3. *Data 149 pesakit kencing manis (5 kes pertama dipaparkan)*

Pesakit	*Status*	*Masa Hayat (tahun)*	*Umur (tahun)*	*BMI*	*Umur semasa Diagnosis (tahun)*	*Status Merokok*	*SBP (mmHg)*	*DPB (mm Hg)*	*ECG*	*CHD*
1	*0*	*12.4*	*44*	*34.2*	*41*	*0*	*132*	*96*	*1*	*0*
2	*0*	*12.4*	*49*	*32.6*	*48*	*2*	*130*	*72*	*1*	*0*
3	*0*	*9.6*	*49*	*22.0*	*35*	*2*	*108*	*58*	*1*	*1*
4	*0*	*7.2*	*47*	*37.9*	*45*	*0*	*128*	*76*	*2*	*1*
5	*0*	*14.1*	*43*	*42.2*	*42*	*2*	*142*	*80*	*1*	*0*

Perhatikan bahawa:

Status (0 = hidup 1, = mati)
Status merokok (0 = tidak, 1 = bekas perokok, 2 = semasa)
ECG (1 = normal, 2 = sempadan, 3 = tidak normal)
CHD (0 = tidak, 1 = ya)

[100 markah]

APPENDIX

Summary of Reliability Formulae

$$F(t) = \int_0^t f(t)dt$$

$$R(t) = 1 - F(t)$$

$$f(t) = \frac{dF(t)}{dt} = -\frac{dR(t)}{dt}$$

$$h(t) = \frac{f(t)}{R(t)}$$

$$H(t) = \int_0^t h(t)dt$$

$$R(t) = e^{-H(t)}$$

$$H(t) = -\ln R(t)$$

$$MTTF = \int_0^\infty tf(t)dt = \int_0^\infty R(t)dt$$

APPENDIKS

Ringkasan Rumus-Rumus Kebolehpercayaan

$$F(t) = \int_0^t f(t)dt$$

$$R(t) = 1 - F(t)$$

$$f(t) = \frac{dF(t)}{dt} = -\frac{dR(t)}{dt}$$

$$h(t) = \frac{f(t)}{R(t)}$$

$$H(t) = \int_0^t h(t)dt$$

$$R(t) = e^{-H(t)}$$

$$H(t) = -\ln R(t)$$

$$MTTF = \int_0^\infty tf(t)dt = \int_0^\infty R(t)dt$$

APPENDIX (contd)

Summary of Reliability Formulae (contd)

Lifetime following an **Exponential Distribution:**

$$f(t) = \lambda e^{-\lambda t}$$
$$F(t) = 1 - e^{-\lambda t}$$
$$R(t) = e^{-\lambda t}$$
$$h(t) = \lambda$$
$$H(t) = \lambda t$$
$$MTTF = \frac{1}{\lambda}$$

Lifetime following a **Weibull Distribution:**

$$f(t) = \beta \alpha^{-\beta} t^{\beta-1} \exp\left[-\left(\frac{t}{\alpha}\right)^{\beta}\right]$$

$$F(t) = 1 - e^{-\left(\frac{t}{\alpha}\right)^{\beta}}$$

$$R(t) = e^{-\left(\frac{t}{\alpha}\right)^{\beta}}$$

$$h(t) = \beta \alpha^{-\beta} t^{\beta-1}$$

$$H(t) = \left(\frac{t}{\alpha}\right)^{\beta}$$

$$MTTF = \alpha \Gamma\left(1 + \frac{1}{\beta}\right)$$

$$\text{Design Life} = t_R = \alpha(-\ln R)^{1/\beta}$$

APPENDIKS (sambung)

Ringkasan Rumus-Rumus Kebolehpercayaan (sambung)

Masahayat mengikut ***Taburan Eksponen:***

$$f(t) = \lambda e^{-\lambda t}$$
$$F(t) = 1 - e^{-\lambda t}$$
$$R(t) = e^{-\lambda t}$$
$$h(t) = \lambda$$
$$H(t) = \lambda t$$
$$MTTF = \frac{1}{\lambda}$$

Masahayat mengikut ***Taburan Weibull:***

$$f(t) = \beta \alpha^{-\beta} t^{\beta-1} \exp\left[-\left(\frac{t}{\alpha}\right)^{\beta}\right]$$

$$F(t) = 1 - e^{-\left(\frac{t}{\alpha}\right)^{\beta}}$$

$$R(t) = e^{-\left(\frac{t}{\alpha}\right)^{\beta}}$$

$$h(t) = \beta \alpha^{-\beta} t^{\beta-1}$$

$$H(t) = \left(\frac{t}{\alpha}\right)^{\beta}$$

$$MTTF = \alpha \Gamma\left(1 + \frac{1}{\beta}\right)$$

$$\text{Design Life} = t_R = \alpha(-\ln R)^{1/\beta}$$

APPENDIX (contd)

Summary of Reliability Formulae (contd)

Lifetime following a **Normal Distribution:**

$$f(t)=\frac{1}{\sqrt{2\pi}\sigma}\exp\left[-\frac{1}{2}\frac{(t-\mu)^2}{\sigma^2}\right]$$

$$F(t)=\Phi\left(\frac{t-\mu}{\sigma}\right)$$

$$R(t)=1-\Phi\left(\frac{t-\mu}{\sigma}\right)$$

$$h(t)=\frac{f(t)}{1-\Phi\left(\frac{t-\mu}{\sigma}\right)}$$

Lifetime following a **Lognormal Distribution:**

$$f(t)=\frac{1}{\sqrt{2\pi}st}\exp\left[-\frac{1}{2s^2}\left(\ln\frac{t}{t_{median}}\right)^2\right]$$

$$F(t)=\Phi\left(\frac{1}{s}\ln\frac{t}{t_{median}}\right)$$

$$R(t)=1-\Phi\left(\frac{1}{s}\ln\frac{t}{t_{median}}\right)$$

$$MTTF=t_{median}\exp\left(\frac{s^2}{2}\right)$$

$$t_R=t_{median}\exp\left(sz_{1-R}\right)$$

APPENDIKS (sambung)

Ringkasan Rumus-Rumus Kebolehpercayaan (sambung)

Masahayat mengikut ***Taburan Normal:***

$$f(t) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{1}{2} \frac{(t-\mu)^2}{\sigma^2} \right]$$

$$F(t) = \Phi\left(\frac{t-\mu}{\sigma} \right)$$

$$R(t) = 1 - \Phi\left(\frac{t-\mu}{\sigma} \right)$$

$$h(t) = \frac{f(t)}{1 - \Phi\left(\frac{t-\mu}{\sigma} \right)}$$

Masahayat mengikut ***Taburan Lognormal:***

$$f(t) = \frac{1}{\sqrt{2\pi} st} \exp\left[-\frac{1}{2s^2} \left(\ln \frac{t}{t_{median}} \right)^2 \right]$$

$$F(t) = \Phi\left(\frac{1}{s} \ln \frac{t}{t_{median}} \right)$$

$$R(t) = 1 - \Phi\left(\frac{1}{s} \ln \frac{t}{t_{median}} \right)$$

$$MTTF = t_{median} \exp\left(\frac{s^2}{2} \right)$$

$$t_R = t_{median} \exp\left(s z_{1-R} \right)$$

- ooo O ooo -